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Natural frequencies of isotropic and orthotropic rectangular plates under static in-plane loading (including shear loading)

Associated software: ESDUpac A9016

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes ESDU 87029, was monitored and guided by the Acoustic Fatigue Panel on behalf of the Vibration and Structural Dynamics Steering Group. The constitution of the Acoustic Fatigue Panel was:

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NATURAL FREQUENCIES OF ISOTROPIC AND ORTHOTROPIC RECTANGULAR PLATES UNDER STATIC IN-PLANE LOADING (INCLUDING SHEAR LOADING)

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1. NOTATION

A_{rs}	displacement parameter [see Equation (2.2)]	m	in
a	length of plate in x-direction (see Sketch 1.1)	m	in
b	length of plate in y-direction (see Sketch 1.1)	m	in
D	flexural rigidity of isotropic plate $\{Et^3/[12(1-\nu^2]\}$	N m	lbf in
E	Young's modulus of isotropic plate material	N/m^2	lbf/in^2
E_x, E_y	orthotropic plate moduli of elasticity in x- and y-directions, respectively	N/m^2	lbf/in^2
f	natural frequency of vibration of plate	Hz	Hz
G	shear modulus of isotropic plate material	N/m^2	lbf/in^2
G_{xy}	shear stiffness of orthotropic plate in x, y plane	N/m^2	lbf/in^2
m, n	mode numbers in x- and y-directions, respectively		
N_x, N_y	static direct forces per unit width in x- and y-directions, respectively (tensile forces positive)	N/m	lbf/in
N_{xy}	static shear force per unit width (see Sketch 1.1 for sign convention)	N/m	lbf/in
N^*_{xy}	shear buckling force per unit width	N/m	lbf/in
r, s	number of orthogonal polynomial functions used to describe displacements in x- and y-directions, respectively		
t	thickness of plate	m	in
W	displacement parameter [see Equation (2.2)]	m	in
w	plate displacement normal to x, y plane	m	in
x, y	plate coordinate system (see Sketch 1.1)		
α	non-dimensional distance, x/a		
λ	isotropic plate frequency parameter [see Equation (2.3)]		
ν	Poisson's ratio of isotropic plate material		
ν_{xy}	Poisson's ratio of orthotropic plate associated with load applied in x-direction		

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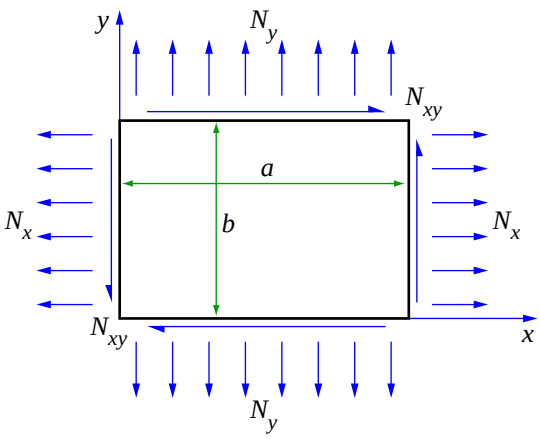
ρ	density of plate material	kg/m^3	$^*lb/in^3$
τ	time	s	s
$\phi_r(x)$	r^{th} orthogonal polynomial function representing plate displacements on line parallel to x-direction		
$\psi_s(y)$	s^{th} orthogonal polynomial function representing plate displacements on line parallel to y-direction		
ω	circular natural frequency $(2\pi f)$	rad/s	rad/s

* See footnote to Section 3.1.2.



Sketch 1.1 Plate geometry and positive loading system

2. NOTES

2.1 General

This Item provides a means of estimating the lower natural frequencies of isotropic or orthotropic, flat rectangular plates under static in-plane loading. The static in-plane loading, uniformly distributed along the plate edges, may be any combination of direct and shear loading that does not cause the plate to buckle (see Section 2.4). Applied forces are assumed to act on the middle surface of the plate with direct loads acting normal to the plate edges. The data are applicable to specially orthotropic plates for which the principal axes of orthotropy are parallel to the plate edges. The data are therefore applicable to many fibre-reinforced composite laminated plates. The required orthotropic plate elastic properties for such plates may be estimated using ESDU 83035*. Data are provided for plates having any combination of clamped, simply-supported and free edges.

