



# ESDU 90010

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## Pitching moment and lift force derivatives due to rate of pitch for aircraft at subsonic speeds

Associated software: ESDUpac A9010



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## PITCHING MOMENT AND LIFT FORCE DERIVATIVES DUE TO RATE OF PITCH FOR AIRCRAFT AT SUBSONIC SPEEDS

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## PITCHING MOMENT AND LIFT FORCE DERIVATIVES DUE TO RATE OF PITCH FOR AIRCRAFT AT SUBSONIC SPEEDS

### 1. NOTATION AND UNITS (see Sketch 1.1)

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in ESDU 86021. Coefficients and aerodynamic derivatives are evaluated in aerodynamic body axes with origin on the body centre-line and with the aerodynamic mean chord of the wing as the characteristic length. The derivatives  $M_q$ ,  $M_{\dot{w}}$  and  $Z_q$  are often written as  $C_{mq}$ ,  $C_{m\dot{\alpha}}$  and  $C_{zq}$  in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming  $C_{mq}$ ,  $C_{m\dot{\alpha}}$  and  $C_{zq}$  differentiation of  $C_m$  and  $C_z$  may be carried out with respect to  $q\bar{c}/2V$  and  $\dot{\alpha}\bar{c}/2V$  instead of  $q\bar{c}/V$  and  $\dot{\alpha}\bar{c}/V$  as implied in the Hopkin system. It is also to be noted that a constant datum value of  $V$  is employed by Hopkin.

|           |                                                                                                                                                    | SI                | British           |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------|-------------------|-------------------|
| $A$       | wing aspect ratio, $b^2/S_W$                                                                                                                       |                   |                   |
| $A_T$     | tailplane aspect ratio, $b_T^2/S_T$                                                                                                                |                   |                   |
| $(a_1)_0$ | spanwise average of two-dimensional lift-curve slope of wing section normal to wing quarter-chord line for incompressible flow, see Equation (3.8) | rad <sup>-1</sup> | rad <sup>-1</sup> |
| $a_{1B}$  | lift-curve slope of body based on planform area $S_B$                                                                                              | rad <sup>-1</sup> | rad <sup>-1</sup> |
| $a_{1T}$  | lift-curve slope of tailplane based on planform area $S_T$                                                                                         | rad <sup>-1</sup> | rad <sup>-1</sup> |
| $a_{1W}$  | lift-curve slope of wing based on planform area $S_W$                                                                                              | rad <sup>-1</sup> | rad <sup>-1</sup> |
| $b$       | wing span                                                                                                                                          | m                 | ft                |
| $b_T$     | tailplane span                                                                                                                                     | m                 | ft                |
| $c$       | chord of wing section normal to wing quarter-chord line, for use in ESDU Aero W.01.01.05                                                           | m                 | ft                |
| $\bar{c}$ | aerodynamic mean chord of wing                                                                                                                     | m                 | ft                |
| $F$       | empirical correction factor in Equation (3.8), see Figure 3                                                                                        |                   |                   |
| $h_B$     | local body height                                                                                                                                  | m                 | ft                |
| $h_{Bm}$  | value of $h_B$ in cross-sectional plane that contains $w_{Bm}$                                                                                     | m                 | ft                |
| $i$       | integer involved in summation to replace integral for $(M_q)_B$ , see examples in Section 8                                                        |                   |                   |

