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Airframe-induced upwash at subsonic speeds

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AIRFRAME-INDUCED UPWASH AT SUBSONIC SPEEDS

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AIRFRAME-INDUCED UPWASH AT SUBSONIC SPEEDS

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A	wing aspect ratio		
a_{vm}	influence coefficient for wing upwash, indicating influence of spanwise loading at station m on upwash at spanwise station v		
a_1	wing lift-curve slope	degree ⁻¹	degree ⁻¹
C_L	wing overall lift coefficient		
C_{LL}	wing local lift coefficient		
c	wing local chord	m	ft
$\bar{c}$	wing geometric mean chord	m	ft
$\bar{\bar{c}}$	wing aerodynamic mean chord	m	ft
D	body maximum diameter, $2r_{max}$	m	ft
$F_{\zeta W}$	height factor for wing, Equation (3.8)		
K_1, K_2	coefficients in Equation (3.12)		
L_B, L_F, L_N	length of body, fuselage, nacelle	m	ft
M	free-stream Mach number		
m	integer defining spanwise station on wing ¼-chord line for spanwise loading		
$R_{\bar{\bar{c}}}$	Reynolds number based on wing aerodynamic mean chord		
R_D	Reynolds number based on body maximum diameter		
r	body local radius	m	ft
r_e	effective local radius for non-axisymmetric body, see Section 3.3.2 and Appendix D	m	ft
r_{emax}	value of r_{max} for effective axisymmetric body (or fuselage or nacelle)	m	ft

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r_{max}	maximum radius of body (or fuselage or nacelle)	m	ft
r_p	body radius at $x'_B = 0$, see Sketch 3.2b	m	ft
S	body local cross-sectional area	m ²	ft ²
S_{max}	body maximum cross-sectional area	m ²	ft ²
s	wing semi-span	m	ft
t	number of body segments		
x_B, y_B, z_B x_F, y_F, z_F x_N, y_N, z_N	coordinate system with origin at body/fuselage/nacelle leading edge and x axis aligned with body/fuselage/nacelle axis, as shown in Sketches 3.2a and 3.2b for body	m	ft
x'_B, x'_F, x'_N		m	ft
$x'_{B0}, x'_{F0}, x'_{N0}$		m	ft
x_W	longitudinal distance ahead of wing ¼-chord line at station y_W , measured parallel to aircraft plane of symmetry, see Sketch 3.1	m	ft
y_W	lateral distance measured from aircraft plane of symmetry, see Sketch 3.1	m	ft
z_w	vertical distance measured normal to wing reference plane, see Sketch 3.1	m	ft
$\alpha_B, \alpha_F, \alpha_N$	effective angle of attack for calculating upwash at body, fuselage, nacelle, see Sketch 3.2a for body and Equations (3.2) to (3.4) for fuselage and nacelle	degree	degree
α_e	effective angle of attack of thrust axis for propeller normal force and yawing moment calculations, taken as $\gamma_{0.7}$	degree	degree
$\alpha_{gB}, \alpha_{gF}, \alpha_{gN}$	geometric angle of attack of body, fuselage, nacelle, as shown in Sketch 3.2 for body	degree	degree
$\alpha_{gNi}, \alpha_{gNo}$	value of α_{gN} relating to inboard, outboard nacelle	degree	degree
α_{gP}	geometric angle of attack of propeller thrust axis	degree	degree
$\alpha_{Ni}, \alpha_{gNo}$	value of α_N for inboard, outboard nacelle; see Equation (3.3), (3.4)	degree	degree
α_W	wing angle of attack, see Equation (3.6)	degree	degree
α_{W0}	value of α_W at $C_L = 0$	degree	degree

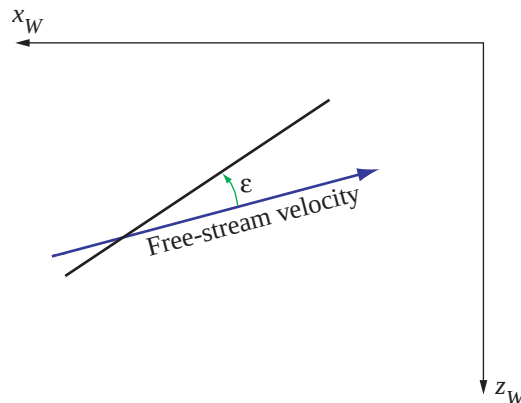
β	compressibility parameter, $(1 - M^2)^{1/2}$		
γ	upflow angle, angle between local velocity (in plane parallel to x_N, z_N plane) and propeller thrust axis at any point along horizontal centre line of propeller disc, Equation (4.1); positive anticlockwise when viewed in y_N direction, see Sketch 4.1	degree	degree
$\gamma_{0.7}$	value of γ at 0.7 of propeller disc radius for propeller-nacelle combination	degree	degree
$\bar{\gamma}_{0.7}$	average value of $\gamma_{0.7}$ at $\Omega = 90^\circ$ and 270° for propeller-airframe combination	degree	degree
ε	total upwash angle, measured from free-stream direction in plane parallel to aircraft plane of symmetry; positive anticlockwise when viewed in y_W direction, see Sketch 1.1	degree	degree
$\varepsilon_B, \varepsilon_F, \varepsilon_N$	upwash angle induced by body, fuselage, nacelle	degree	degree
ε_F^{Ni}	fuselage-induced upwash angle on inboard nacelle axis at longitudinal location of point of interest	degree	degree
ε_{Ni}^{No}	upwash angle induced by inboard nacelle on axis of outboard nacelle at longitudinal location of point of interest	degree	degree
ε_T	upwash angle induced by tailplane	degree	degree
ε_W	upwash angle induced by wing	degree	degree
ε_W^F	upwash angle induced by wing on fuselage centre line at longitudinal location of point of interest	degree	degree
$\varepsilon_W^{Ni}, \varepsilon_W^{No}$	upwash angle induced by wing on nacelle axis for inboard, outboard nacelle at longitudinal location of point of interest	degree	degree
ε_{W1}	wing-induced upwash angle for wing represented by single swept horseshoe vortex, see Appendix A	degree	degree
η_{Wv}	$\cos(v\pi/8)$		
θ	body polar coordinate relative to arbitrary point P , see Sketch 3.2b	degree	degree
θ_t	value of θ at body trailing edge, see Sketch 3.2b	degree	degree
θ_{max}	value of θ at $r = r_{max}$	degree	degree
θ_0	value of θ at body leading edge, see Sketch 3.2b	degree	degree

