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Procedures for balancing planar linkages using rotating counterweights

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VIEWpac 9007A**

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Mechanisms Committee. This Committee first met in 1964 and now has the following membership:

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Prof. J. Rees Jones	— Unilever Research, Merseyside
Dr M.R. Smith	— University of Newcastle upon Tyne
Dr C.W. Stammers	— University of Bath.

* Corresponding Member

The technical work involved in the assessment of the available information and the construction and subsequent development of the Item was undertaken by

Mr P.J. Carden — Engineer, Mechanical Motion and System Dynamics Group.

The person with overall responsibility for the work in this subject area is Mr C.J. Loughton, Head of the Mechanical Motion and System Dynamics Group.

The Mechanisms Committee, which first met in 1975, is continuing the work that previously was the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

PROCEDURES FOR BALANCING PLANAR LINKAGES USING ROTATING COUNTERWEIGHTS

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PROCEDURES FOR BALANCING PLANAR LINKAGES USING ROTATING COUNTERWEIGHTS

1. NOTATION AND UNITS

Units are given in SI. However, any coherent system of units may be used.

		Units
A_k	Fourier coefficient of k^{th} even term of Fourier series	–
B_k	Fourier coefficient of k^{th} odd term of Fourier series	–
C_k	amplitude (modulus) of k^{th} term of Fourier series $\left\{ = (A_k^2 + B_k^2)^{1/2} \right\}$	–
d	distance between frame shaking moment reference point and axis of counterweight shaft	m
F_{pq}	“average” value of residual frame shaking force	N
F_r	residual frame shaking force (i.e. frame shaking force remaining after addition of one counterweight)	N
F_{rp}, F_{rq}	components of residual frame shaking force, parallel and perpendicular, respectively, to second counterweight datum (p -axis)	N
F_s	frame shaking force	N
F_{su}, F_{sv}	components of frame shaking force, parallel and perpendicular, respectively, to crank datum (u -axis)	N
F_{sw}	component of frame shaking force in direction of w -axis	N
F_{sx}, F_{sy}	components of frame shaking force in x - and y -axis directions, respectively	N
F_{uv}	“average” value of frame shaking force	N
MR_{CW1}	mass-moment of first counterweight	kg m
MR_{CW2}	mass-moment of second counterweight	kg m
M_s	frame shaking moment	N m
n	number of intervals of crank angular position per cycle	–
p, q	moving axes with same origin as x, y axes	–
u, v	moving axes with same origin as x, y axes	–

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w	fixed axis of symmetry of Lanchester Balancer	–
x, y	axes fixed in machine frame	–
$(\beta_{CW})_k$	angle between w -axis and line joining each counterweight shaft axis to centre of mass of counterweight for Lanchester Balancer	degree
β_{CW1}, β_{CW2}	angles between datums and lines joining counterweight shaft axes to centres of mass of first and second counterweights, respectively	degree
δ	angle between line of action of residual frame shaking force and second counterweight datum	degree
η	angle between x -axis and axis of symmetry of Lanchester Balancer	degree
θ_2	angle between x -axis and crank datum (anticlockwise positive)	degree
ϕ	angle between line of action of frame shaking force and crank datum	degree
μ	angle between line parallel to x -axis and line joining two counterweight axes	degree
ψ_k	phase angle of k^{th} term of Fourier series $\left\{ = \tan^{-1} (A_k/B_k) \right\}$	degree
Ω_r	angle between u -axis and residual frame shaking force	degree
Ω_s	angle between x -axis and frame shaking force	degree
ω_2	angular velocity of input crank	rad/s

Subscripts and dressings

i	refers to discrete position of input crank
k	refers to order of term in Fourier series
opt	optimum
$ \dots $	bars either side of a quantity indicate the modulus of that quantity

A bar above a quantity indicates the arithmetic mean of that quantity taken over n intervals of crank angular position.

2. INTRODUCTION

This Data Item presents four procedures for the design of counterweights which, when fixed to shafts rotating with constant angular velocity, improve the balance of machines that incorporate single degree-of-freedom, planar, multibar linkage mechanisms. The frame shaking force[†] and/or the frame shaking moment[†] may be reduced by using the procedures contained in this Item to determine the optimum sizes and locations of the counterweights which form the following devices.

- (a) Lanchester Balancer – in which two rotating masses are sized and aligned to eliminate a single frequency term of the frame shaking force in a chosen direction fixed with respect to the frame and in the plane of the linkage (see Section 3).
- (b) Single Counterweight Balancer – in which one rotating mass is sized and aligned to minimise the average frame shaking force (see Section 4).
- (c) Generalised Lanchester Balancer – in which two rotating masses are sized and aligned to eliminate a single frequency term of the frame shaking force in the plane of the linkage (see Section 5).
- (d) Moment Balancer – in which two rotating masses are sized and aligned to eliminate a single frequency term of the frame shaking moment (see Section 6).

Computer program, ESDUpac A9007, supplied with this Data Item can be used to carry out all the required calculations (see Sections 9 and 10). The criteria for choosing an appropriate method are included in ESDU 89043 (Reference 6). The use of rotating counterweights to improve balance has, in general, the following advantages.

- (i) Often these methods require little extra space and so are especially suited to retrofitting existing machinery.
- (ii) Linkage joint forces, other than those connecting counterweighted shafts to the fixed frame are unchanged and so bearing re-design is not usually necessary.
- (iii) Peak-to-peak input torque during steady-state operation is not altered.

It should be noted, however, that balancing by the methods presented in this Item will not necessarily provide the best possible improvement in dynamic performance. The use of rotating counterweights can give only a partial reduction in frame shaking force. The attainable degree of reduction is dependent on the particular application and is difficult to predict without detailed calculation. If extra shafts, gears and bearings are necessary then costs may be high and space may not be available. The torque required to accelerate and decelerate the machine will be increased so more powerful prime movers and more effective brakes may be required.