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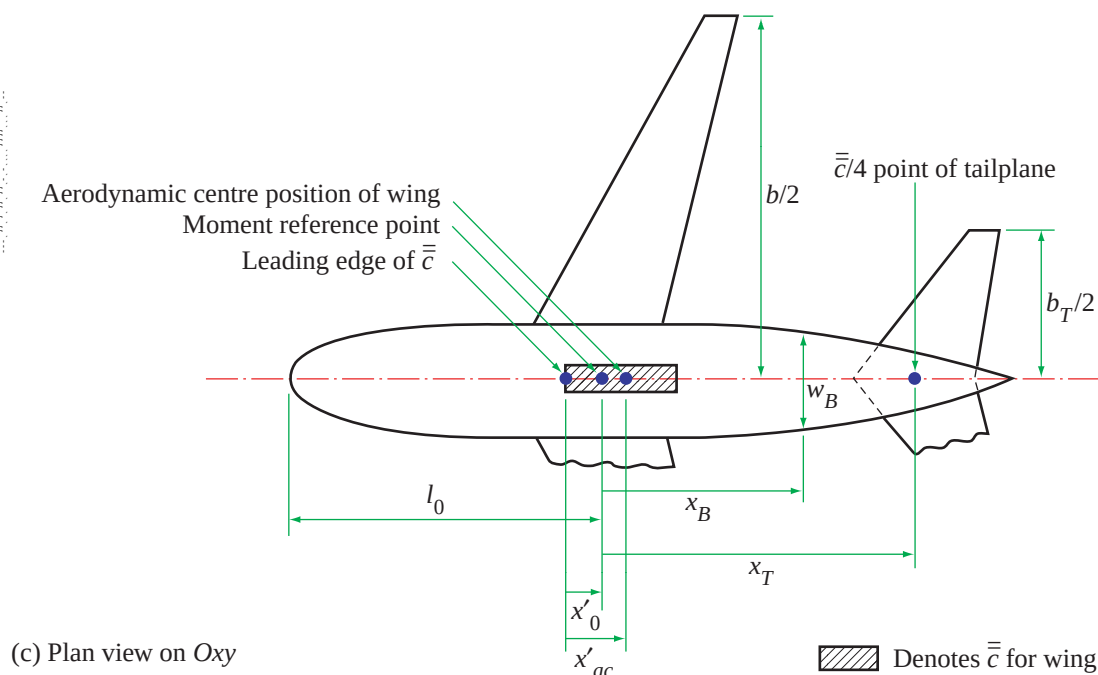
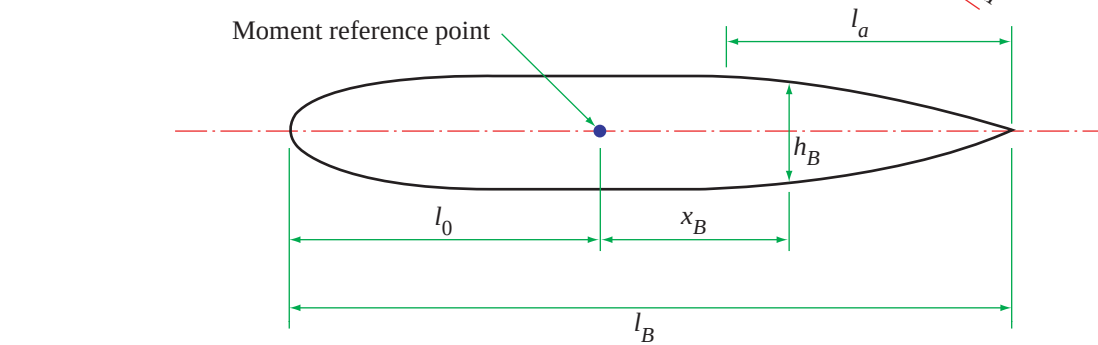
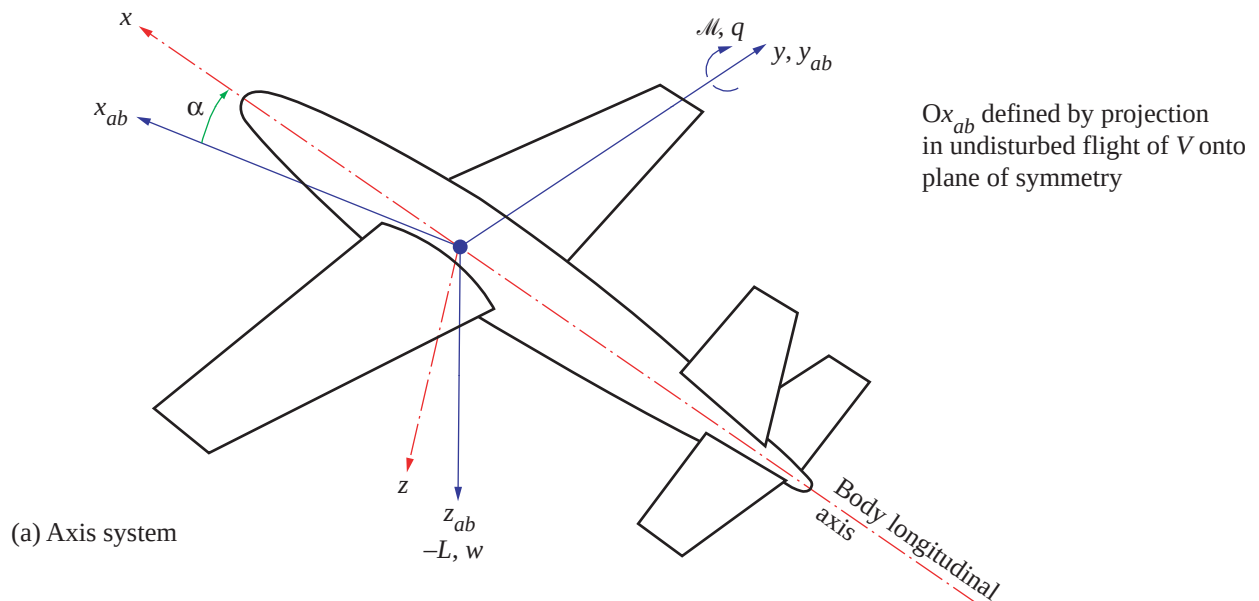
|                             |                                                                                                                                         |                |                 |
|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|----------------|-----------------|
| $k_a$                       | parameter in calculation of afterbody correction factor $(1 - k_a)$ , see Figure 2                                                      |                |                 |
| $k_f$                       | correction factor for body fineness ratio, see Figure 1                                                                                 |                |                 |
| $k_s$                       | correction factor for body cross-sectional shape, see Equations (3.3) and (3.4)                                                         |                |                 |
| $L$                         | lift                                                                                                                                    | N              | lbf             |
| $l_a$                       | length of tapered afterbody                                                                                                             | m              | ft              |
| $l_B$                       | body length                                                                                                                             | m              | ft              |
| $l_0$                       | distance of moment reference point aft of body nose                                                                                     | m              | ft              |
| $\mathcal{M}$               | pitching moment about axis $Oy_{ab}$ , positive nose up                                                                                 | N m            | lbf ft          |
| $M$                         | Mach number                                                                                                                             |                |                 |
| $M_q$                       | pitching moment derivative due to rate of pitch, $(\partial \mathcal{M} / \partial q) / (\frac{1}{2} \rho V S_W \bar{c}^2)$             |                |                 |
| $M_{q^{1/4}}$               | value of $M_q$ for moment reference point and pitching axis at quarter-chord point of wing aerodynamic mean chord                       |                |                 |
| $M_{\dot{w}}$               | pitching moment derivative due to acceleration in heave, $(\partial \mathcal{M} / \partial \dot{w}) / (\frac{1}{2} \rho S_W \bar{c}^2)$ |                |                 |
| $Ox, Oy, Oz$                | body axis system, origin $O$ at moment reference point, see Sketch 1.1a                                                                 |                |                 |
| $Ox_{ab}, Oy_{ab}, Oz_{ab}$ | aerodynamic-body axis system, origin $O$ at moment reference point, see Sketch 1.1a                                                     |                |                 |
| $q$                         | rate of pitch about axis $Oy_{ab}$ , positive nose up                                                                                   | rad/s          | rad/s           |
| $R$                         | Reynolds number based on flow quantities normal to the wing quarter-chord line and $c$ , for use in ESDU Aero W.01.01.05                |                |                 |
| $S_B$                       | planform area of body on plane $Oxy$                                                                                                    | m <sup>2</sup> | ft <sup>2</sup> |
| $S_m$                       | cross-sectional area of body in plane that contains maximum body width                                                                  | m <sup>2</sup> | ft <sup>2</sup> |
| $S_T$                       | planform area of gross tailplane, see Section 3.3                                                                                       | m <sup>2</sup> | ft <sup>2</sup> |
| $S_W$                       | planform area of gross wing, see Section 4                                                                                              | m <sup>2</sup> | ft <sup>2</sup> |
| $S_1$                       | cross-sectional area of body base                                                                                                       | m <sup>2</sup> | ft <sup>2</sup> |
| $t$                         | maximum thickness of wing section normal to wing quarter-chord line, for use in ESDU Aero W.01.01.05                                    | m              | ft              |