θ_β	equivalent value of θ for compressible flow, see Equation (3.10)	degree	degree
$\theta_{\beta t}$	value of θ_β at body trailing edge	degree	degree
$\theta_{\beta 0}$	value of θ_β at body leading edge	degree	degree
Λ_0	sweepback of wing leading edge	degree	degree
$\Lambda_{1/4}$	sweepback of wing $1/4$ -chord line	degree	degree
$\Lambda_{\beta 1/4}$	equivalent value of $\Lambda_{1/4}$ for compressible flow, $\tan^{-1}[(\tan \Lambda_{1/4})/\beta]$	degree	degree
λ	wing taper ratio, tip chord/centre-line chord		
v	integer defining specific point on $3/4$ -chord line where boundary condition of no flow through wing is applied		
ξ_B, η_B, ζ_B ξ_F, η_F, ζ_F ξ_N, η_N, ζ_N	dimensionless coordinates for body $x_B/r_{max}, y_B/r_{max}, z_B/r_{max}$, and similarly for fuselage/nacelle		
ξ'_B, ξ'_F, ξ'_N	dimensionless longitudinal coordinate for body, x'_B/r_{max} and similarly for fuselage or nacelle		
$\xi'_{B0}, \xi'_{F0}, \xi'_{N0}$	value of ξ'_B for body leading edge and similarly for fuselage and nacelle		
$\xi'_{\beta B}, \xi'_{\beta F}, \xi'_{\beta N}$	equivalent value of ξ'_B (or ξ'_F or ξ'_N) in compressible flow, as given in Equation (3.11) for body		
ξ_W, η_W, ζ_W	dimensionless coordinates for wing, $x_W/s, y_W/s, z_W/s$		
$\xi_{\beta W}$	equivalent value of ξ_W in compressible flow, ξ_W/β		
Ω	angular position about body axis, measured anticlockwise from upper vertical as viewed from front, see Sketch 3.2b	degree	degree

Subscripts

B	denotes value for body
F	denotes value for fuselage
j	denotes axial location of body segment; $j = 0$ denotes leading edge, $j = t$ denotes trailing edge
N	denotes value for nacelle

t	denotes body trailing edge
β	denotes equivalent value for compressible flow
ζ_{W0}	denotes value for $\zeta_W = 0$



Sketch 1.1 Definition of upwash angle

2. INTRODUCTION

This Item provides a simple theoretically-based method for predicting the airframe-induced upwash at any point in the flowfield ahead of the wing at subsonic speeds. The airframe can be considered to consist of “wing-like”, essentially planar, lifting components, and “body-like”, essentially non-lifting, components. The method, detailed in Section 3, consists of a means of predicting the upwash due to these two classes of airframe component in isolation, together with an approximate means whereby account is taken of interference effects between the various components. The method is therefore capable of application to a wide range of airframe configurations.

The method, commonly called the “Yaggy-Rogallo” method, although originally developed for application to the propeller vibration and flutter problem (Derivation 4), is applicable to any area requiring a knowledge of upwash flow fields. Such areas could include jet engine inlet flows, the local flow around canards, boom-mounted sensors and fuselage-mounted or underwing-mounted stores, for example.

The applicability and accuracy of the method are discussed in Section 4. In particular, Section 4.2.3 describes two extremes of application for which the method has already been used. The Derivation and References are given in Section 5 and worked examples illustrating two of the many possible uses of the method are provided in Section 6. Material supporting the implementation of the method is given in Appendices A, B and D. Details of a computer program for the method, ESDUpac A9020, are given in Appendix C.

3. AIRFRAME-INDUCED UPWASH

3.1 General

The airframe can be considered to consist of a combination of components which can be classed as either “wing-like” or “body-like”. The former are essentially planar lifting components such as wings, tailplanes or foreplanes (canards), while the latter are essentially non-lifting components such as fuselages, nacelles or externally-mounted stores.

Methods of predicting the upwash ahead of wing-like and body-like components are given in Sections 3.2 and 3.3 respectively. In order to illustrate the way in which the upwash contributions arising from the various airframe components are combined, with approximate allowance for interference effects where necessary, the case of an aircraft with a conventional configuration having a wing and tailplane with wing-mounted powerplants will be used. However, the same basic principles can be applied to any other configuration of interest. For the proposed configuration the total upwash angle at some point in the flow field ahead of the wing is given by

$$\varepsilon = \varepsilon_W + \varepsilon_T + \varepsilon_F + \Sigma \varepsilon_N, \quad (3.1)$$

where ε_W and ε_T are the wing-induced and tailplane-induced upwash angles, ε_F is the fuselage-induced upwash angle and $\Sigma \varepsilon_N$ is the sum of the nacelle-induced upwash angles.

Since upwash ahead of a wing decays quite rapidly with increasing distance from the contributing source (roughly an inverse square law) some of the upwash contributions will be of negligible significance compared with those of the major contributors. One obvious example in the proposed configuration is the contribution of the tailplane, ε_T , which is likely to be negligible compared with that from the wing, ε_W . Likewise, if the point at which total upwash is required is well ahead of the nacelles then some, or all, of the contributions to $\Sigma \varepsilon_N$ might also be insignificant. Each situation has to be treated on its own merits, but if in doubt all contributions should be considered.

3.1.1 Interference effects

A further complication is introduced by the possible need to consider interference effects caused by the influence of the upwash field of one airframe component on adjacent components. To determine ε , the total angle of attack of each airframe component must be known. The total angle of attack of each component is the sum of its geometric angle of attack and the angle induced by the upwash of other components. On the face of it, therefore, the determination of the exact component angle of attack is an iterative process. In practice, however, a first approximation is found to be sufficiently accurate. The points made earlier, with respect to the relative significance of the upwash contributions from various components, also hold true with respect to the decision as to whether to consider interference effects or not.

In order to explain how interference is dealt with, an important case requiring corrections for interference will be considered, i.e. that of the upwash flow field just ahead of a nacelle in the multi-nacelle configuration to which Equation (3.1) is applicable.

For the wing, ε_W is adequately predicted by considering only an isolated wing at its geometric angle of attack. It was found in Derivation 2 that although nacelles and fuselages had a significant influence on the wing spanwise loading locally, their influence on upwash was minimal.

For the fuselage or nacelles in the proposed problem situation, however, the upwash induced by the wing cannot be ignored in the determination of their total angle of attack, nor in general can that induced by other adjacent bodies. An approximate general rule is that any body which extends ahead of “the point of interest”, *i.e.* that point at which airframe-induced upwash is required, can have a significant influence on the angle of attack of any body directly aft of that point, depending on their relative proximity (both longitudinal and lateral). As an example, the interference effect of the wing on the fuselage is taken into account by defining an effective angle of attack for the fuselage, α_F , given by the sum of the fuselage geometric angle of attack, α_{gF} , and the value of the wing upwash angle, ε_W^F , on the fuselage axis at the longitudinal location of the point of interest, *i.e.*

$$\alpha_F = \alpha_{gF} + \varepsilon_W^F. \quad (3.2)$$

The point of interest here is in the propeller disc plane, and so ε_W^F would be calculated on the fuselage axis at the longitudinal location of the propeller disc plane.

If an inboard nacelle is relatively large it might be necessary to add its upwash effect to that of the wing in Equation (3.2). However, it is usually sufficient to consider only the wing effect.

The definition of the interference correction for an airframe component such as that for the wing effect on the fuselage, ε_W^F , is that adopted by Yaggy and Rogallo in Derivation 4. It is seen as an approximation which is of adequate accuracy for the present purpose.

For the inboard nacelle of a four-engined configuration (or the nacelle of a twin-engined configuration) the effective angle of attack in the proposed problem situation would be

$$\alpha_{Ni} = \alpha_{gNi} + \varepsilon_W^{Ni} + \varepsilon_F^{Ni}, \quad (3.3)$$

in which the upwash angle due to the fuselage on the nacelle axis at the longitudinal location of the point of interest, ε_F^{Ni} , as well as that due to the wing, ε_W^{Ni} , would need to be accounted for. The outboard nacelle of a four-engined configuration would usually be unimportant in its effect on α_{Ni} , for a swept wing at least, since it would then lie wholly aft of the point of interest.