The number of variables associated with orthotropic plate vibrations is too large to allow graphical presentation of data. A program contained in ESDUpac A9016 is therefore provided with example input and output files. For rectangular isotropic plates, ESDU 75030† provides a graphical presentation of natural frequency data for initially unstressed flat plates. That Item also provides an equation that relates the natural frequencies of plates under biaxial in-plane loads to those of the initially unstressed plate. For

* Estimation of the stiffnesses and apparent elastic properties of laminated flat plates.
† Natural frequencies of rectangular flat plates with various edge conditions.

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fibre-reinforced composite laminated plates, ESDU 83036* provides a computer program for the estimation of natural frequencies of initially unstressed plates having combinations of simply-supported and clamped edges. Also for laminated plates, ESDU 89011† provides a computer program for the estimation of natural frequencies of singly-curved plates with simply-supported edges. In that Item, static direct forces, uniformly distributed along the plate edges, may be considered but in-plane shear forces are assumed to be zero. As with this Item, the use of Item No. 89011 is limited for compressive in-plane loading to a load that does not exceed 30 per cent of the buckling load. The programs in both Item Nos ESDU 83036 and ESDU 89011 allow direct input of the lamina properties from which the orthotropic stiffnesses are evaluated.

In this Item general guidance on the effect of in-plane loading on natural frequencies of plates is given in Section 2.4, and in Section 2.5 the special square plate modes are discussed. The special square plate modes, which occur only for certain combinations of boundary conditions, have nodal lines that are not parallel, or approximately parallel, to the plate edges.

2.2 Method of Analysis

The method of analysis is based on that given in Derivation 14. Natural frequencies, and buckling loads, are obtained by using beam characteristic orthogonal polynomials in the Rayleigh-Ritz method. The plate deflection w, normal to the xy-plane, is expressed as

$$w = W \sin \omega \tau \ , \tag{2.1}$$

where the displacement parameter W is expressed in the form of a series

$$W = \sum_r \sum_s A_{rs} \phi_r(x) \psi_s(y) \, , \tag{2.2}$$

where $\phi_r(x)$ and $\psi_s(y)$ are appropriate polynomial functions. Substitution of the displacement relationship in the Rayleigh Quotient and minimising with respect to the coefficients A_{rs} leads to the eigenvalue equation.

Solution of the eigenvalue/eigenvector problem leads to the natural frequencies and mode shapes for the plate. Because the method of analysis is applicable only to plates in the prebuckled state, a check is made to ensure that the applied loading does not exceed the buckling load (*i.e.* the zero natural frequency condition for the mathematical model used in this Item – see Section 2.4.1 for a discussion of the effects of physical edge constraints on buckling).

The set of orthogonal polynomials, used in Equation (2.2), are sequentially derived from a starting polynomial function that satisfies the boundary conditions on the opposite edges of the plate under consideration. The starting polynomials used in this Item, and the method of derivation of the second and subsequent polynomical functions, are detailed in Reference 16. Also in that Reference the derivation of the eigenvalue equation and the computations involving the orthogonal polynomial functions are detailed. Additionally in Reference 16 notes are provided on the computation procedure used in the program in ESDUpac A9016.

* Natural frequencies of singly-curved laminated plates with simply-supported edges.
† Natural frequencies of rectangular, specially orthotropic laminated plates.

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2.3 Mode Numbers

Each natural frequency is identified by a pair of mode numbers m and n, or in some cases (discussed later) a combination of pairs of mode numbers. Mode numbers can be explicitly defined for the case of initially unstressed simply-supported plates. That definition applies in general to plates having other edge conditions, but there are then some complicating factors that must be considered when identifying modes. The mode number definition for simply-supported plates will first be given and then the complicating factors will be discussed.