|                  |                                                                                                                                                                |     |      |
|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|------|
| $V$              | velocity of aircraft relative to air in undisturbed flight, datum velocity                                                                                     | m/s | ft/s |
| $w$              | aircraft velocity in $Oz_{ab}$ direction, velocity in heave                                                                                                    | m/s | ft/s |
| $w_B$            | local body width                                                                                                                                               | m   | ft   |
| $w_{Bm}$         | maximum value of $w_B$                                                                                                                                         | m   | ft   |
| $w_{B1}$         | local body width at body base                                                                                                                                  | m   | ft   |
| $x'_{ac}$        | longitudinal distance along body axis of wing aerodynamic centre, measured positive aft of leading-edge of aerodynamic mean chord                              | m   | ft   |
| $\Delta x'_{ac}$ | rearward shift in aerodynamic centre due to effect of body on wing loading, see Section 4                                                                      | m   | ft   |
| $x_B$            | longitudinal distance along body axis to general body station, measured positive aft of moment reference point                                                 | m   | ft   |
| $x'_0$           | longitudinal distance along body axis of moment reference point, measured positive aft of leading-edge of aerodynamic mean chord                               | m   | ft   |
| $x_t$            | chordwise location of boundary layer transition, measured from leading edge of wing section normal to wing quarter-chord line, for use in ESDU Aero W.01.01.05 | m   | ft   |
| $x_T$            | longitudinal distance along body axis of quarter-chord point of aerodynamic mean chord of tailplane, measured positive aft of moment reference point           | m   | ft   |
| $Z_q$            | lift force derivative due to rate of pitch, $-(\partial L/\partial q)/(\frac{1}{2}\rho V S_w \bar{c})$                                                         |     |      |
| $Z_{q^{1/4}}$    | value of $Z_q$ for pitching axis at quarter-chord point of wing aerodynamic mean chord                                                                         |     |      |
| $\alpha$         | angle of attack                                                                                                                                                | rad | rad  |
| $\beta$          | Mach number similarity parameter, $(1 - M^2)^{1/2}$                                                                                                            |     |      |
| $\beta'$         | Mach number similarity parameter, $(1 - M^2 \cos^2 \Lambda_{1/4})^{1/2}$                                                                                       |     |      |
| $\Lambda_0$      | sweepback of wing leading edge                                                                                                                                 | deg | deg  |
| $\Lambda'_0$     | $\tan^{-1}(\beta^{-1} \tan \Lambda_0)$                                                                                                                         | deg | deg  |
| $\Lambda_{1/4}$  | sweepback of wing quarter-chord line                                                                                                                           | deg | deg  |
| $\Lambda_{1/2}$  | sweepback of wing half-chord line                                                                                                                              | deg | deg  |

|                                           |                                                                                                                                                                                                                                               |                   |                      |
|-------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------|----------------------|
| $\Lambda_{\frac{1}{2}T}$                  | sweepback of tailplane half-chord line                                                                                                                                                                                                        | deg               | deg                  |
| $\lambda$                                 | taper ratio of wing, ratio of tip chord to centre-line chord                                                                                                                                                                                  |                   |                      |
| $\lambda_T$                               | taper ratio of tailplane, ratio of tip chord to centre-line chord                                                                                                                                                                             |                   |                      |
| $\rho$                                    | density of air                                                                                                                                                                                                                                | kg/m <sup>3</sup> | slug/ft <sup>3</sup> |
| $\tau$                                    | trailing-edge angle of wing section normal to wing quarter-chord line, for use in ESDU Aero W.01.01.05                                                                                                                                        | deg               | deg                  |
| $\tau_a$                                  | trailing-edge angle of wing section normal to wing quarter-chord line, defined as the angle between straight lines passing through points at 90 and 99 per cent of the chord on the upper and lower surfaces, for use in ESDU Aero W.01.01.05 | deg               | deg                  |
| $(\partial \epsilon / \partial \alpha)_T$ | mean effective gradient of downwash at tailplane                                                                                                                                                                                              |                   |                      |

Subscripts

|        |                                  |
|--------|----------------------------------|
| $B$    | denotes body                     |
| $T$    | denotes tailplane                |
| $W$    | denotes wing                     |
| $W(B)$ | denotes wing in presence of body |
| $WB$   | denotes wing-body                |



Sketch 1.1

## 2. INTRODUCTION

This Item gives a semi-empirical method for estimating  $M_q$  and  $Z_q$ , the pitch-rate derivatives of moment and force, for a rigid aircraft at subsonic speeds and low angles of attack. The derivatives are expressed in aero-normalised form and for an aerodynamic-body axis system. Those concepts are explained in ESDU 86021 (Reference 104), which also gives the relationships for conversion of the derivatives into other forms. Equations for transformations to other axis systems are contained in ESDU 86041 (Reference 105). The separate contributions of the body, wing and tailplane are estimated in Section 3, and interference effects are discussed in Section 4. In the analysis of aircraft motion it is sometimes only possible to consider  $M_q$  in combination with  $M_w$ , the pitching moment derivative due to acceleration in heave. This is always the case with data from oscillatory-rig tests. Therefore, Section 5 provides an expression for the major contribution to  $M_w$  from the tailplane. The accuracy of the various parts of the method is reviewed in Section 6.1 and its applicability is discussed in Section 6.2. The Derivation and References are given in Section 7. The first subsection of the Derivation lists the ESDU Items that will be required for use with the present Item. Worked examples are provided in Section 8. Details of a computer program for the present method, ESDUpac A9010, are given in Appendix A.