For the outboard nacelle of a four-engined configuration the appropriate angle of attack would be

$$\alpha_{No} = \alpha_{gNo} + \varepsilon_W^{No} + \varepsilon_{Ni}^{No}, \quad (3.4)$$

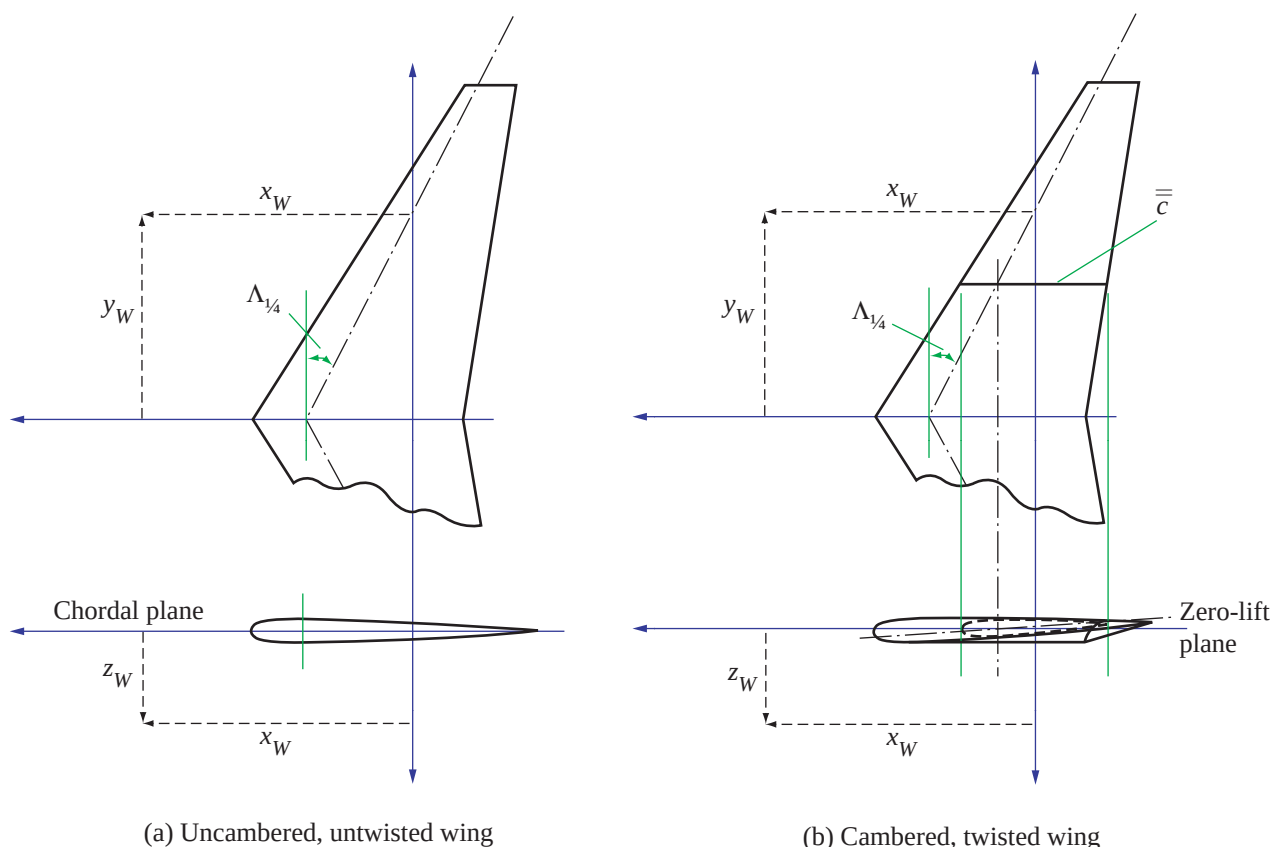
in which, in addition to the appropriate wing contribution ε_W^{No} , the upwash angle due to the inboard nacelle on the axis of the outboard nacelle at the longitudinal location of the point of interest, ε_{Ni}^{No} , would usually be of importance, for a swept wing at least, since it would lie wholly forward of the point of interest. The fuselage would usually be too remote, laterally, to have a significant effect on an outboard nacelle. Appendix B gives details of a method for estimating the effective angles of attack of the fuselage and nacelles. The method, which is used in the program of Appendix C, applies the principles of this section but includes all interference components irrespective of their magnitudes.

3.2 Wing-induced Upwash

Section 3.2.1 outlines a theoretical method for predicting wing upwash that can be applied to a wide range of straight-tapered wings. However, it is rather time consuming and an approximate method that is simpler and quicker to use, with a negligible loss of accuracy for most wing planforms, is given in Section 3.2.2. It is this simplified method that is being adopted here.

3.2.1 Full method

A method, based on extended lifting-line theory, is provided in Derivation 2 whereby the upwash distribution may be obtained in the chordal plane ahead of the quarter-chord line and between the wing tips of any straight-tapered uncambered, untwisted wing (see Sketch 3.1(a)) using spanwise loading and a set of influence coefficients, a_{vm} .



Sketch 3.1 Wing axis system

In terms of four spanwise points m , the upwash angle at any point ξ_W , η_{Wv} in the chordal plane ($\zeta_W = 0$) is given, for subsonic compressible flow, as

$$\frac{A(\varepsilon_W)_{\zeta W 0}}{C_L} = \frac{90}{\pi} \sum_{m=1}^4 a_{vm} \left(\frac{C_{LL} c}{C_L \bar{c}} \right)_m, \quad (3.5)$$

where A is the wing aspect ratio and C_L is the lift coefficient. The dimensionless longitudinal coordinate ξ_W is the distance ahead of the wing quarter-chord line at lateral station η_W . The corresponding height coordinate is ζ_W .

Derivation 2 provides graphs whereby a_{vm} may be established for $v, m = 1$ to 4 in terms of $\Lambda_{\beta 1/4} (= \tan^{-1}[(\tan \Lambda_{1/4})/\beta])$ up to 70° and $\xi_{\beta W} (= \xi_W/\beta) = 0.15$ and 0.2 to 0.7 in steps of 0.1 . The specific stations at which upwash may be calculated correspond to $v = 1$ to 4 or $\eta_{Wv} = \cos(v\pi/8) = 0.924, 0.707, 0.383$ and 0 respectively. The first-order effects of compressibility are accounted for via the parameter β in the determination of a_{vm} . The effects of compressibility on spanwise loading are quite small and it is

satisfactory to assume incompressible flow for that purpose. The spanwise loading $(C_{LL}c/C_L\bar{c})_m$ in Equation (3.5) can be obtained from Item No. 83040 (Reference 17), which can easily deal with the effects of compressibility if so desired.

If the variation of wing lift coefficient can be assumed to be linear with angle attack, then Equation (3.5) can be written in the alternative form

$$\frac{A(\varepsilon_W)_{\zeta W0}}{\alpha_W - \alpha_{W0}} = \frac{90}{\pi} a_1 \sum_{m=1}^4 a_{vm} \left(\frac{C_{LL}c}{C_L\bar{c}} \right)_m, \quad (3.6)$$

where α_W and α_{W0} are respectively the wing angle of attack and wing angle of attack at $C_L = 0$ and a_1 is the wing lift-curve slope. Values of α_{W0} and a_1 may be obtained from Item Nos 87031 (Reference 18) and 70011 (Reference 13) respectively.