For each mode of vibration of an initially unstressed plate having all edges simply-supported, the nodal lines (*i.e.* lines of zero displacement w) are parallel to the plate edges. The mode numbers m and n are defined as one plus the number of nodal lines crossing any line parallel to the x-axis and y-axis, respectively. In this definition plate edges are not counted as nodal lines. A compatible definition for simply-supported plates is that m and n are the number of half waves spanning the plate in the x- and y-directions, respectively. For example, for the plate having nodal lines as shown in Sketch 2.1a, m is 3 and n is 2 so that the mode of vibration is referred to as the (3, 2) mode.

Mode numbers for plates not having all edges simply-supported, and those subjected to in-plane loading, are generally defined as above although the following factors must be considered. For plates having one, or more free edges, the definition in terms of half waves is not meaningful. Further nodal lines may not be parallel to the plate edges so that the number of nodal lines crossed by a line parallel to the plate edges will depend on the location of the line. This is illustrated in Sketch 2.1b where, for a particular mode of vibration of a cantilevered plate, line AA crosses the dotted nodal line in two places and line BB does not cross the nodal line.

When assigning mode numbers to modes having nodal lines that are approximately parallel to the plate edges, it is recommended that one plus the number of nodal line crossings of a line drawn parallel to the plate edges should still be used. However, in determining the value of m, count only the crossing of nodal lines that span the plate in the y-direction and ignore crossings of nodal lines that span the plate in the x-direction. Similarly in the determination of n, count only the crossing of nodal lines that span the plate in the x-direction and ignore crossings in the y-direction. When deriving values of m and n in this way, plate edges either simply-supported or clamped are not counted as nodal lines.

For particular combinations of plate edge conditions of square plates there are a number of modes that have nodal lines that are not parallel to the plate edges. Nevertheless these modes degenerate, as the plate a/b ratio is increased or reduced, into modes having nodal lines that are approximately parallel to the plate edges. Square plate modes are considered in Section 2.5.

There is another class of modes that can only be defined in terms of a combination of pairs of mode numbers. Such modes occur when there is a transition between two modes that can be individually described using the (m, n) modal numbering system. Such transitions may occur for an unstressed plate over a range of plate a/b ratios. This is illustrated in Sketch 2.2 where the variation in natural frequencies with a/b ratio is shown for two modes of vibration of a cantilever plate. On that Sketch, nodal line plots are given at discrete values of a/b ratio. Clearly, in Sketch 2.2, to the left of the transition region modes A and B are the (1, 4) and (2, 2) modes, respectively, and to the right of that region modes A and B are interchanged. Within the transition region the two modes can only be described as combinations of the (1, 4) and (2, 2) modes. Similar transitions may occur when plates are subject to in-plane loading as illustrated in Sketch 2.3. In that Sketch, natural frequencies of the (1, 1) and (1, 3) modes of an isotropic plate with all edges clamped (a/b = 1/3) are plotted against the shear loading ratio N_{xy}/N^*_{xy}.

When using the computer program in this Item, natural frequencies are output in ascending order of frequency and it is necessary to investigate the mode shapes in order to determine the mode number.