## 3. METHODS FOR INDIVIDUAL COMPONENTS

### 3.1 Body Contribution

For an isolated body, Derivations 8 and 9 quote a semi-empirical formula, first given in Reference 100, for the yawing moment derivative due to rate of yaw. This may be readily adapted for a general prediction of  $(M_q)_B$ . A description of the original formulation of the equation is unavailable, but Derivation 8 shows that it can be deduced from a simple strip-theory concept, the strips being parallel to the axis of rotation. If the equation is rewritten to give the body pitch-rate derivative based on the reference area  $S_W$  and length  $\bar{c}$ , then

$$(M_q)_B = - \frac{a_{1B}}{S_W \bar{c}^2} \int_{-l_0}^{l_B - l_0} w_B x_B^2 dx_B \quad (3.1)$$

where  $l_B$  is the total length of the body, and  $x_B$  denotes the longitudinal distance aft of the moment reference point, which is assumed to be a distance  $l_0$  aft of the body nose. The body lift-curve slope  $a_{1B}$  is based on the body planform area  $S_B$ , and  $w_B$  is the local body width at  $x_B$ . The integration is carried out along the length of the body.

Derivations 8 and 9 quote three constant values for  $a_{1B}$ , depending on whether the fuselage cross-sectional shape is circular, elliptical or rectangular. In this Item a more general semi-empirical formula has been developed for  $a_{1B}$  for use in modified fashion with Equation (3.1),

$$a_{1B} = \left( \frac{2S_m}{S_B} \right) k_f (1 - k_a) k_s. \quad (3.2)$$

Three correction factors have been applied to the slender-body-theory prediction of the lift-curve slope of a forebody,  $2S_m/S_B$ , to allow for body fineness ratio, afterbody geometry, and cross-sectional shape. Strictly,  $S_m$  is the maximum cross-sectional area, but in practice it is best to take  $S_m$  as the area in the plane containing the maximum body width  $w_{Bm}$ .

The fineness-ratio correction factor  $k_f$  is taken from Derivation 5, and is given in Figure 1 as a function of  $\beta(w_{Bm} + h_{Bm})/(2l_B)$  where  $h_{Bm}$  is the body height in the plane containing  $w_{Bm}$ . Thus  $a_{1B}$  decreases as the body fineness ratio decreases.

The afterbody-geometry correction factor  $(1 - k_a)$  has been developed empirically, and Figure 2 gives  $k_a/(l_a/l_B)$  as a function of  $(S_1/S_m)^{1/2}$  and  $w_{B1}/w_{Bm}$ , where  $l_a$  is the length of any afterbody taper,  $S_1$  is the cross-sectional area of the body base and  $w_{B1}$  its width. The effect of the factor  $(1 - k_a)$  is to decrease  $a_{1B}$  as the relative length of the afterbody increases or the relative body base area decreases. The variation with  $w_{B1}/w_{Bm}$  provides a correction for afterbodies that taper differently in height and width. It is based on a small number of data presented in Derivation 67 for bodies with  $S_1 = 0$  and  $w_{B1}/w_{Bm} \leq 0.6$ , with a tentative extrapolation to provide a full coverage of possible shapes.

If the body cross-sectional shape is constant along the length of the body, and it has a smooth perimeter such as an ellipse, then the correction factor for cross-sectional shape  $k_s$  is simply

$$k_s = \frac{w_B}{h_B}. \quad (3.3)$$

In Derivation 67 this is shown to work well for elliptically sectioned forebodies over a wide range of major to minor axis ratios.

If, however, the cross-sectional shape is angular, then the discussion in Derivation 67 illustrates how dramatically  $a_{1B}$  can increase. The precise effect will depend in complicated fashion on the sharpness of the irregularities in the cross-sectional shape. For its intended application via Equation (3.1) to the types of shape to be met on the centrebodies and afterbodies of combat aircraft and on light aircraft it has been found best to make the approximation  $S_m = w_{Bm}h_{Bm}$  in the expressions for forebody lift-curve slope and to take

$$k_s = \frac{2w_B}{h_B}. \quad (3.4)$$

This brings in the body maximum width as the critical dimension for  $a_{1B}$ . The true value of  $S_m$  is retained in the evaluation of  $k_a$ .

If the shape of the body cross-section varies along its length, an empirical examination of the different results achieved has shown that, instead of a simple substitution for  $a_{1B}$  in Equation (3.1), it is better to take  $k_s$  into the integrand and make the correct choice of Equation (3.3) or (3.4) at each longitudinal station. The final equation for  $(M_q)_B$  is then

$$(M_q)_B = - \left( \frac{2S_m}{S_B} \right) \frac{k_f(1 - k_a)}{S_w \bar{c}^2} \int_{-l_0}^{l_B - l_0} k_s w_B x_B^2 dx_B \quad (3.5a)$$

if  $w_{Bm}$  lies in a plane where the cross-sectional shape has a smooth perimeter, or

$$(M_q)_B = - \left( \frac{2w_{Bm}h_{Bm}}{S_B} \right) \frac{k_f(1 - k_a)}{S_w \bar{c}^2} \int_{-l_0}^{l_B - l_0} k_s w_B x_B^2 dx_B, \quad (3.5b)$$

if  $w_{Bm}$  lies in a plane where the cross-sectional shape is angular.

The use of Mach number similarity parameters to allow for compressibility effects leaves the equations unaltered, so the prediction of  $(M_q)_B$  varies with Mach number only through changes in  $k_f$ , which are small.