Equation (3.5) is based on extended lifting-line theory which accounts only for planform effects. However, the effects of camber and twist can be incorporated into the spanwise loading using Reference 17, taking as the reference plane, $\zeta_W = 0$, the zero-lift plane, passing through the quarter-chord point of the aerodynamic mean chord as shown in Sketch 3.1(b).

Equation (3.5) is not applicable to wings with high-lift devices deployed.

3.2.2 Approximate method

Although Equation (3.5) can be used to calculate $(\varepsilon_W)_{\zeta W0}$ its use is somewhat time consuming. It was shown in Derivation 6 that for $\eta_W = 0$ it is possible to correlate calculated data for $A(\varepsilon_W)_{\zeta W0}/C_L$ in terms of $\xi_{\beta W}$ and $\Lambda_{\beta^{1/4}}$ only, albeit with a certain amount of scatter attributable to A and λ (taper ratio). Further correlation work along these lines was carried out at ESDU. Spanwise loading data (Derivation 5) for a wide range of straight-tapered wings computed using lifting-surface theory and the influence coefficients from Derivation 2 were used to calculate $A(\varepsilon_W)_{\zeta W0}/C_L$ from Equation (3.5). It was found that for $\lambda = 0.25$, at a given value of $\xi_{\beta W}$, the data correlated excellently in terms of $\Lambda_{\beta^{1/4}}$ with no dependency on A for the range $2 \leq A \leq 10$. However, cross-plots against $\xi_{\beta W}$ for a given $\Lambda_{\beta^{1/4}}$ revealed small inconsistencies similar to those found in the work of Derivation 6. With these inconsistencies faired out the resulting correlations are given in Figures 1a to 1d for $|\eta_W| = 0, 0.383, 0.707$ and 0.924 respectively. For intermediate values of $|\eta_W|$ a cross-plot will be required. If Figures 1a to 1d are used for λ values much different from 0.25 there are small errors dependent on $\xi_{\beta W}$, $|\eta_W|$ and A , see Section 4.2.1.

The effect of height ζ_W on wing upwash is dealt with by the application of a factor $F_{\zeta W}$, obtained from Derivation 3 and based on calculations for a single swept horseshoe vortex (Derivation 1), i.e.

$$\varepsilon_W = F_{\zeta W}(\varepsilon_W)_{\zeta W0} \quad (3.7)$$

where

$$F_{\zeta W} = \frac{A\varepsilon_{W1}/C_L}{A(\varepsilon_{W1})_{\zeta W0}/C_L}. \quad (3.8)$$

The lengthy expression for $A\varepsilon_{W1}/C_L$, yielding the upwash angle due to a wing as represented by a single swept horseshoe vortex, is given in Appendix A.

The approximation involved in the use of Equation (3.7) is expected to be good for the small values of ζ_W likely to be required ($\zeta_W < 0.1$, say). A graphical presentation of $F_{\zeta W}$, based on Equation (A1.1) in Appendix A is given in Figure 2 consisting of two charts, Figure 2a being a carpet in ζ_W (from 0 to 0.1)

and $\Lambda_{\beta^{1/4}}$ (from 0 to 60°) for a datum value of $\xi_{\beta W} = 0.15$ and Figure 2b being a set of converging guide lines used as a means of correcting the datum value $[F_{\zeta W}]_{0.15}$ to a value corresponding to any value of $\xi_{\beta W} > 0.15$. It should be noted that Figure 2 is based on calculations for $|\eta_W| = 0.5$. From Equation (A1.1) in Appendix A it is seen that $F_{\zeta W}$ is dependent on $|\eta_W|$; however, calculations show that Figure 2 provides reasonable values for $0.2 \leq |\eta_W| \leq 1.0$. For $\Lambda_{\beta^{1/4}} \leq 10^\circ$, Figure 2 may be used over the full range $0 \leq |\eta_W| \leq 1$. Further, for $|\eta_W| \approx 0$, Figure 2 is acceptable for any value of $\Lambda_{\beta^{1/4}}$ provided that the curve for $\Lambda_{\beta^{1/4}} = 0$ is used irrespective of its true value. For all other cases, i.e. for $0 < |\eta_W| < 0.2$ with $\Lambda_{\beta^{1/4}} > 10^\circ$, it is advisable to use Equation (A1.1).

3.3 Body-induced Upwash

The method for predicting body-induced upwash described in this section is applicable to any “body-like” component. Therefore, although it is presented in terms of a general body (subscript B), it is equally applicable to, say, a fuselage or a nacelle, for which the appropriate subscript, F or N , replaces B throughout.

Section 3.3.1 presents an outline of the method, which requires the evaluation of an integral which is unlikely to be solved analytically. Consequently, a numerical method of evaluating the integral is given in Section 3.3.2.

3.3.1 Method

Derivation 6 provides a detailed development of the upwash angle induced at an arbitrary point P by an axisymmetric body at an effective angle of attack α_B at subsonic speeds, see Sketches 3.2a and 3.2b.

In terms of the dimensionless coordinates $\eta_B (= y_B/r_{max})$ and $\zeta_B (= z_B/r_{max})$ the upwash angle at point P is given by

$$\frac{\varepsilon_B}{\alpha_B} = \frac{\eta_B^2 - \zeta_B^2}{2(\eta_B^2 + \zeta_B^2)^2} \int_{\theta_{\beta 0}}^{\theta_{\beta t}} \left(\frac{r}{r_{max}} \right)^2 \sin \theta_{\beta} d\theta_{\beta}, \quad (3.9)$$

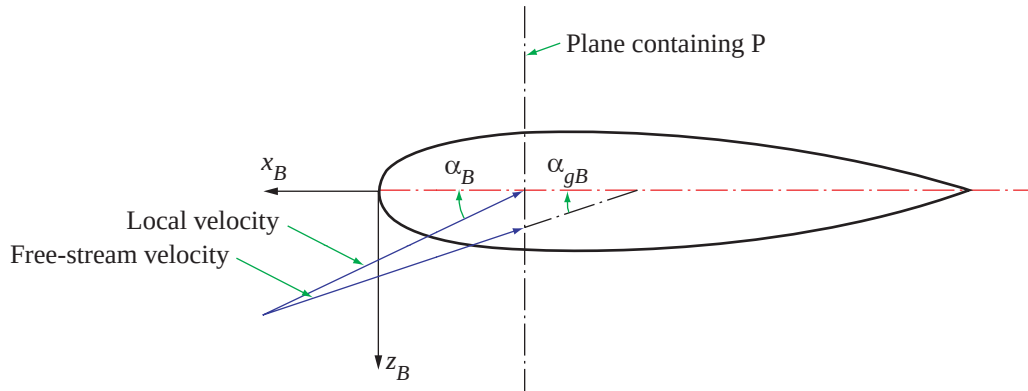
where the use* of

$$\left. \begin{aligned} \theta_{\beta} &= \tan^{-1} \left[(\eta_B^2 + \zeta_B^2)^{1/2} / \xi'_{\beta B} \right] & \text{for } \xi'_{\beta B} \geq 0 \\ \text{and } \theta_{\beta} &= 180 + \tan^{-1} \left[(\eta_B^2 + \zeta_B^2)^{1/2} / \xi'_{\beta B} \right] & \text{for } \xi'_{\beta B} < 0 \end{aligned} \right\} \quad (3.10)$$