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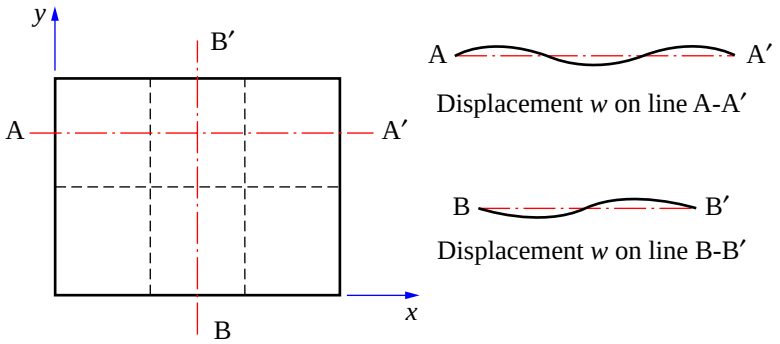
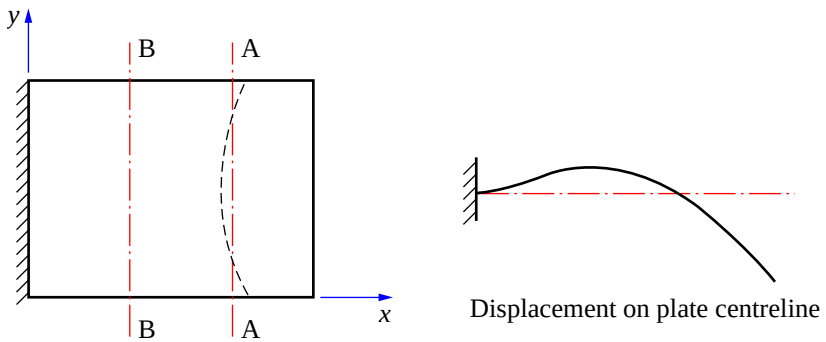


Sketch 2.1a Mode number illustration. Plate with all edges simply-supported, (3,2) mode



Sketch 2.1b Mode number illustration. Cantilevered plate (2,1) mode

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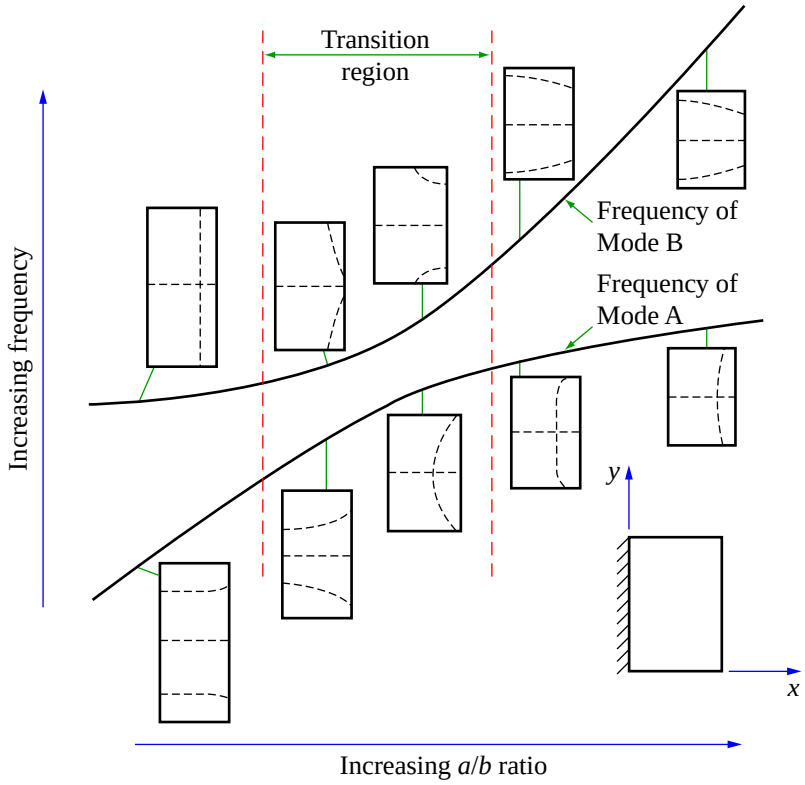
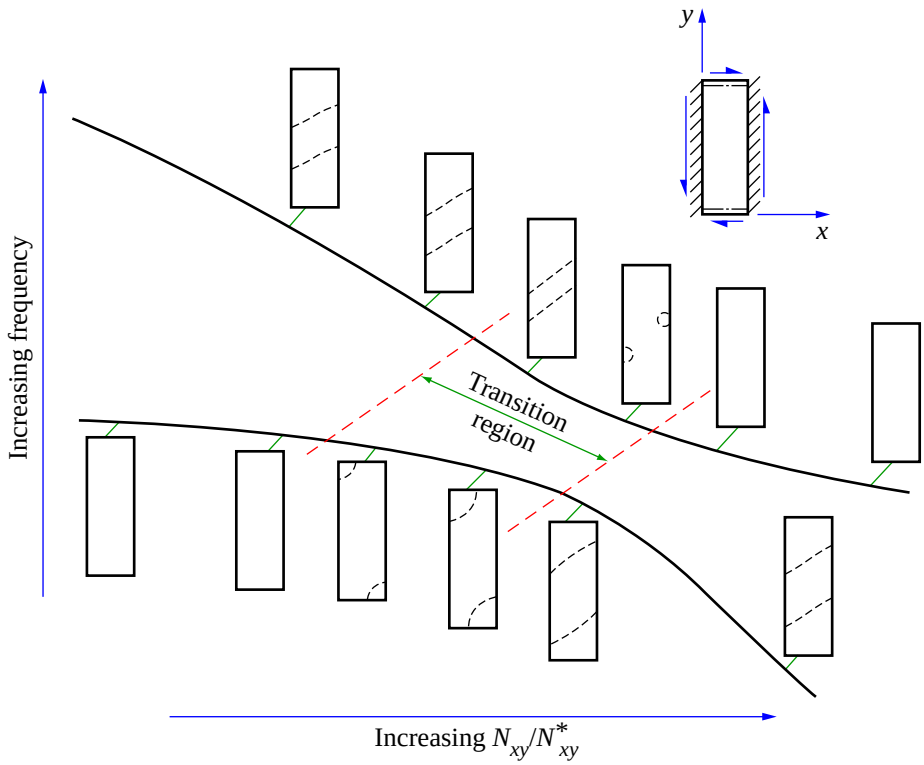