For a discussion of the experimental data used in the verification of Equations (3.2), (3.5a) and (3.5b), see Section 6.1.1.

The body contribution  $(Z_q)_B$  is sufficiently small to be neglected.

## 3.2 Wing Contribution

The derivatives for a wing depend on  $x'_0$ , the distance of the moment reference point aft of the leading edge of the aerodynamic mean chord. Sections 3.2.1 and 3.2.2 give  $(M_{q^{1/4}})_W$  and  $(Z_{q^{1/4}})_W$ , the values appropriate to a moment reference point and pitching axis situated at the quarter-chord point of the aerodynamic mean chord. Axis transformation equations then give the results for arbitrary  $x'_0$ ,

$$(M_q)_W = (M_{q^{1/4}})_W - (Z_{q^{1/4}})_W \left( \frac{x'_0 - 0.25\bar{c}}{\bar{c}} \right) - a_{1W} \left( \frac{x'_0 - x'_{ac}}{\bar{c}} \right) \left( \frac{x'_0 - 0.25\bar{c}}{\bar{c}} \right) \quad (3.6)$$

$$\text{and} \quad (Z_q)_W = (Z_{q^{1/4}})_W + a_{1W} \left( \frac{x'_0 - 0.25\bar{c}}{\bar{c}} \right), \quad (3.7)$$

where  $a_{1W}$  the wing lift-curve slope and  $x'_{ac}$  the position of its aerodynamic centre measured from the leading edge of the aerodynamic mean chord may be obtained from ESDU 70011 (Derivation 2), and should be evaluated at the Mach number of interest.

### 3.2.1 Straight-tapered wings

For a straight-tapered wing the formulae developed in Derivations 6 and 7 on the basis of strip theory give

$$(M_{q^{1/4}})_W = -a_{1W} \left[ \frac{1}{4} \left( \frac{x'_{ac} - 0.25\bar{c}}{\bar{c}} \right) + \left( \frac{x'_{ac} - 0.25\bar{c}}{\bar{c}} \right)^2 \right] - F \left\{ \frac{(a_1)_0 \cos \Lambda_{1/4}}{48\beta'} \left[ \frac{\beta' A^3 \tan^2 \Lambda_{1/4}}{(\beta' A + 6 \cos \Lambda_{1/4})} + 3 \right] \right\}, \quad (3.8)$$

$$\text{and} \quad (Z_{q^{1/4}})_W = -a_{1W} \left[ \frac{1}{4} + \left( \frac{x'_{ac} - 0.25\bar{c}}{\bar{c}} \right) \right]. \quad (3.9)$$

As before,  $a_{1W}$  and  $x'_{ac}$  may be obtained from ESDU 70011. In Equation (3.8),  $(a_1)_0$  is strictly the spanwise average of the two-dimensional lift-curve slope for wing sections normal to the wing quarter-chord line. However, for the purposes of this Item the section for the determination of  $(a_1)_0$  can be taken as that at the mid-semi-span and normal to the wing quarter-chord line. The value of  $(a_1)_0$  may be obtained from ESDU Aero W.01.01.05 (Derivation 1) for simple sections, or from other sources. The value for incompressible flow must be used as similarity parameters have been introduced to allow for the first-order effects of compressibility.

Equation (3.8) was derived for untapered wings but in the absence of other methods it is generally used for all straight-tapered wings. Theory predicts  $F = 1$ , but in Derivation 14 it is shown by comparisons with experimental data for swept and unswept untapered wings ( $1 \leq A \leq 5$ ;  $0 \leq \Lambda_{1/4} \leq 60^\circ$ ) that in the low to moderate aspect ratio range predictions are much improved by using lower values of  $F$ . For higher aspect ratios the assumptions of the theory are more valid and  $F$  should approach its theoretical value. Figure 3 shows a general variation of  $F$  with  $1/(\beta A)$  that has been constructed from the available data.

See Section 6.1.2 for comparisons with experimental data.

### 3.2.2 Delta and cropped-delta wings

In the particular case of delta and cropped-delta wings (unswept trailing edge), published theoretical results from modified versions of Multhopp's lifting-surface theory are available for the prediction of the pitch-rate derivatives. The theoretical data in Derivations 11 and 12 have been used to construct Figures 4 and 5 that give  $\beta(M_{q^{1/4}})_W$  and  $\beta(Z_{q^{1/4}})_W$  in terms of the taper ratio  $\lambda$  and  $\Lambda'_0$ , the similarity parameter for leading-edge sweepback. Those Figures should be used instead of Equations (3.8) and (3.9) for planforms that fall within their scope.

Comparisons published in Derivations 11, 31, 34, 39 and 40 show that Multhopp's method provides good estimates of the composite oscillatory-damping derivative  $(M_q + M_w \cos \alpha)_W$ , for a range of  $x'_0$  values. Data with which to assess the prediction of  $(M_q)_W$  alone are scarce. A single set of data is available from the curved-flow tests reported in Derivation 16, but they cover only a single reference point,  $x'_0 = 0.25\bar{c}$ . Comparison between predicted values and those experimental results, see Section 6.1.2, suggests that  $(M_{q^{1/4}})_W$  is underestimated by about 25% and  $(Z_{q^{1/4}})_W$  overestimated by about 30%. In view of the success with which the oscillatory-damping results are predicted these errors seem large. Furthermore, if correction factors are applied to correct any bias, the variation of  $(M_q)_W$  with  $x'_0$  shows less similarity to the shape of the oscillatory-damping results than might be expected (see Derivation 11). For those reasons, Figures 4 and 5 present theoretical values without modification.

### 3.2.3 Other planforms

Derivatives may be calculated for an isolated wing of any planform by first using the method of ESDU 76003 (Derivation 3) to construct an 'equivalent' straight-tapered wing. But it is for a wing in combination with a body that such a construction will normally be needed to produce a straight-tapered gross planform, see Section 4.