[†] See Section 2.1.1 for definition.

2.1 Assumptions and Prerequisite Information

When using the methods presented in this Item it is necessary to assume that:

- (a) the machine contains at least one shaft that rotates with nominally constant angular velocity (this is often the linkage input crankshaft),
 - (b) the linkage is planar – this means that all the link centres of mass are in the same plane and if there is more than one set of identical links there must be a central plane of symmetry,
 - (c) the moving links and fixed frame are rigid
- and (d) the joints have negligible clearance.

Instantaneous values of frame shaking force and, in some cases, frame shaking moment must be known at equal intervals of crank angular position throughout one operating cycle of the linkage before using the procedures in this Item. These quantities are defined below and may be calculated using force analysis methods outlined in References 3 and 4. Knowledge of these forces implies that the sizes, shapes and materials of each of the moving links are determined. It is also necessary to know the spatial constraints imposed by linkage geometry, other machine members and casings. The frequency terms referred to in this Item are the frequency terms of an infinite trigonometric series expansion of any periodically varying quantity (such as frame shaking moment). The first-order frequency term is the lowest frequency term and has the same period (*i.e* the linkage cycle time) as the original quantity. The frequency terms may be obtained by Fourier analysis.

2.1.1 Definition of frame shaking force and frame shaking moment

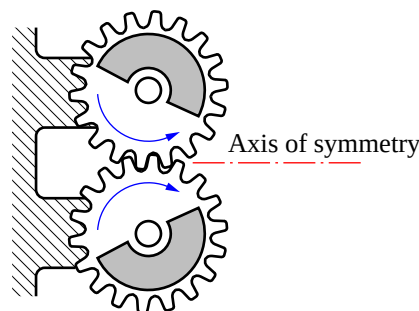
Forces acting at an interface between moving links and a fixed frame may be represented by a single resultant force acting at a specified reference point and a moment about that point. The instantaneous values of this force and moment, which vary in both magnitude and direction throughout the machine cycle, are known as the frame shaking force, F_s , and the frame shaking moment, M_s . The input link shaft centre is often chosen to be the reference point but for some applications another point may be more suitable. In most practical high speed machines the frame shaking force and frame shaking moment are mainly due to inertia forces associated with accelerations of moving links. Complete definitions and discussion of these terms with examples of their use are included in Reference 4.

3. THE DESIGN OF A LANCHESTER BALANCER

A Lanchester Balancer comprises two identical masses attached to parallel shafts that are driven synchronously in opposite directions (see Sketch 3.1). These masses are arranged to have mirror-image symmetry about an axis parallel to the direction in which the frame shaking force is to be reduced. The components of force exerted on the frame by the masses perpendicular to the axis of symmetry always cancel each other. The device must be driven at an angular velocity corresponding to the frequency of the term of the shaking force which is to be balanced. When this term is the first-order frequency term the Lanchester Balancer is driven such that it completes one cycle in the same time as the linkage. The Lanchester Balancer may be especially useful if:

- (a) the component of frame shaking force due to the motion of a translating link dominates the overall frame shaking force,
- or
- (b) the machine frame is configured so that the component of shaking force in a particular direction, and with a particular frequency, causes unwanted vibrations by exciting a natural frequency.

The Lanchester Balancer is a limiting case of the Generalised Lanchester Balancer (see Section 5). For most practical applications use of the Generalised Lanchester Balancer results in a greater overall reduction in frame shaking force as demonstrated in Example 2.



Sketch 3.1 Lanchester Balancer

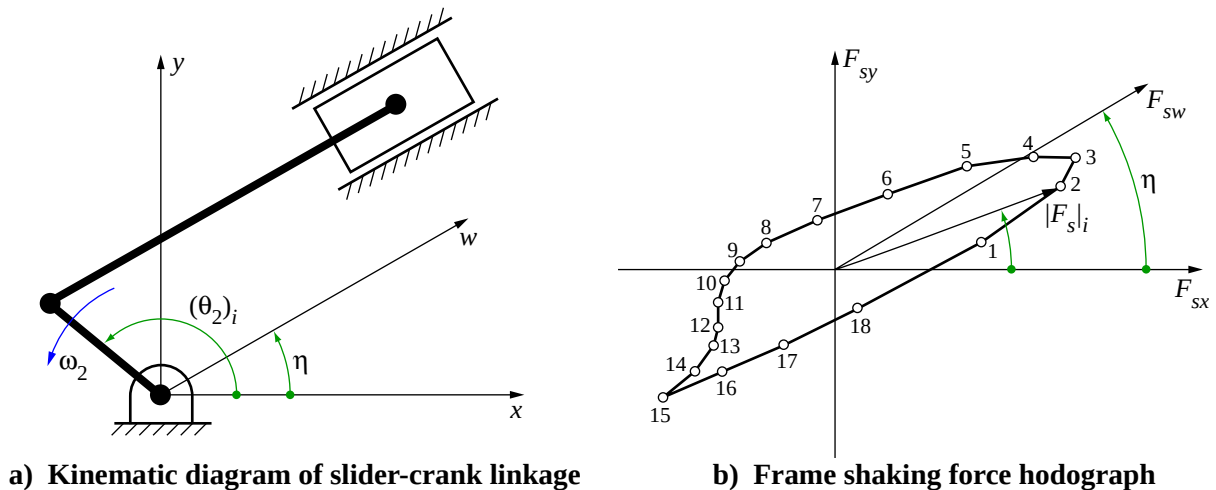
3.1 Determination of Design Parameters

The required design parameters are the mass-moment of each counterweight, MR_{CW} , the orientation of each counterweight relative to the axis of symmetry of the Balancer, β_{CW} , and the orientation of the axis of the Balancer relative to a chosen fixed frame axis, η . The parameters are found by following Steps 1 to 5 in Sections 3.1.1 to 3.1.5.