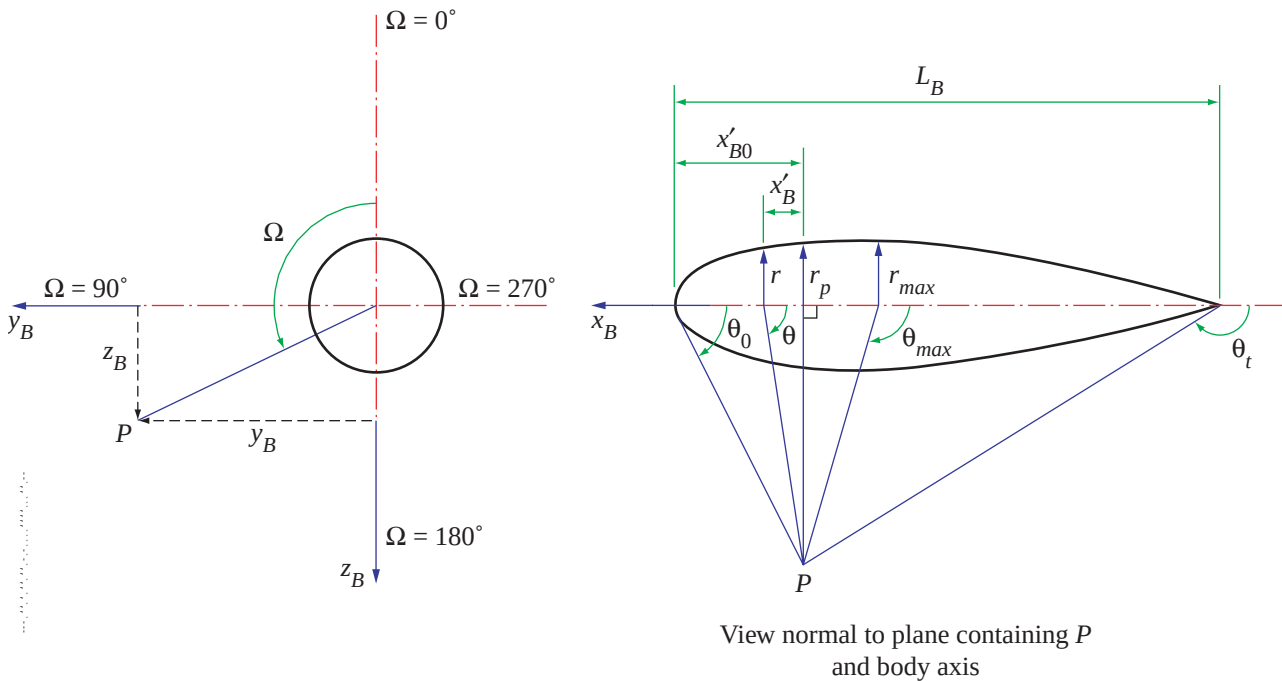
$$\left. \begin{aligned} \text{in which } \xi'_{\beta B} &= \beta \xi'_B & \text{for } \xi'_B \geq 0 \\ \text{and } \xi'_{\beta B} &= \xi'_B / \beta & \text{for } \xi'_B < 0 \end{aligned} \right\} \quad (3.11)$$

is the means whereby allowance is made for compressibility.

* The form of the definition of θ_{β} has been chosen for use with computers that return the principal value of arctan between -90° and $+90^\circ$.



Sketch 3.2a Body angle of attack



Sketch 3.2b Body geometry

In the extreme case of an infinite cylinder, Equation (3.9) reduces to

$$\frac{\varepsilon_B}{\alpha_B} = \frac{\eta_B^2 - \zeta_B^2}{(\eta_B^2 + \zeta_B^2)^2}.$$

In practice this is a reasonable approximation if the point P is such that $\beta_{\xi_{B0}}' \geq 6 r_p/r_{max}$, where r_p is the body local radius at the longitudinal location of P . In such cases (for a fuselage, for example) η_B and ζ_B should be defined as $\eta_B = y_B/r_p$ and $\zeta_B = z_B/r_p$.

3.3.2 Approximate evaluation of Equation (3.9)

The practical evaluation of Equation (3.9) was tried in Derivation 6 using several approximate methods in which the body was divided into a number of segments (t) ranging from 8 to 82. The method eventually adopted assumed that the variation of body cross-sectional area over each segment was linear in $\cot\theta_\beta$ and then summed the contributions from analytical integration. Equation (3.9) then becomes

$$\frac{\varepsilon_B}{\alpha_B} = \frac{\eta_B^2 - \zeta_B^2}{2(\eta_B^2 + \zeta_B^2)^2} \sum_{j=1}^t [K_1(\sin\theta_{\beta j} - \sin\theta_{\beta(j-1)}) + K_2(\cos\theta_{\beta(j-1)} - \cos\theta_{\beta j})] \quad (3.12)$$

$$\text{where} \quad K_1 = \frac{(r_{j-1}/r_{max})^2 - (r_j/r_{max})^2}{\cot\theta_{\beta(j-1)} - \cot\theta_{\beta j}} \quad (3.13)$$

$$\text{and} \quad K_2 = -K_1 \cot\theta_{\beta j} + (r_j/r_{max})^2. \quad (3.14)$$

In these Equations $j-1 = 0$ is the body leading edge and $j = t$ is the body trailing edge. The form of solution given by Equation (3.12) was found in Derivation 6 to be relatively insensitive to the number of segments; with judicious location about 20 segments should suffice, see examples in Sections 6.1 and 6.2. Note that where discontinuities in cross-sectional area occur other than at the body leading or trailing edge (due to engine intakes and exhausts on fuselages for example) the summation in Equation (3.12) would have to be terminated just before and restarted just after each discontinuity with the appropriate values of r/r_{max} .

A non-axisymmetric body must be converted to an equivalent axisymmetric body.

In the early literature, Derivation 6, and in the original issue of this Data Item, an equivalent axisymmetric body of local radius, r_e , was formed so that at any axial station the local cross-sectional area, S , defined r_e as $(S/\pi)^{1/2}$. However, although satisfactory results were obtained, there were no decisive experimental data to support this approach. The theoretical argument presented in Appendix D indicates that an axisymmetric body with r_e equal to half the local planform width in the plane perpendicular to the cross-flow can be expected to give best results, and this is now the recommendation of this Data Item.

4. APPLICABILITY AND ACCURACY

4.1 Applicability

The method of predicting airframe-induced upwash presented in this Item, although originally developed for application to propeller vibration and flutter problems, is applicable to any area requiring a knowledge of upwash flow fields. Such areas could include jet engine inlet flows, the local flow around canards, boom-mounted sensors and fuselage-mounted or wing-mounted stores, for example.