### 3.3 Tailplane Contribution

The tailplane contributions to the derivatives are

$$(M_q)_T = -a_{1T} \frac{S_T x_T^2}{S_W \bar{c}^2} \quad (3.10)$$

$$\text{and} \quad (Z_q)_T = -a_{1T} \frac{S_T x_T}{S_W \bar{c}}. \quad (3.11)$$

To evaluate the equations, the tailplane is treated as an isolated surface with a planform formed by extending its leading and trailing edges to the plane of symmetry;  $S_T$  denotes the area of that planform and  $x_T$  the distance of the quarter-chord point of its aerodynamic mean chord aft of the moment reference point. The

lift-curve slope of the planform may be found by using ESDU 70011. If the tailplane planform is not straight-tapered, the method of ESDU 76003 must first be used to construct an 'equivalent' straight-tapered planform.

To obtain an accurate estimate of aircraft lift and pitching moment it is necessary to employ an installation factor to allow for any loss of tailplane effectiveness caused by the presence of the body and possible changes in local kinetic pressure (see ESDU 89029, Reference 106). In the calculation of  $M_q$ , however, comparisons with experimental data clearly indicate that, within the present method, significant underestimates are obtained from Equation (3.10) if an installation factor is introduced. This is particularly apparent from the study of bodies of relatively large width at the tailplane, when an installation factor of 0.6 to 0.9 would be expected. For bodies of smaller relative width, and installation factors greater than 0.9, the conclusion is less clear due to the scatter of the data, but there is no indication of a need to allow for installation effects. Therefore no factor appears in Equations (3.10) and (3.11) and none should be introduced for bodies of any width at the tailplane.

## 4. INTERFERENCE EFFECTS

To take account of the presence of the wing, it is possible that the integral of Equations (3.5a) and (3.5b) should be modified over that part of the body length covered by the wing root chord, but the moment arm in this region is so small for practical choices of the moment reference point that any effect is minimal.

To allow for the interference effect of a body on any straight-tapered wing, including delta and cropped-delta wings, a gross planform is formed by extrapolating the leading and trailing edges of the exposed wing panels through the body to the plane of symmetry. If the wing is not straight-tapered the simple geometric technique of ESDU 76003 must be used to construct 'equivalent' straight-tapered planforms to represent the exposed wing panels before extrapolation. Addendum A of ESDU 76003 gives a full demonstration of this process. With the gross planform defined, ESDU 70011 is used to determine its lift-curve slope  $a_{1W}$  and the position of its aerodynamic centre  $x'_{ac}$ . A rearward shift  $\Delta x'_{ac}$  in the position of the aerodynamic centre is then calculated to allow for the effect of the body on the wing loading. The method of Derivation 13, which is similar in its treatment of body carry-over lift to that in Derivation 10, has been used to calculate  $\Delta x'_{ac}$ . The results are given in Figures 6a to 6d in terms of  $w_{Bm}/b, \lambda$  and  $Atan\Lambda_{1/2}$ . With  $(M_{q^{1/4}})_W$  and  $(Z_{q^{1/4}})_W$  evaluated for the gross wing, the pitch-rate derivatives for the wing in the presence of the body are

$$\begin{aligned} (M_q)_{W(B)} = & (M_{q^{1/4}})_W - (Z_{q^{1/4}})_W \left( \frac{x'_0 - 0.25\bar{c} - \Delta x'_{ac}}{\bar{c}} \right) \\ & - a_{1W} \left( \frac{x'_0 - x'_{ac} - \Delta x'_{ac}}{\bar{c}} \right) \left( \frac{x'_0 - 0.25\bar{c} - \Delta x'_{ac}}{\bar{c}} \right) \end{aligned} \quad (4.1)$$

$$\text{and} \quad (Z_q)_{W(B)} = (Z_{q^{1/4}})_W + a_{1W} \left( \frac{x'_0 - 0.25\bar{c} - \Delta x'_{ac}}{\bar{c}} \right). \quad (4.2)$$

See Section 6.1.3 for comparisons with experimental data.

Apart from the provision made through Equations (4.1) and (4.2), and the comments made in respect of tailplane installation factors in Section 3.3, no further allowance is made for interference effects. The experimental data were neither sufficiently systematic nor copious to permit the ready development of additional empirical corrections.

Therefore, the derivatives for a wing-body-tailplane combination are obtained by the summations

$$M_q = (M_q)_B + (M_q)_{W(B)} + (M_q)_T \quad (4.3)$$

and 
$$Z_q = (Z_q)_{W(B)} + (Z_q)_T, \quad (4.4)$$

with  $(M_q)_B$  from Equation (3.5a) or (3.5b),

$(M_q)_{W(B)}$  and  $(Z_q)_{W(B)}$  from Equations (4.1) and (4.2)

and  $(M_q)_T$  and  $(Z_q)_T$  from Equations (3.10) and (3.11).

See Section 6.1.4 for comparisons with experimental data.

## 5. TAILPLANE CONTRIBUTION TO $M_{\dot{w}}$

In the analysis of data from oscillatory-rig tests and from some flight tests it is only possible to determine the composite derivative due to pitch-rate and acceleration in heave,  $M_q + M_{\dot{w}} \cos \alpha$ . For aircraft with tailplanes, the lag in downwash at that surface gives rise to the most significant contribution to  $M_{\dot{w}}$ , which may be evaluated as

$$(M_{\dot{w}})_T = \left\{ 1 - \left( \frac{x'_{ac} + \Delta x'_{ac} - x'_0}{x_T} \right) \right\} \left( \frac{\partial \varepsilon}{\partial \alpha} \right)_T (M_q)_T, \quad (5.1)$$

where the mean effective gradient of downwash at the tailplane  $(\partial \varepsilon / \partial \alpha)_T$  may be obtained at the appropriate Mach number from ESDU 80020 (Derivation 4).