Sketch 2.2 Modal transition on an initially unstressed plate



Sketch 2.3 Mode transition on a shear loaded plate

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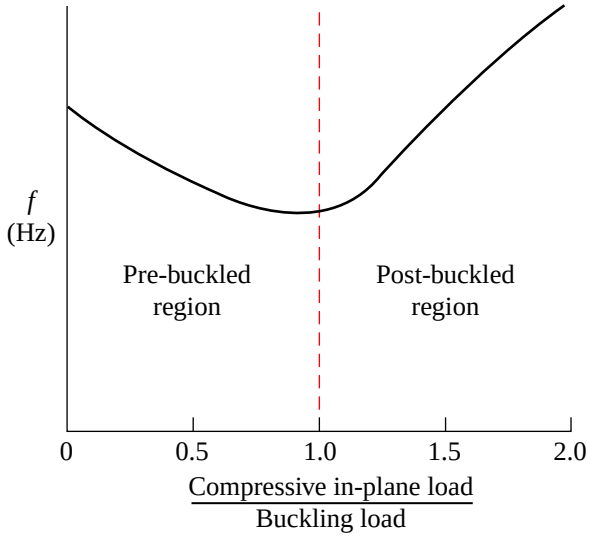
2.4 Effect of In-plane Loading

The effects of in-plane loading are considered in two parts. Effects of direct in-plane loading only (*i.e.* $N_{xy} = 0$) are first considered and then the effects of shear loading only.

2.4.1 Plates under direct loading

Under direct loading the natural frequency of a mode of vibration of a plate increases with increasing static tensile in-plane loading (*i.e.* N_x, N_y positive), but with the application of, and initial increase in, compressive loading the natural frequency decreases. However, as the buckling load is approached under compressive in-plane loading, plate natural frequencies may increase with increasing loading. In the post-buckled region, natural frequencies usually increase with increasing compressive loading as illustrated in Sketch 2.4. The minimum shown in the curve in Sketch 2.4 will occur when the value of the ratio (compressive in-plane load)/(buckling load) is in the region of unity. This Item is not applicable to in-plane loading near the buckling load or in the post-buckled region (see Section 2.5). Information on the vibration of plates in the post-buckled region is given in Derivation 13.