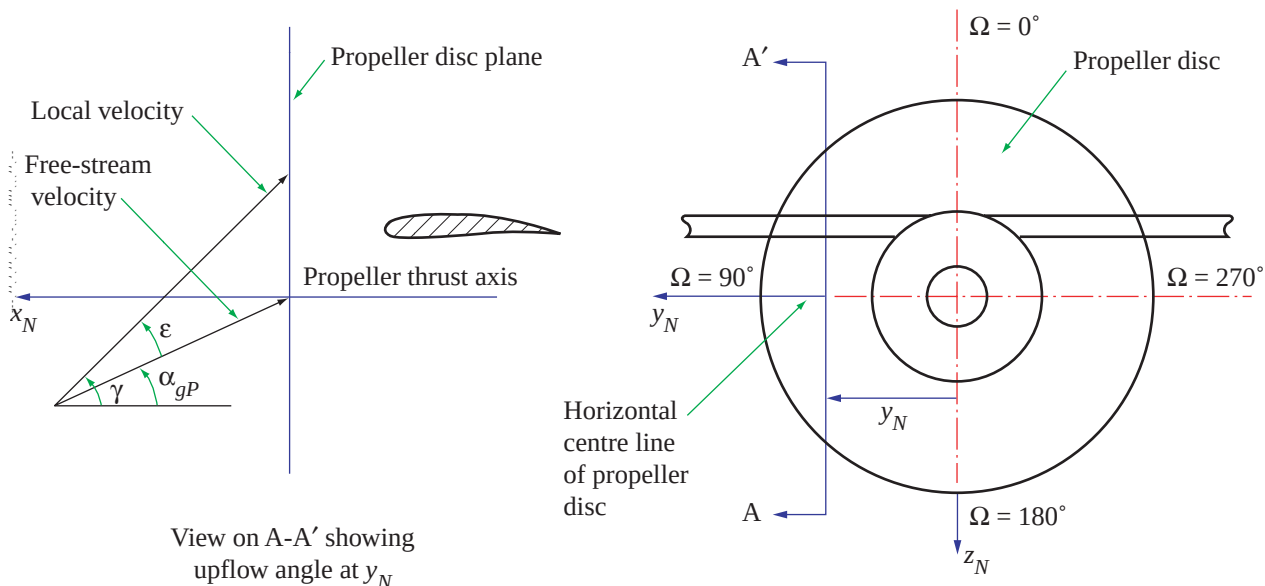
The method is applicable to a wide range of airframe configurations having straight-tapered lifting surfaces. Lifting surfaces that are not straight tapered can be brought within the scope of the method by the use of Item No. 76003 (Reference 15) to construct equivalent straight-tapered planforms. Although the method was developed assuming idealised planar lifting surfaces and axisymmetric bodies, means are provided whereby it can be applied to more practical designs.

4.1.1 Application to propeller normal force and yawing moment

A special application of the method is in the prediction of the effective angle of attack, α_e , of an inclined propeller installed on an airframe, in order to determine the propeller normal force and yawing moment using Item No. 89047 (Reference 19). For this purpose, as for the application to propeller vibration and flutter, the variation of the upflow angle γ , given by

$$\gamma = \alpha_{gP} + \varepsilon \quad (4.1)$$

along the horizontal centre line of the propeller disc, see Sketch 4.1, is required. The propeller geometric angle of attack, α_{gP} , is often the same as that of the nacelle, α_{gN} .



Sketch 4.1 Definition of upflow angle

Following the recommendations in the appendix of Reference 10, α_e for propellers installed in isolated nacelles is taken as the value of γ at 0.7 of the propeller disc radius, i.e.

$$\alpha_e = \gamma_{0.7} \quad (4.2)$$

When the nacelle is installed in an airframe the interference effects of other components are such that asymmetry in γ is introduced across the horizontal centre line of the propeller disc, so that $\gamma_{0.7}$ at $\Omega = 90^\circ$ is no longer equal to that at $\Omega = 270^\circ$. One simple means of allowing for this in determining α_e might be to use the average, $\bar{\gamma}_{0.7}$, of the values of $\gamma_{0.7}$ at $\Omega = 90^\circ$ and 270° , *i.e.*

$$\alpha_e = \bar{\gamma}_{0.7}.$$

In order to check this hypothesis test data for the propeller contribution to normal force and yawing moment in an aircraft model would be required, and no such tests have so far been found.

4.2 Accuracy

4.2.1 Wing-induced upwash

The simplified method for wing-induced upwash prediction, presented in Figures 1a to 1d gives very good agreement with values derived from the full method of Equation (3.5) provided that $\lambda = 0.25$. Use of these Figures for other values of λ involves errors that are greatest for the extremes, $\lambda = 0$ and 1, and at $\xi_{\beta W} = 0.15$ can amount to about $\pm 15\%$ near $|\eta_W| = 0$ or 1 for $A = 10$. The errors decrease with reduction in A and with increase in $\xi_{\beta W}$. However, for $0.25 \leq \lambda \leq 0.5$, which encompasses most practical planforms, the errors are not likely to be much in excess of $\pm 0.5\%$, which is felt to be satisfactory for the present purpose. If a better accuracy is required then use of Equation (3.5) and Derivation 2 as detailed in Section 3.2 is advised.

The data in Figures 1a to 1d have been extrapolated to useful upper limits of $\xi_{\beta W} = 4$ for Figure 1a and $\xi_{\beta W} = 2$ for the others using, as a guide, the trends given by Equation (A1.1) in Appendix A. Further linear extrapolation beyond this value is acceptable. It is inadvisable to attempt extrapolation beyond the lower limit $\xi_{\beta W} = 0.15$.

The effect of variation in ζ_W is discussed in Section 4.2.3.

4.2.2 Body-induced upwash

Predictions using the method of Equation (3.9) have been compared with wind-tunnel test data for a number of bodies in isolation. In Reference 7 the method is compared with test data for upwash angle variation along the horizontal centre line of the propeller disc for three large axisymmetric nacelles at low speed ($M = 0.18$, $R_D = 6.5 \times 10^6$) in the Ames 40 ft $\times$ 80 ft tunnel. The comparisons, for α_{gN} up to 16° , are good (within about $\pm 1^\circ$ on ε_N) even close to the inlet for the two nacelles typical of those commonly used to house radial piston engines; the third nacelle had its inlet blocked with a conical fairing.

Similar comparisons were made in Reference 9 for a slender asymmetric nacelle typical of a turbo-propeller installation, but with no inlet*. The test data were obtained from the Ames 12 ft tunnel over a range of Mach numbers up to 0.92 ($R_D = 1.4 \times 10^6$ at $M = 0.12$, $R_D = 0.7 \times 10^6$ for $M \geq 0.6$). The calculations were made assuming that the lateral plane of symmetry was that of the forebody (ahead of the faired inlet). The original calculations in Reference 9 took no account of compressibility but gave good agreement (to within about ± 0.03) with the test data for $d\varepsilon_N/d\alpha_{gN}$ at three radial locations on the horizontal centre line of the propeller disc, showing substantial independence of M for $M \leq 0.6$. Subsequent calculations allowing for compressibility and using Equation (3.12) predicted the decrease with Mach number above $M = 0.6$ quite well, even up to the extreme of $M = 0.9$.

* For the comparisons made in Reference 9, the asymmetric nacelle was represented as an axisymmetric body with the same distribution of local cross-sectional area from the nose of the spinner up to the point of nacelle maximum cross-sectional area and represented as an infinite untapered circular cylinder aft of that point. See Section 3.3.2 for the current recommendation on modelling of asymmetric bodies.

4.2.3 Airframe-induced upwash

The method of this Item for predicting airframe-induced upwash was originally developed for application to propeller vibration and flutter. Consequently, most of the available test data for airframe upwash relate to that application and involve upwash angle variation along the horizontal centre line of the propeller disc.