3.1.1 Obtain values of frame shaking force (STEP 1)

Consider the mechanism of Sketch 3.2(a) in which the chosen fixed machine axes, x and y , and the crank angular position, $(\theta_2)_i$, are illustrated. Discrete values of frame shaking force of magnitude, $|F_s|_i$, and direction, $(\Omega_s)_i$, at successive 20° intervals of crank angular position are known and are shown diagrammatically on the vector plot (or hodograph) of Sketch 3.2(b). It may be noted that the 20° interval that was chosen when the force data were originally obtained is small enough to define “sufficiently” the shape of the curve joining the data points. This “sufficiency” means that a halving of the interval of crank

angular position with a consequent doubling of the number of data points would produce a curve that differs from the original curve by a negligible amount. Appendix A contains the equations used to calculate the frame shaking force due to inertia forces in a four-bar crank-rocker linkage. For other linkage types see References 3 and 4.



Sketch 3.2 Mechanism and force data

3.1.2 Choose direction of axis of symmetry (STEP 2)

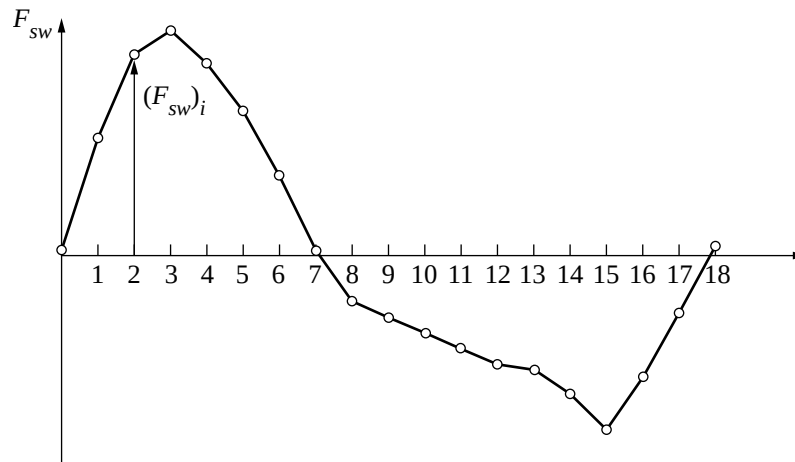
The Lanchester Balancer should be aligned so that its axis of symmetry coincides with the direction in which reduction of the frame shaking force is most advantageous. For minimum additional frame shaking moment locate the Balancer such that its axis of symmetry passes through the frame shaking moment reference point. If the Lanchester Balancer is a suitable choice of balancing method the direction of the axis of symmetry will usually be apparent by inspection of the frame shaking force hodograph. For example the hodograph (see Sketch 3.2(b)) associated with the slider-crank mechanism of Sketch 3.2(a) has an elongated shape so the direction of the axis of symmetry is chosen to pass through the extreme points of the hodograph. The positive direction of the axis of symmetry relative to the positive x-axis is called the w-axis and is defined by the angle, η . The direction in which the frame shaking force should be reduced may be obvious by considering the machine configuration. For example when balancing a mechanism situated effectively at the free end of a horizontal cantilever (e.g. a domestic sewing machine needle drive) the most advantageous axis of symmetry for the Lanchester Balancer may be perpendicular to the cantilever if a significant component of frame shaking force acts in that direction.

3.1.3 Find component of frame shaking force parallel to axis of symmetry (STEP 3)

The component of frame shaking force, $(F_{sw})_i$, parallel to the axis of symmetry must be determined for each crank angular position, $(\theta_2)_i$, $i = 1, 2, \dots, n$, so that the Balancer can be sized to provide an opposing force. This component is given by

$$(F_{sw})_i = |F_s|_i \cos \{ \eta - (\Omega_s)_i \} \quad (3.1)$$

and is illustrated in Sketches 3.2 and 3.3.



Sketch 3.3 Component of frame shaking force parallel to axis of symmetry

3.1.4 Find unwanted frequency term of frame shaking force (STEP 4)

Since a Lanchester Balancer can eliminate only one frequency term of the component of frame shaking force parallel to the axis of symmetry, it is necessary to identify the unwanted frequency term. The first-order frequency term of the frame shaking force is usually associated with undesirable vibration of the frame. However it may be worth calculating the higher order terms to ensure that these do not have large amplitudes or frequencies that closely correspond to natural frequencies of the machine frame. Calculate the Fourier coefficients of the frequency terms of the frame shaking force from

$$A_k = \frac{2}{n} \sum_{i=1}^n (F_{sw})_i \cos \{k(\theta_2)_i\} \quad (3.2)$$

and

$$B_k = \frac{2}{n} \sum_{i=1}^n (F_{sw})_i \sin \{k(\theta_2)_i\}. \quad (3.3)$$

These coefficients are used to obtain an expression for the unwanted frequency term from