## 6. ACCURACY AND APPLICABILITY

### 6.1 Accuracy

The derivative  $Z_q$  is relatively unimportant except for its role in axis transformations. The predicted values will be sufficiently accurate for that purpose. Consequently, comparisons of experimental data and predicted values are presented only for  $M_q$  in Sketches 6.1 to 6.4 at the end of this Section. All comparisons are made for angles of attack close to zero.

#### 6.1.1 Body pitch-rate moment derivative

Comparisons between experimental data and the predictions of Equations (3.5a) and (3.5b) are shown in Sketch 6.1, with the derivative normalised by the body reference quantity  $S_B l_B^2$ .

Only very few test data are available on the pitch-rate (or yaw-rate) derivatives of isolated bodies. Those from curved-flow tests furnish  $(M_q)_B$  directly and those from oscillatory-rig tests only give the composite derivative  $(M_q + M_{\dot{w}} \cos \alpha)_B$  but it can be assumed that the acceleration derivative is comparatively small. Additional test data may be deduced from yaw-rate tests on wing-body combinations with subtraction of a small theoretical contribution from the wing (ESDU 71017, Reference 102).

In the present study, data for isolated bodies have been taken from Derivations 15, 20 and 22 (curved-flow tests) and from Derivations 33, 36 and 55 (oscillatory-rig tests). Data extracted from wing-body yaw-rate tests have been taken from Derivations 17, 18, 19, 23, 24 and 26 (curved-flow tests) and from Derivations 45, 48 to 51, 54 and 55 (oscillatory-rig tests).

The measured values of the body derivatives are small and sometimes difficult to determine with precision, certainly from some of the presented experimental data. Because of this and the minor importance of the body contribution to the value of  $M_q$  for an aircraft, the fairly large scatter shown in Sketch 6.1 is not considered to be excessive. This is particularly so since the band includes values for the analytically problematical shapes of some combat aircraft in addition to those for simply-shaped bodies.

## *Body lift-curve slope*

The empirical equation for  $a_{1B}$  has been developed only for its intended use in the prediction of  $(M_q)_B$ , and it has proved satisfactory in that regard. However, it is not considered to be sufficiently accurate or versatile for use in the determination of body lift-curve slope in its own right.

The lift-curve slopes of bodies are relatively small and seldom reported in sufficient accuracy for easy analysis. More accessible data are often contained within reports that deal primarily with the measurement of static characteristics in sideslip because they present a force derivative, not just a small-scale curve of force coefficient against angle. Furthermore, additional results to expand the database may be obtained from tests on bodies with plane wings mounted at the mid-body position since the effect of such wings is small in sideslip. As a result of those considerations, the experimental data of Derivations 68 to 99, together with the discussions in Derivation 67, have been used to guide the construction of Equation (3.2)

### 6.1.2 Wing pitch-rate moment derivative

Comparisons between predicted and experimental results for straight-tapered wings are shown in Sketch 6.2a, and for delta and cropped-delta wings in Sketch 6.2b.

The test data for straight-tapered wings have been taken from Derivations 14 and 21 (curved-flow tests) and from Derivations 30, 34, 39 and 40 (oscillatory-rig tests). For delta and cropped-delta wings, data have been taken from Derivation 16 (curved-flow tests) and from Derivations 34, 39, 40 and 44 (oscillatory-rig tests).

Values for the straight-tapered wings are shown in Sketch 6.2a to be predicted well; the oscillatory-rig results show slightly greater scatter than the curved-flow results, as might be expected.

As discussed previously in Section 3.2.2, the curved-flow data for the delta and cropped-delta wings are under-predicted in most cases. For the moment reference points of the tests, the measured oscillatory-rig results would be expected to be greater in magnitude than the predicted value of  $-(M_q)_W$ , and this is the case except at small values.

### 6.1.3 Wing-body pitch-rate moment derivative

Sketch 6.3 compares experimental results for wing-body combinations with the predictions formed by adding  $(M_q)_B$  and  $(M_q)_{W(B)}$ .

Data for combinations with straight-tapered wings have been taken from Derivation 21 (curved-flow tests) and Derivations 33, 36, 37, 49, 53 and 55 (oscillatory-rig tests). Data for combinations with delta and cropped-delta wings have been taken from Derivations 27, 28, 29, 32 and 35 (oscillatory-rig tests). The general level of agreement is reasonable.

Within the present method, the predicted shift in aerodynamic centre position  $\Delta x'_{ac}$  provides a simple allowance for wing-body interference for a wide range of geometries. However, the prediction is too approximate to be generally recommended for use in estimation of wing-body static stability, for which the method of ESDU 76015 (Reference 103) is preferred.

## 6.1.4 Wing-body-tailplane pitch-rate moment derivative

Sketch 6.4 shows experimental and predicted values for wing-body-tailplane combinations.

The experimental data for curved-flow tests are compared with the predictions of  $M_q$  from Equation (4.3). The data from oscillatory-rig tests and the available flight-test data provide only the composite derivative  $M_q + M_{\dot{w}} \cos \alpha$ . Therefore, in such cases, the estimated tailplane contribution  $(M_{\dot{w}})_T \cos \alpha$  has been added to the predicted value of  $M_q$  in order to permit appropriate comparison.

Data for a parametric set of wing-body-tailplane configurations are available from the curved-flow tests of Derivation 21. The configurations have bodies that are relatively small in volume. With that restriction, the overall correlation with prediction is excellent for a variety of body lengths and tailplanes of different sizes mounted either on the body or on a fin. A small selection of those test data are plotted in Sketch 6.4 together with the result from the curved-flow test of Derivation 25.

Also plotted are oscillatory-rig test data from Derivations 33, 36 to 38, 41 to 43, 46 to 49, 52 and 55, and flight-test data from Derivations 56 to 66.

The level of agreement is generally good, although some values for aircraft with long tail arms are predicted less well.