Sketch 2.4 Typical variation of frequency under unidirectional compression

Under increasing or decreasing direct in-plane loading, the rate of change of a natural frequency is a function of both the number of waves in the direction of loading and the plate aspect ratio (a/b). For example, under increasing compressive loading, the rate of decrease becomes increasingly great with an increase in half waves in the direction of loading. It follows that, with increase in in-plane loading, the sequence of natural frequencies of some modes may change. This is illustrated in Sketch 2.5. which shows typical variations of frequency with increasing static in-plane loading for the lower modes of a rectangular plate with all edges fixed. Near the buckling load, the mode of vibration associated with the lowest frequency is always the same as the buckling mode. It should be noted that as buckling is approached the amplitude of vibration may become large so that the theory on which this Item is based could no longer apply.

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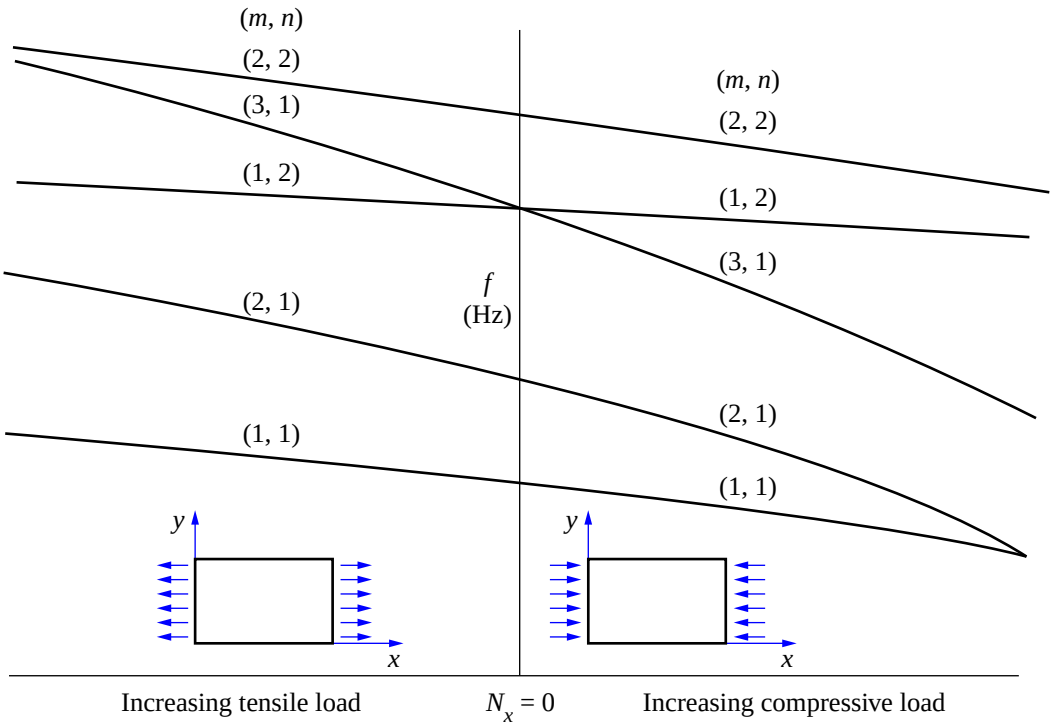

Sketch 2.5 Typical variation in natural frequencies with changing direct load. (Note: a compressive load input into the computer program must have a negative value.)

Some experimental investigations of the effects of in-plane compressive loading on natural frequencies of plate vibration have shown that, in some cases, there can be a marked divergence between theoretical and measured values with increasing compressive force. In those cases the measured natural frequencies do not fall as rapidly with compressive load as the theory predicts. Sketch 2.4 shows the behaviour in practice in which the natural frequency does not fall to zero, as predicted by the theory used in this Item, at the buckling load. The reason for the measured natural frequencies exceeding the theoretical values has been attributed to an increase in the stiffness of the plate due to curvature and effects associated with initial geometrical imperfections in the plate (*e.g.* the plate is not perfectly flat). The theoretical natural frequencies are evaluated assuming uniform in-plane stresses but, in practical plates, there is a redistribution of in-plane stress due to the growth of initial geometric imperfections with increasing compressive load. As the relative plate thickness increases (*i.e.* b/t reduced), the out-of-plane geometric imperfections will have a decreasing effect on the uniformity of the in-plane stress. Also increasing the edge fixing of the plate tends to inhibit the growth of the initial imperfections so retaining a more uniform in-plane stress distribution. It has been shown (Derivations 11 and 13) that by accounting for the initial plate geometric imperfections, the upward trend of natural frequencies with increased compression at high values of in-plane loading can be predicted. A further reason for differences between measured and estimated natural frequencies for a plate under in-plane loading is that the actual boundary conditions are not exactly modelled in the theoretical prediction procedure. In particular, in the theoretical analysis used in this Item, no restraint is provided on the plate edges in the in-plane direction.