$$(F_{sw})_k = A_k \cos \{k\theta_2\} + B_k \sin \{k\theta_2\}, \quad (3.4a)$$

or

$$(F_{sw})_k = C_k \sin \{k\theta_2 + \psi_k\}, \quad (3.4b)$$

where

$$C_k = (A_k^2 + B_k^2)^{1/2} \quad (3.4c)$$

and

$$\psi_k = \tan^{-1}(A_k/B_k). \quad (3.4d)$$

3.1.5 Determine mass-moment, orientation and velocity of counterweights (STEP 5)

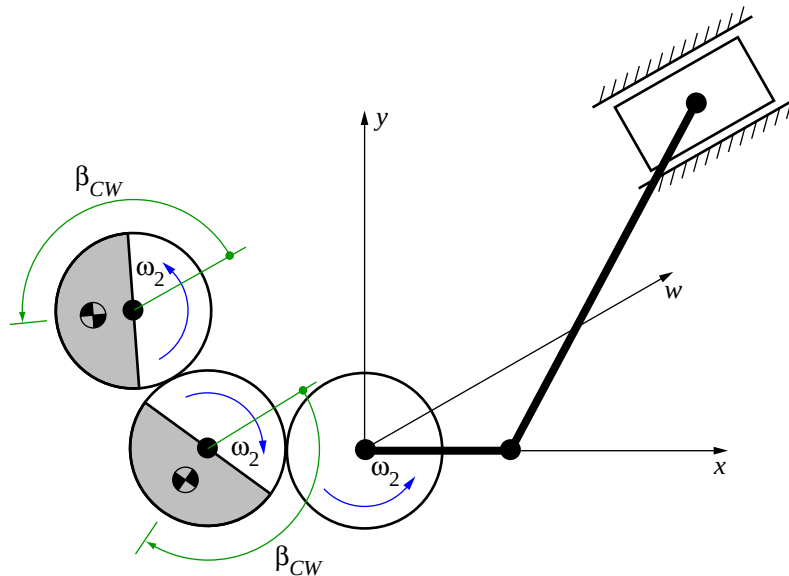
If the Lanchester Balancer is to eliminate the first-order frequency term of the component of frame shaking force parallel to the axis of symmetry then the counterweights must complete one cycle in the same time as the linkage. If the k th-order frequency term is to be removed the counterweights must rotate with k times the angular velocity of the input crank. The mass-moment of each counterweight is given by

$$(MR_{CW})_k = 0.5C_k/(k\omega_2)^2 \quad (3.5)$$

and the angles between the w -axis and the lines joining the counterweight centres of rotation to the counterweight centres of mass at the starting position of the linkage cycle, $(\theta_2)_i = 0^\circ$, as shown in Sketch 3.4 are given by

$$(\beta_{CW})_k = \psi_k + 90^\circ. \quad (3.6)$$

Counterweights of any shape may be installed provided they have the calculated values of mass-moment and orientation. Equation (B.1) in Appendix B may be used to check the feasibility of fitting sectorial counterweights.



Sketch 3.4 Counterweight orientation at start of linkage cycle

4. THE DESIGN OF A SINGLE COUNTERWEIGHT BALANCER

A single counterweight mounted on a shaft perpendicular to the plane of the linkage and rotating at nominally constant angular velocity may be designed so that the average value of frame shaking force is reduced. This is achieved by finding the average value of the frame shaking force and designing the counterweight so that it exerts an opposing force on the machine frame. Usually it is most convenient to locate the counterweight on the linkage input crankshaft. This ensures that the counterweight rotates synchronously with the linkage and the cost of extra shafts, bearings and gears is avoided. Conventionally the frame shaking moment is referred to the axis of the input crankshaft and so remains unaltered by the addition of a counterweight to that shaft. Alternatively, if the frame shaking moment is referred to some other point the counterweight may be sized to minimise a weighted sum of frame shaking force and frame shaking moment. This method is described in Reference 10.

A further method (see Steps 1 to 4 in Sections 4.1.1 to 4.1.4) in which a reduction of shaking moment as well as shaking force may be achieved is to mount the counterweight on a separate shaft parallel to the linkage input crankshaft, rotating in bearings which form part of the linkage frame. This arrangement is recommended only if the counterweight cannot be mounted on the linkage crankshaft. If the cost of an extra shaft can be justified it is usual to add two counterweights, one on the crankshaft and one on the balancing shaft (see Generalised Lanchester Balancer in Section 5). This section includes the method for locating the balancing shaft such that the first-order frequency term of the frame shaking moment, about a given reference point, is eliminated (see Steps 5 to 7 in Sections 4.2.1 to 4.2.3).

4.1 Determination of the Design Parameters for Minimum Frame Shaking Force

The design parameters to be determined are the counterweight mass-moment, MR_{CW1} , and the orientation, β_{CW1} , of the counterweight relative to the linkage crank. These parameters are found by following Steps 1 to 4 in Sections 4.1.1 to 4.1.4.

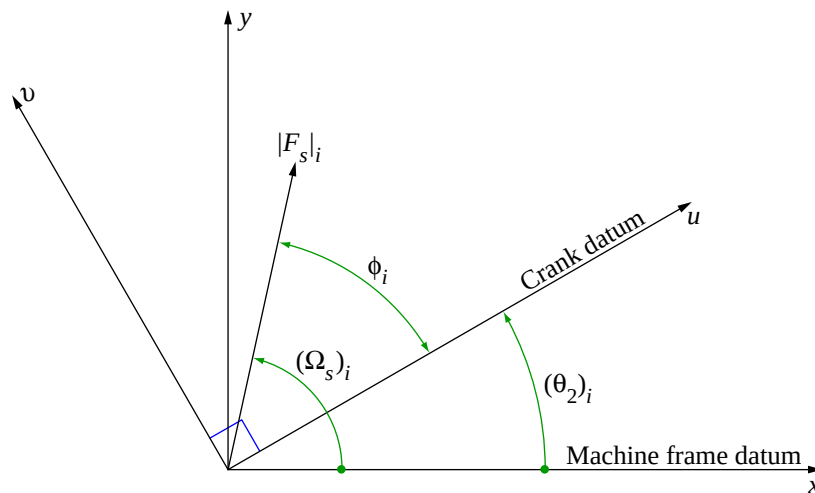
4.1.1 Obtain values of frame shaking force (STEP 1)

Follow Step 1 of the Lanchester Balancer design (Section 3.1.1) to obtain appropriate values of magnitude, $|F_s|_i$, and direction, $(\Omega_s)_i$, of the frame shaking force at each crank angular position, $(\theta_2)_i$, $i = 1, 2, \dots, n$.