Experimental data are also available from rocket-launched models (see the summary in Reference 101 for example) but those results are mainly confined to supersonic speeds or to the transonic region where the derivatives often show significant variations in magnitude as Mach number changes. The values at high subsonic speeds were not considered to be of sufficient reliability to be included in the present study.

## 6.2 Applicability

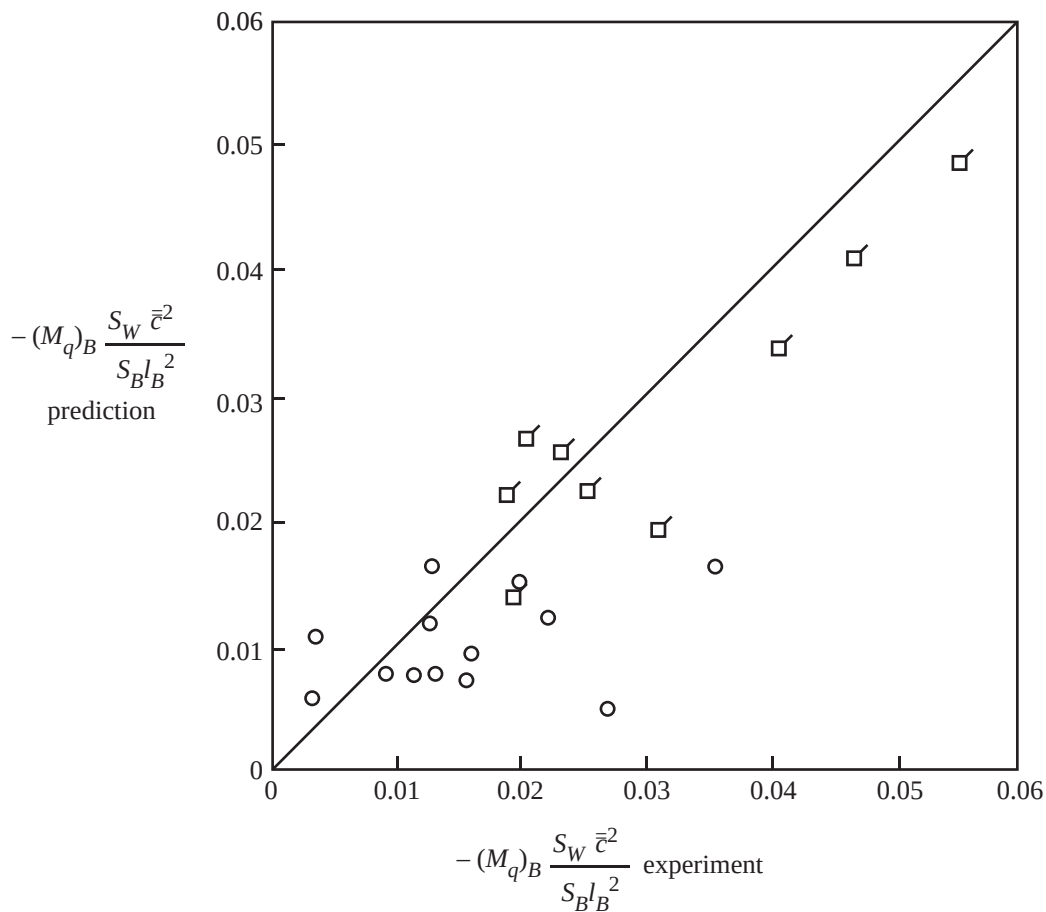
The method can be expected to apply satisfactorily for most conventional aircraft configurations. Wings and tailplanes with planforms that are not straight-tapered are brought within the scope of the method by the use of ESDU 76003 to construct 'equivalent' straight-tapered planforms. The neglect of aircraft flexure may be a source of error for aircraft with long tail arms. The aircraft is assumed to be at low angles of attack, with no high-lift devices deployed.

Strictly, the method is restricted to  $\alpha = 0$ . However, experimental values of  $M_q$  do not usually change rapidly in magnitude as  $\alpha$  increases to modest values, up to  $10^\circ$  say. The method is expected to be most reliable when confined to the angle of attack range where the static pitching moment coefficient varies linearly with  $\alpha$ .

The effects of compressibility on  $M_q$  are modelled only through the use of similarity parameters, which should be acceptable up to  $M = 0.8$ . Comparisons have been made with a small number of available experimental data. Derivations 33 and 36 each report oscillatory-rig tests on an aircraft model which was also tested without the tailplane and without both wing and tailplane. The test measurements confirm that the body contribution remains largely constant as Mach number increases. However, measurements for the wing-body combinations show an increase in magnitude due to Mach number that is noticeably greater than that predicted. This suggests that the calculated increase of  $(M_q)_{W(B)}$  is too small, at least for the low aspect ratio wings of those tests ( $A = 3.6$ ,  $\Lambda_{1/4} = 44^\circ$  and  $A = 2.4$ ,  $\Lambda_{1/4} = 20^\circ$ ). No empirical correction

is proposed on the basis of this small sample. For wing-body-tailplane configurations the predicted values of  $M_q$  show trends that are a reasonable first approximation to the measured values reported in Derivations 33 and 36, and to those in Derivations 41 and 47 (oscillatory-rig tests) and Derivation 61 (flight-test).

If the tailplane derivative  $(M_{\dot{w}})_T$  is to be calculated, then it is possible that the restrictions on the application of ESDU 80020 (Derivation 4) to determine the mean effective gradient of downwash from the wing may exclude some configurations with the tailplane too close behind the wing, and the discussion of applicability in that Item should be consulted.



□ Bodies of angular cross-section.

○ Bodies of smooth cross-section.

Primed symbols denote oscillatory-rig tests for which experimental values of  $-(M_q + M_{\dot{w}} \cos \alpha)_B (S_W \bar{c}^2 / S_B l_B^2)$  are plotted.

**Sketch 6.1 Pitch-rate moment derivatives for bodies**