Additional information on the effect of direct in-plane loading on the square plates modes is given in Section 2.5.

2.4.2 Plates under shear loading

Information given in this Section on the effect of shear loading on the natural frequencies of plates is restricted to the pre-buckled region. Natural frequencies are identical for equal positive or negative shear

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loads; however, the nodal patterns for positive and negative shear loads are mirror images. As shear loading is increased, the natural frequencies of some modes decrease and, at a sufficiently high loading, shear buckling will occur. As for the direct loading case, the mode of vibration associated with the lowest frequency in the region of the buckling load is always the same as the buckling mode. For some modes, particularly for square plates, natural frequencies initially increase with increasing shear loading.

To illustrate the effect of shear loading on plate natural frequencies, theoretical variations of the frequency parameter, are plotted against loading ratio N_{xy}/N^*_{xy} in Figures 1 to 3. The non-dimensional isotropic plate frequency parameter is given by

$$\lambda = \frac{2a^2 f}{\pi}\sqrt{\frac{\rho t}{D}}. \tag{2.3}$$

On Figures 1 to 3, data for isotropic plates having all edges clamped are presented for a/b ratios 1.0, 1.5 and 2.0, respectively. Modes are identified by their mode numbers except on Figure 1 (see Section 2.5) where some modes cannot be defined in terms of mode numbers. Nodal line patterns, at specific loading ratios, are shown on that figure.

No experimental investigations have been reported on the effect of shear loading on natural frequencies of plates.

2.5 Special Square Plate Modes of Vibration

For plates that are square, or nearly square, particular combinations of edge conditions and mode numbers give rise to modes of vibration in which the nodal lines are not parallel, or approximately parallel, to the plate edges. Such modes may occur when the plate is not simply-supported on all edges but there is symmetry of boundary conditions about a diagonal. The following five boundary conditions can therefore give rise to the square plate modes that are characterised by nodal lines that are not parallel to the plate edges.

(i) Free on all edges.

(ii) Free at $x = 0$ and $y = 0$, and simply-supported at $x = a$ and $y = b$.

(iii) Clamped at $x = 0$ and $y = 0$, free at $x = a$ and $y = b$.

(iv) Clamped at $x = 0$ and $y = 0$, and simply-supported at $x = a$ and $y = b$.

(v) Clamped on all edges.

The square plate modes can be described in terms of combinations of mode number m and n, where mode numbers are defined, in terms of nodal lines, as stated in Section 2.3. The square plate modes exist (Derivation 1*) when $m - n = \pm 2, 4, 6, ...$*etc*. for boundary conditions (i) and (v) stated earlier, and for $m \neq n$ for boundary conditions (ii), (iii) and (iv). Each appropriate pair of mode numbers leads to two modes of vibration that, as the ratio of side lengths a/b moves away from unity (*i.e*. away from an exact square plate), degenerate into two modes having the following nodal lines:

(a) $(m - 1)$ nodal lines parallel to the x-axis and $(n - 1)$ nodal lines parallel to the y-axis,

and (b) $(n - 1)$ nodal lines parallel to the x-axis and $(m - 1)$ nodal lines parallel to the y-axis.

* It should be noted that the definitions of mode number in Derivation 1 are not consistent with those in this Item. Mode number definitions, used in this Item, are given in Section 2.3.