4.1.2 Find relative angle of frame shaking force (STEP 2)

To find the angle of the frame shaking force relative to the angular position of the crank a pair of moving (u, v) axes having the same origin as the fixed (x, y) axes are introduced. The u -axis corresponds to the crank datum (see Sketch 4.1) and the moving axes rotate with the crank for one linkage motion cycle. For each crank position, $(\theta_2)_i$, $i = 1, 2, \dots, n$, find the angle, ϕ_i , between the crank datum (u -axis) and the corresponding value of frame shaking force from

$$\phi_i = (\Omega_s)_i - (\theta_2)_i. \quad (4.1)$$



Sketch 4.1 Frame shaking force relative to crank position

4.1.3 Find average values of frame shaking force components (STEP 3)

For each crank position find the components of the frame shaking force parallel to both the u -axis and the v -axis. These components are given by

$$(F_{su})_i = |F_s|_i \cos \phi_i \quad (4.2)$$

and
$$(F_{sv})_i = |F_s|_i \sin \phi_i. \quad (4.3)$$

The arithmetic mean values of these components are given by

$$\bar{F}_{su} = \frac{1}{n} \sum_{i=1}^n (F_{su})_i. \quad (4.4)$$

and
$$\bar{F}_{sv} = \frac{1}{n} \sum_{i=1}^n (F_{sv})_i. \quad (4.5)$$

These values are used to determine the “average” value of the frame shaking force using

$$F_{uv} = [(\bar{F}_{su})^2 + (\bar{F}_{sv})^2]^{1/2}. \quad (4.6)$$

4.1.4 Find counterweight mass moment and orientation (STEP 4)

The counterweight should exert a force on the frame which is equal in magnitude and opposite in direction to the “average” frame shaking force. Since the counterweight rotates with the same angular velocity as

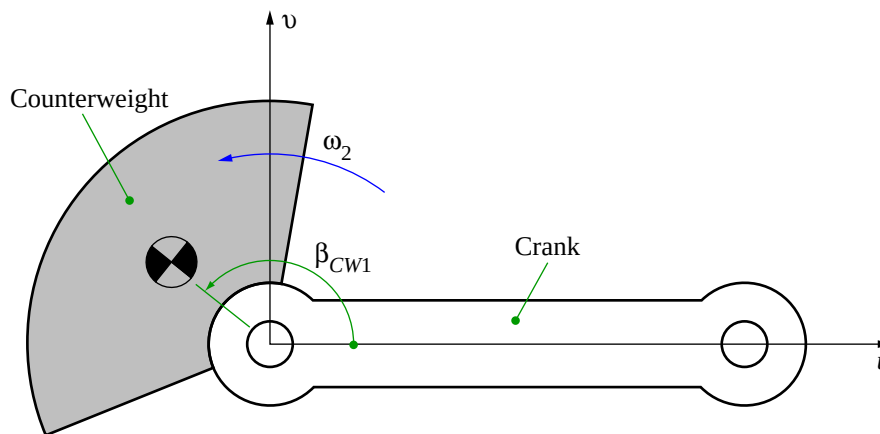
the crank, ω_2 , the required counterweight mass-moment is given by

$$MR_{CW1} = F_{uv}/\omega_2^2. \quad (4.7)$$

The angle (measured as positive anticlockwise for a crank rotating in an anticlockwise direction) between the crank datum and the line joining the crank axis of rotation to the counterweight centre of mass is given by

$$\beta_{CW1} = \tan^{-1}(\bar{F}_{sv}/\bar{F}_{su}) + 180^\circ. \quad (4.8)$$

This angle is illustrated in Sketch 4.2. Ideally the counterweight should be incorporated in the design of the crankshaft. Equation (B.1) in Appendix B may be used to check the feasibility of fitting a sectorial counterweight. If the counterweight is to be mounted on a separate shaft then use Steps 5 to 7 (Section 4.2.1 to 4.2.3) to determine the location of this shaft for minimum frame shaking moment.



Sketch 4.2 Orientation of counterweight

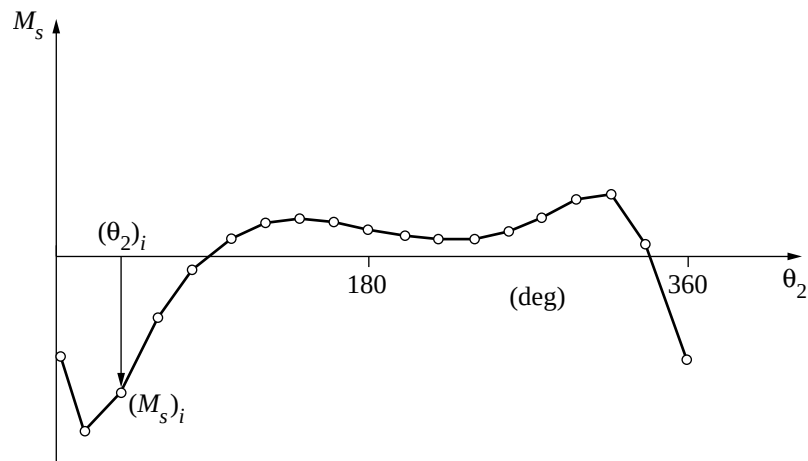
4.2 Determination of Design Parameters for Minimum Frame Shaking Moment

It is theoretically possible to locate the counterweight shaft axis so that the first-order frequency term of the frame shaking moment is eliminated. The required design parameters are the distance, d_{opt} , from the reference point to the counterweight shaft axis and the angle, μ_{opt} , (positive anticlockwise) between the x-axis and the line joining the reference point to the counterweight shaft axis. The parameters are found by following Steps 5 to 7 in Sections 4.2.1 to 4.2.3. This procedure is recommended only if the counterweight cannot be mounted on the crankshaft. If it is not possible to locate the counterweight shaft axis at the position defined by this method then a location near to the ideal may still result in a significant reduction in frame shaking moment.