Airframe upwash into propellers

The test data in Reference 7 refer to a full-scale twin-radial-piston-engined aircraft with an unswept wing tested at low speed ($M \approx 0.2$, $R_{\infty} = 12 \times 10^6$) in the Ames 40 ft $\times$ 80 ft tunnel. The six large-scale half-models with 40° sweep in Reference 8 cover a combination of two wing aspect ratios ($A = 7.3, 10$) with one, or two, axisymmetric nacelles with variations of nacelle longitudinal and vertical location on the wing and nacelle inclination. The tests were again in the Ames 40 ft $\times$ 80 ft tunnel at $M = 0.13$ ($R_{\infty} = 5.5 \times 10^6$ or 5.8×10^6). Finally, Reference 9 deals with tests in the Ames 12 ft tunnel of a half-model of a four-engined transport aircraft with a swept wing ($A = 10$, $\Lambda_{1/4} = 40^\circ$) designed for high subsonic cruise speed. The wing and the nacelles were fully representative, the wing having camber and twist and employing wing fences; the underslung nacelles were asymmetric, to simulate a typical turbo-propeller powerplant and were inclined downwards relative to the local wing chord to alleviate the effects of propeller vibration in the high-speed cruise condition. The tests covered a range of Mach numbers up to 0.92 at $R_{\infty} = 2 \times 10^6$. The test data from these three References, involving 8 models, can therefore be taken as a fairly comprehensive set with which to compare the prediction method.

The comparisons in the relevant References show that the agreement between prediction and experiment is generally good. The trends, in terms of the spanwise variation of upwash angle, are well represented in each case and the magnitude of the local upwash angle is quite well predicted, bearing in mind the simplicity of the prediction method.

The comparisons for the unswept wing aircraft of Reference 7 are particularly good, while for the swept wing models of Reference 8 at the same angle of attack ($\alpha_{gN} = 10^\circ$) the test data are sometimes overpredicted by about 1° on average. However, a more complete survey of the data of Reference 8, in Derivation 3, shows that the discrepancy for these tests is dependent on angle of attack and decreases roughly linearly with decreasing angle of attack. The comparisons show no variation in accuracy for the models in which nacelle vertical location is varied, implying that the approximate allowance for ζ_W via Equation (3.7) is reasonable.

The comparisons for upwash gradient ($d\varepsilon/d\alpha$) for the four-engined swept wing transport in Reference 9 are good (to within ± 0.03) up to $M = 0.6$, while above that value the prediction method provides increasing overestimates. Calculations in support of this Data Item show that in making an allowance for compressibility on the nacelles (not accounted for by the authors of Reference 9) the prediction is marginally improved. The method for the wing, although it allows for the significant effect of compressibility, could not be expected to account for effects beyond the critical Mach number for the wing. Likewise, the method as a whole could not be expected to provide correct values of upwash angle beyond the critical Mach number for the most sensitive component.

Calibration of angle of attack vane

A rather different application of the method is given in Derivation 6. In that report comparisons* are made between predictions, α_{pred} , for the angle of attack at a noseboom-mounted vane on an F-111A aircraft and true values, α_{true} , from flight test (derived from the inertial navigation and pitot static systems) for three leading-edge sweep angles ($\Lambda_0 = 26^\circ, 35^\circ$ and 58°). The test data were obtained from a number of accelerated and decelerated flights at constant altitude over a range of subsonic Mach numbers.

* For the comparisons made in Reference 6 the fuselage was represented as an axisymmetric body with the same distribution of local cross-sectional area. See Section 3.3.2 for the current recommendation on modelling of asymmetric bodies.

The angle of attack indicated by the vane, α_{ind} , was corrected to remove local flow effects (due to boundary layer, shock wave and other interference effects) via a local flow correction, $\Delta\alpha_{lf}$, obtained from wind-tunnel tests on the boom assembly to give the corrected value,

$$\alpha_{corr} = \alpha_{ind} + \Delta\alpha_{lf}. \quad (4.3)$$

Finally, the corrected value was further modified to allow for wing/fuselage upwash, $\varepsilon_W + \varepsilon_F$, using the method of Figure 1a and Equation (3.12), to give the predicted value,

$$\alpha_{pred} = \alpha_{corr} - (\varepsilon_W + \varepsilon_F). \quad (4.4)$$

In the calculations of Derivation 6 no allowance was made for the small effect of wing height (ζ_W). Neither was any allowance made for interference between the wing and the fuselage, it being considered negligible for the vane location (see also Section 6.1).

The results, shown in Derivation 6 as comparisons between α_{true} and each of α_{ind} , α_{corr} and α_{pred} for each of the three wing sweep angles, show that the successive corrections $\Delta\alpha_{lf}$ and $(\varepsilon_W + \varepsilon_F)$ effectively eradicate the error of about + 8% in α_{ind} for values up to about 14°.

Similar comparisons using the same technique (References 11, 12, 14 and 16) are referred to in Derivation 6.

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. SPREITER, J.R.
SACKS, A.H. The rolling up of the trailing vortex sheet and its effect on the downwash behind wings.
J. Aeronaut. Sci., Vol. 18, No.1, January 1951, pp. 21 to 32.
2. ROGALLO, V.L. Effects of wing sweep on the upwash at the propeller planes of multi-engine airplanes.
NACA tech. Note 2795, 1952.
3. ROGALLO, V.L.
McCLOUD, J.L. Calculation of upwash in the region above or below the wing-chord planes of swept-back wing-fuselage-nacelle combinations.
NACA tech. Note 2894, 1952.
4. ROGALLO, V.L.
YAGGY, P.F.
McCLOUD, J.L. An analysis of once-per-revolution oscillating aerodynamic thrust loads on single-rotation propellers on tractor airplanes at zero yaw.
NACA Rep. 1295, 1955.
5. GARNER, H.C.
INCH, S.M. Subsonic theoretical lift-curve slope, aerodynamic centre and spanwise loading for arbitrary aspect ratio, taper ratio and sweepback.
ARC CP 1137, 1970.
6. RAWLINGS, K. A method of estimating upwash angle at noseboom-mounted vanes.
AFFTC-TIM-81-1, Air Force Flight Test Center, Edwards AFB, CA, 1981.

5.2 References

The References list sources of information supplementary to that given in this Item.

7. YAGGY, P.F. A method for predicting the upwash angles induced at the propeller plane of a combination of bodies with an unswept wing. NACA tech. Note 2528, 1951.
8. ROGALLO, V.L.
McCLOUD, J.L. Surveys of the flow fields at the propeller planes of six 40° sweptback wing-fuselage-nacelle combinations. NACA tech. Note 2957, 1953.
9. LOPEZ, A.E.
DICKSON, J.K. The effects of compressibility on the upwash at the propeller planes of a four-engine tractor airplane configuration having a wing with 40° of sweepback and an aspect ratio of 10. NACA tech. Note 3675, 1953.
10. SUTTON, F.B.
DEMELE, F.A. The effects of operating propellers on the longitudinal characteristics at high subsonic speeds of a four-engine tractor airplane configuration having a wing with 40° of sweepback and an aspect ratio of 10. NACA RM A53J23 (TIL 4054), 1954.
11. HORTON, V.W.
ELDREDGE, R.C.
KLEIN, R.E. Flight-determined low-speed lift and drag characteristics of the light-weight M2-F1 lifting body. NASA tech. Note D-3021, 1965.
12. PYLE, J.S.
SWANSON, R.H. Lift and drag characteristics of the M2-F2 lifting body during subsonic gliding flight. NASA tech. Memor. X-1931, 1967.
13. ESDU Lift-curve slope and aerodynamic centre position of wings in inviscid subsonic flow. Item No. 70011, ESDU International, 1970.
14. PYLE, J.S. Lift and drag characteristics of the HL-10 lifting body during subsonic gliding flight. NASA tech. Note D-6263, 1971.
15. ESDU Geometrical properties of cranked and straight tapered wing planforms. Item No. 76003, ESDU International, 1976.
16. SAKAMOTO, G.M. Aerodynamic characteristics of a vane flow angularity sensor system capable of measuring flight path accelerations for the Mach number range from 0.4 to 2.54. NASA tech. Note D-8242, 1976.
17. ESDU Method for the rapid estimation of spanwise loading of wings with camber and twist in subsonic attached flow. Item No. 83040, ESDU International, 1983.
18. ESDU Wing angle of attack for zero lift at subcritical Mach numbers. Item No. 87031, ESDU International, 1987.
19. ESDU In-plane forces and moments on installed inclined propellers at low forward speeds. Item No. 89047, ESDU International, 1989.