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To illustrate the variation in natural frequencies and nodal patterns as the ratio a/b deviates from unity, data for an isotropic plate are presented in Figures 4 to 8. Frequency data are presented in terms of the non-dimensional plate frequency parameter, λ [defined in Equation (2.3)] for each of the five boundary conditions that give rise to the square plate modes for each boundary condition; the combination of mode numbers was chosen to satisfy the requirements for square plate modes and provide the lowest square plate mode natural frequency. Nodal line plots are shown in Figures 4 to 8 at discrete values of the ratio a/b. For some rectangular orthotropic plates, it is possible that certain combinations of orthotropy and plate ratio a/b may generate modes similar to those of the square isotropic plate.

The application of even low initial direct loading, and combined tensile and compressive loading, causes the square plate modes to degenerate into modes having nodal lines parallel to the plate edges. To illustrate the effect of direct in-plane loading on natural frequencies, data for an isotropic square plate are presented in Figures 9 to 12. Figures 9 and 10 show the variation of the frequency parameter with load ratio, N_x/N_y, for compressive biaxial in-plane loading for two boundary conditions. When deriving data presented in Figures 9 and 10, the value of N_x was held constant at 25 per cent of the buckling value for biaxial compression ($N_x = N_y$). Figures 11 and 12 show similar data for square plates under tensile biaxial in-plane loading. Also for Figures 11 and 12, the value of N_x was kept constant at the same magnitude that was used in Figures 9 and 10. The effect of increasing shear loading on the square plate modes of a plate having all edges clamped is illustrated in Figure 1. Reversal of the shear loading causes the pair of modes having single diagonal nodal lines (*i.e.* modes having $\lambda = 7.44$ at $N_{xy}/N^*_{xy} = 0$) to be interchanged; the two modes having $\lambda = 16.7$ at $N_{xy}/N^*_{xy} = 0$ behave similarly.

For isotropic plates, it is possible that some combinations of the ratios $a/b \neq 1$ and loading ratio $N_x/N_y \neq 1$ will lead to the generation of modes having similar shapes to those of a square plate with biaxial loading $N_x = N_y$. Also for orthotropic plates, certain combinations of orthotropy, values of a/b and in-plane loading may lead to modes of vibration similar to the square plate modes.

2.6 Accuracy and Limitations of Data

In this Item a Rayleigh-Ritz method of analysis is used to predict natural frequencies and mode shapes of the flexural modes of vibration of rectangular plates subjected to in-plane loading. The data are applicable to flat plates of isotropic or orthotropic materials. For orthotropic plates the method of analysis is applicable only to specially orthotropic plates in which the principal axes of orthotropy are parallel to the plate edges. In the analysis the plate displacements are represented by a series of orthogonal polynomial functions that satisfy the appropriate boundary condition at the plate edges. Using the Rayleigh-Ritz method of analysis with mode shapes that do not exactly represent the true shapes in effect subjects the plates to geometrical constraints so that the estimated frequencies are upper bounds. However, in practice, for plates under compressive loading the computed natural frequencies may underestimate the measured values (see Section 2.4.1).

The effects of shear deformation and rotary inertia have been neglected in the analysis. This simplification is usually acceptable if the ratio of half wave length to plate thickness is greater than 10. However, for a unidirectional fibre-reinforced plate, the criterion for neglecting shear deformation requires a higher value of the ratio (half wavelength)/(plate thickness). Also displacements are assumed to be small relative to the plate thickness and the plate is assumed to be thin relative to the plate dimensions. The latter assumption is satisfied if the smallest value of the ratio (half wavelength)/(plate thickness) exceeds 20.

To illustrate the accuracy of prediction of the square plate modes, the correlation between measured and estimated natural frequencies for composite plates is presented in Figure 13. Data on that Figure are for initially unstressed, fibre-reinforced laminated square plates either free on all edges (Derivation 10) or with all edges clamped (Derivations 3 and 6). Predicted nodal lines for those plates are in close agreement with measurements.

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