The shaking moment exerted on the fixed frame by the action of the moving links is dependent on the point to which it is referred. If the fixed frame is configured so that an unbalanced moment is likely to cause unwanted vibration about some point then that point should be used as the frame shaking moment reference point for calculation purposes. Otherwise the frame shaking moment, by convention, is calculated assuming that the input shaft axis is the reference point.

4.2.1 Obtain values of frame shaking moment (STEP 5)

Discrete values of frame shaking moment, $(M_s)_i$, at successive 20° intervals of crank angular position are known and are shown diagrammatically in Sketch 4.3. The calculation methods are described in References 3 and 5.



Sketch 4.3 Frame shaking moment data

4.2.2 Find first-order frequency term of frame shaking moment (STEP 6)

To locate the counterweight it is necessary to find the first-order frequency term of the frame shaking moment. The Fourier coefficients of this term are given by

$$A_1 = \frac{2}{n} \sum_{i=1}^n (M_s)_i \cos(\theta_2)_i \quad (4.9)$$

and

$$B_1 = \frac{2}{n} \sum_{i=1}^n (M_s)_i \sin(\theta_2)_i. \quad (4.10)$$

These coefficients are used to obtain an expression for the first-order frequency term from

$$(M_s)_1 = A_1 \cos \theta_2 + B_1 \sin \theta_2 \quad (4.11a)$$

or

$$(M_s)_1 = C_1 \sin(\theta_2 + \psi_1) \quad (4.11b)$$

where

$$C_1 = (A_1^2 + B_1^2)^{1/2} \quad (4.11c)$$

and

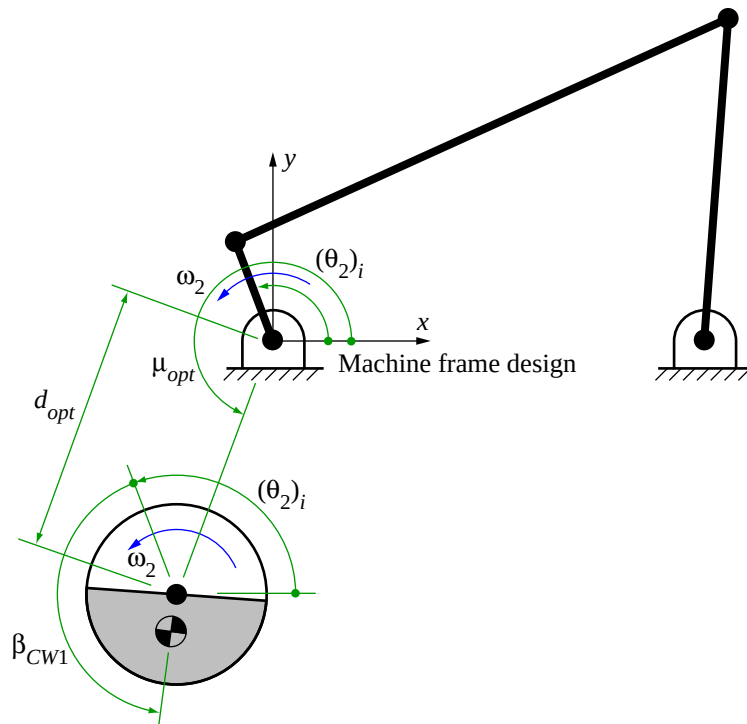
$$\psi_1 = \tan^{-1}(A_1/B_1). \quad (4.11d)$$

4.2.3 Find location of counterweight (STEP 7)

The counterweight should ideally provide a moment which is equal and opposite to the first-order frequency term of the frame shaking moment. For a counterweight of fixed size, MR_{CW1} , and orientation, β_{CW1} , the location of the counterweight shaft axis for minimum frame shaking moment is characterised by the distance, d_{opt} , and the angle, μ_{opt} . These parameters are illustrated in Sketch 4.4 and are given by

$$d_{opt} = C_1 / F_{uv} \quad (4.12)$$

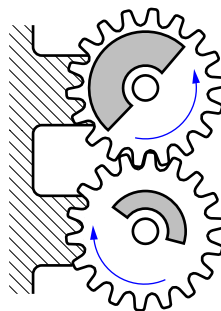
and
$$\mu_{opt} = \beta_{CW1} - \psi_1. \quad (4.13)$$



Sketch 4.4 Counterweight location

5. THE DESIGN OF A GENERALISED LANCHESTER BALANCER

A Generalised Lanchester Balancer comprises two unequal masses attached to parallel shafts that are driven synchronously in opposite directions as shown in Sketch 5.1. The design of this device is an extension of the Single Counterweight Balancer design presented in Section 4. That procedure (Steps 1 to 4) may be used to determine the size and orientation of the counterweight which rotates in the same angular sense as the crank. The procedure in this section determines the size and orientation of the contra-rotating counterweight. The combined effect of the two counterweights is the elimination of the first-order frequency term of the frame shaking force. If the locations of the contra-rotating counterweight shafts are both arbitrarily chosen then, generally, an alternating frame shaking moment of crankshaft frequency remains unbalanced. To avoid this unbalance the location of only one shaft should be chosen arbitrarily. There is then a unique location for the other shaft that results in the elimination of the first-order term of the frame shaking moment. This second location is determined in Section 5.2. Usually it will be convenient to make the crankshaft one of the two contra-rotating shafts so that only one additional shaft is required.



Sketch 5.1 Generalised Lanchester Balancer

5.1 Determination of Design Parameters for Minimum Frame Shaking Force

This section comprises a procedure to determine the required mass-moments, MR_{CW1} and MR_{CW2} , and orientations, β_{CW1} and β_{CW2} , of two counterweights mounted on rotating shafts. These parameters are found by following Steps 1 to 5 in Sections 5.1.1 to 5.1.5.

5.1.1 Determine mass-moment and orientation of first counterweight (STEP 1)

Follow Steps 1 to 4 of the single counterweight design (Sections 4.1.1 to 4.1.4) to find the size, MR_{CW1} , and orientation, β_{CW1} , of the first counterweight which rotates in the same angular direction as the linkage.

5.1.2 Find residual frame shaking force (STEP 2)

To size the second counterweight it is necessary to find the components of the frame shaking force which remain after the first counterweight has been attached to the linkage. These components, $(F_{ru})_i$ and $(F_{rv})_i$, relative to the u, v axis system, are calculated for each crank position, $(\theta_2)_i$, $i = 1, 2, \dots, n$, from

$$(F_{ru})_i = (F_{su})_i - \bar{F}_{su} \quad (5.1)$$

$$\text{and} \quad (F_{rv})_i = (F_{sv})_i - \bar{F}_{sv} \quad (5.2)$$

The residual frame shaking force is most usefully expressed in polar form as

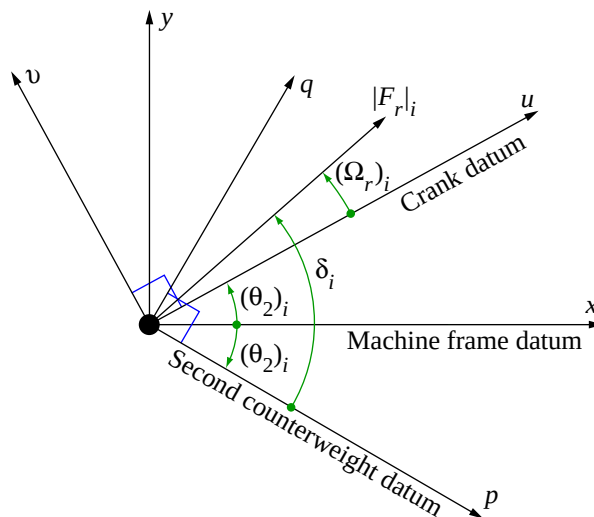
$$|F_r|_i = [(F_{ru})_i^2 + (F_{rv})_i^2]^{1/2} \quad (5.3)$$

and $(\Omega_r)_i = \tan^{-1}[(F_{rv})_i / (F_{ru})_i]. \quad (5.4)$

5.1.3 Find relative angle of residual frame shaking force (STEP 3)

It is necessary to introduce a second set of moving axes, the p , q axes, which share the same origin as the x , y axes and rotate in the opposite direction to the u , v axes at the same angular velocity (see Sketch 5.2). The p -axis forms the datum for the second counterweight and the angle from the p -axis to the residual frame shaking force is given by

$$\delta_i = (\Omega_r)_i = 2(\theta_2)_i. \quad (5.5)$$



Sketch 5.2 Residual frame shaking force

5.1.4 Find average values of residual frame shaking force components (STEP 4)

For each crank position find the components of the residual frame shaking force parallel to the p -axis and the q -axis. These components are given by

$$(F_{rp})_i = |F_r|_i \cos \delta_i \quad (5.6)$$

and $(F_{rq})_i = |F_r|_i \sin \delta_i. \quad (5.7)$

The arithmetic mean values of these components are given by

$$\bar{F}_{rp} = \frac{1}{n} \sum_{i=1}^n (F_{rp})_i \quad (5.8)$$

and

$$\bar{F}_{rq} = \frac{1}{n} \sum_{i=1}^n (F_{rq})_i. \quad (5.9)$$

These values are used to determine the “average” magnitude of the residual frame shaking force using

$$F_{pq} = [(\bar{F}_{rp})^2 + (\bar{F}_{rq})^2]^{1/2}. \quad (5.10)$$

5.1.5 Find mass-moment and orientation of second counterweight (STEP 5)

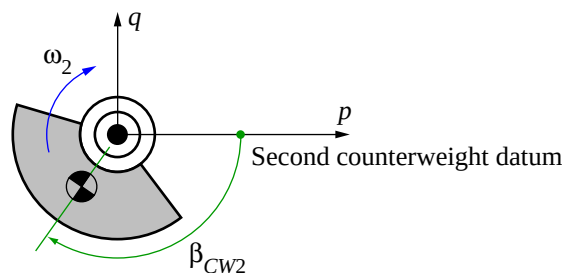
The second counterweight should exert on the frame a force which is equal in magnitude and opposite in direction to the “average” residual frame shaking force. Since the counterweight rotates with the same angular velocity as the crank the required mass-moment of the counterweight is given by

$$MR_{CW2} = F_{pq}/\omega_2^2. \quad (5.11)$$

The angle (measured as positive clockwise for a counterweight rotating in a clockwise direction) between the counterweight datum (see Sketch 5.3) and the line joining the counterweight axis of rotation to the counterweight centre of mass is given by

$$\beta_{CW2} = 180^\circ - \tan^{-1}(\bar{F}_{rq}/\bar{F}_{rp}). \quad (5.12)$$

Use Equation (B.1) in Appendix B to check the feasibility of incorporating a sectorial counterweight.



Sketch 5.3 Orientation of second counterweight

5.2 Determination of Design Parameters for Minimum Frame Shaking Moment

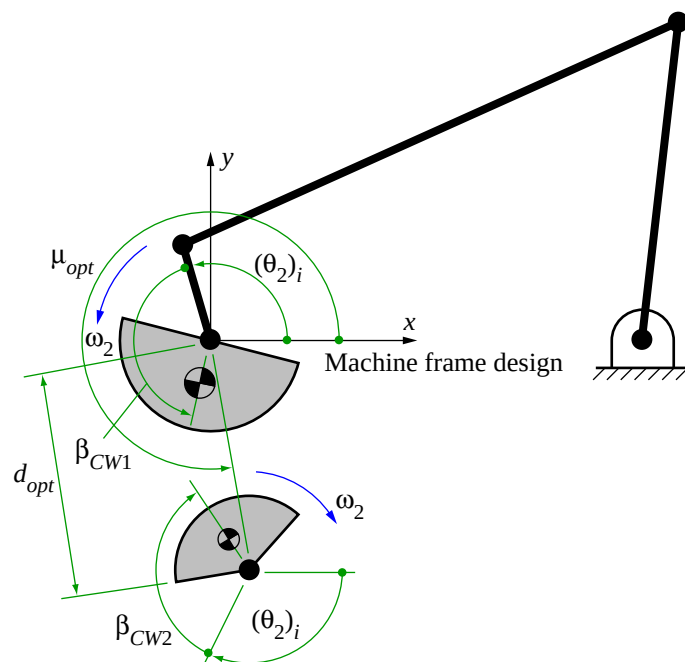
When the frame shaking moment is referred to the axis of rotation of the crankshaft and the first counterweight is attached to the crankshaft the first counterweight has no effect on the frame shaking moment. The second counterweight is attached to a shaft which is separate from but parallel to the input crankshaft so it affects the frame shaking moment. It is theoretically possible to locate the second counterweight so that the first-order frequency term of the frame shaking moment is eliminated. The required design parameters are the distance, d_{opt} , from the reference point to the counterweight shaft axis and the angle, μ_{opt} (positive anticlockwise), between the x-axis and the line joining the reference point to the second counterweight shaft axis. These parameters are found by following Step 6 (Section 5.2.1). If, for practical reasons, the second shaft cannot be positioned at this location the remaining frame shaking moment will be minimised by installing the shaft as close as possible to its ideal location.

5.2.1 Find location of second counterweight (STEP 6)

Follow Steps 5 and 6 of the Single Counterweight Balancer design (Sections 4.2.1 and 4.2.2) to find the Fourier coefficients of the first-order frequency term of the frame shaking moment. The location of the second counterweight is illustrated in Sketch 5.4 and given by

$$d_{opt} = C_1 / F_{pq} \quad (5.13)$$

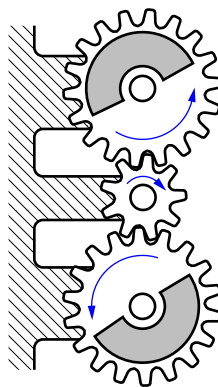
and
$$\mu_{opt} = \beta_{CW2} + \psi_1 + 180^\circ. \quad (5.14)$$



Sketch 5.4 Counterweight location

6. THE DESIGN OF A MOMENT BALANCER

The Moment Balancer comprises two identical masses attached to parallel shafts that are driven synchronously in the same direction (see Sketch 6.1). The device is used to eliminate the first-order frequency term of the frame shaking moment without affecting the frame shaking force. The counterweights may be located on any convenient shafts that are parallel to each other and perpendicular to the plane of the linkage. This device has limited practical application because the Generalised Lanchester Balancer, which can provide the same reduction in frame shaking moment as well as eliminating the first-order frequency term of the frame shaking force, is usually a preferred solution.



Sketch 6.1 Moment Balancer

6.1 Determination of Design Parameters

This section details a procedure to determine the required mass-moments, MR_{CW1} and MR_{CW2} , and orientations, β_{CW1} and β_{CW2} , of the two counterweights.

6.1.1 Find Fourier coefficients of frame shaking moment (STEP 1)

Follow Steps 5 and 6 of the Single Counterweight Balancer design (Sections 4.2.1 and 4.2.2) to find the Fourier coefficients of the first-order frequency term of the frame shaking moment about the chosen reference point. See Section 4.2 for comments on the choice of reference point.

6.1.2 Choose counterweight locations (STEP 2)

Since the Moment Balancer exerts a pure moment on the machine frame the counterweight shaft locations can be chosen for convenience. The distance, d , between the counterweight shaft axes and the angle, μ (measured as positive in the anticlockwise direction), between the x -axis and the line joining the counterweights must be specified at the outset. If the counterweights can be mounted on shafts which are far apart, preferably shafts which exist to perform some other function, then the counterweights will be smaller than those required for close shafts.

6.1.3 Determine the mass-moment and orientation of counterweights (STEP 3)

The force exerted on the frame by each counterweight is given by the magnitude of the required first-order frequency term of the frame shaking moment divided by the distance between the counterweights. Hence the mass-moment of each counterweight is given by

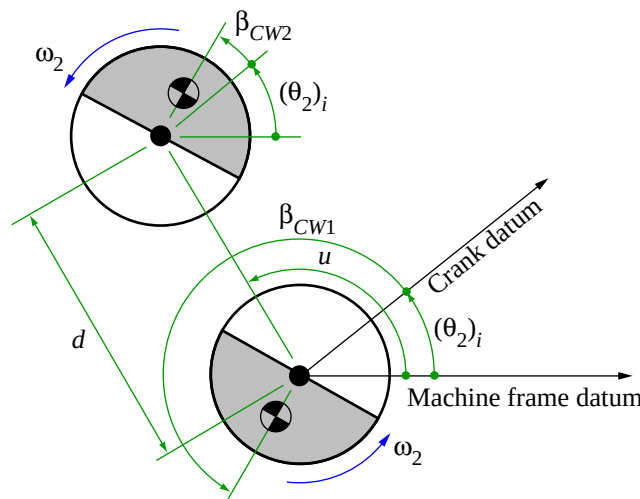
$$MR_{CW1} = MR_{CW2} = C_1 / (d\omega_2^2). \quad (6.1)$$

The angle from the crank datum (see Sketch 6.2) to the line joining the counterweight shaft axis to the centre of mass of the first counterweight is given by

$$\beta_{CW1} = \mu + \psi_1 \quad (6.2)$$

and since the second counterweight exerts an equal and opposite force on the frame its corresponding angle is given by

$$\beta_{CW2} = \beta_{CW1} - 180^\circ. \quad (6.3)$$



Sketch 6.2 Counterweight mass-moments and orientations