6. EXAMPLES

6.1 Example 1

It is required to calibrate a noseboom-mounted angle of attack vane installed on a combat aircraft as shown in Sketch 6.1. As part of the calibration process (described in Section 4.2.3) estimates are required of the airframe-induced upwash angle at the vane for ranges of aircraft angle of attack and Mach number in steady level flight. Find the upwash angle for one such combination, $\alpha_W = 5^\circ$ * and $M = 0.8$, given that $C_L = 0.39$, $\alpha_{gF} = 5^\circ$, $A = 3.5$, $\Lambda_{1/4} = 45^\circ$ and $\lambda = 0.3$. The contribution of the tailplane will be ignored as being negligible compared to that of the wing.

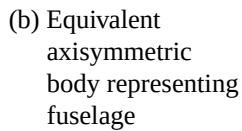
The required geometry is detailed in Sketch 6.1. The fuselage must be represented by an equivalent axisymmetric body. Following the recommendation in Section 3.3.2 the equivalent body has a local radius, r_e , equal to half the local planform width of the fuselage, as set out in Table 6.1 and plotted in Sketch 6.1, and $r_{emax}/L_F = 0.074$.

In order to simplify the problem for the present purpose it will be assumed that (i) the effect of the noseboom itself will have been established in a wind tunnel and (ii) $\Omega = 90^\circ$ for the vane relative to the equivalent axisymmetric body representation of the fuselage. In practice, because of body asymmetry, the value of Ω may require adjustment based on an in-flight calibration (see Derivation 6).

TABLE 6.1 Equivalent axisymmetric body representing fuselage

j	$-x_F/L_F$	r_e/r_{emax}	j	$-x_F/L_F$	r_e/r_{emax}
0	0	0	10	0.34	0.960
1	0.02	0.136	11	0.35	1.000
2	0.04	0.232	12	0.40	0.960
3	0.06	0.304	13	0.46	0.784
4	0.10	0.384	14	0.50	0.720
5	0.16	0.416	15	0.60	0.720
6	0.20	0.416	16	0.67	0.720
7	0.24	0.416	17	0.72	0.840
8	0.28	0.440	18	0.82	0.840
9	0.32	0.448	19	0.86	0.680
10	0.34	0.456	19	0.86	0.320
			20	0.90	0.240
			21	0.96	0.080
			22	1.0	0

* Note that α_W is not in fact required for the calculation since C_L has been given. It has simply been provided for information.



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(i) Wing contribution

The value of β is given by: $\beta = (1 - M^2)^{1/2} = (1 - 0.8^2)^{1/2} = 0.6$.

For the wing: $\Lambda_{\beta 1/4} = \tan^{-1}(\tan \Lambda_{1/4} / \beta) = \tan^{-1}(\tan 45^\circ / 0.6) = 59.04^\circ$.

For the vane location: $\xi_{\beta W} = \xi_W / \beta = 1.95 / 0.6 = 3.2$,

$$\eta_W = 0.03$$

and $\zeta_W = 0.08$.

From Figure 1a for $\eta_W = 0$ (i.e. assuming the effect due to $\eta_W = 0.03$ is small enough to be neglected), with $\Lambda_{\beta 1/4} = 59^\circ$ and $\xi_{\beta W} = 3.2$,

$$A(\varepsilon_W)_{\zeta W 0} / C_L = 0.3 \text{ degrees.}$$

Therefore $(\varepsilon_W)_{\zeta W 0} = 0.3 \times 0.39 / 3.5 = 0.033$ degrees.

Figure 2b shows that for $\xi_{\beta W} = 3.2$, $F_{\zeta W} = 1$,

so that $\varepsilon_W = F_{\zeta W} \times (\varepsilon_W)_{\zeta W 0} = 1 \times 0.033 = 0.033$ degrees.

(ii) Fuselage contribution

For the fuselage contribution the numerical method of Section 3.2.2 will be used with α_F given by Equation (3.2) in which $\varepsilon_W^F (= \varepsilon_W$ from Section 6.1 (i)) = 0.033 degrees, so that

$$\alpha_F = \alpha_{gF} + \varepsilon_W^F = 5 + 0.033 = 5.03 \text{ degrees.}$$

This illustrates that in practice, for this type of application, the wing interference effect on the fuselage angle of attack is negligible as noted in Derivation 6 (see Section 4.2.3).

Equation (3.12) is used with subscript B (for a general body) replaced by F (for the fuselage), $\eta_F = 0.136$ and $\zeta_F = 0$ (since $\Omega = 90^\circ$) and $t = 22$. However, because there are two discontinuities in r_e / r_{emax} , at $j = 10$ and 19 , caused by the powerplant intakes and exits, the summation in Equation (3.12) has to be carried out in three parts, with the breaks at values of r_e / r_{emax} just forward or aft of the discontinuity as appropriate. Thus Equation (3.12) becomes

$$\frac{\varepsilon_F}{\alpha_F} = \frac{1}{2 \times (0.136)^2} \sum_{j=1}^{22} I_j = 27.03 \left[\sum_{j=1}^{10} I_j + \sum_{j=11}^{19} I_j + \sum_{j=20}^{22} I_j \right] \quad (6.1)$$

$$\text{where } I_j = K_1 (\sin \theta_{\beta j} - \sin \theta_{\beta(j-1)}) + K_2 (\cos \theta_{\beta(j-1)} - \cos \theta_{\beta j}), \quad (6.2)$$

$$\text{with } K_1 = \frac{(r_{e(j-1)} / r_{emax})^2 - (r_{ej} / r_{emax})^2}{\cot \theta_{\beta(j-1)} - \cot \theta_{\beta j}} \quad (6.3)$$

$$\text{and } K_2 = -K_1 \cot \theta_{\beta j} + (r_{ej} / r_{emax})^2. \quad (6.4)$$

In evaluating Equation (6.1) θ_β is determined using Equations (3.10) and (3.11) and for $\xi'_F < 0$ taken over the total length of the fuselage, i.e. $((\xi'_{F0} - L_F/r_{emax}) \leq \xi'_F \leq \xi'_{F0})$, in which

$$\xi'_F = \xi_F + \xi'_{F0} = \left(\frac{x_F}{L_F} + \frac{x'_{F0}}{L_F} \right) \frac{L_F}{r_{emax}} \quad (6.5)$$

with $x'_{F0}/L_F = -0.1$ and $r_{emax}/L_F = 0.074$, so that

$$\xi'_F = \left(\frac{x_F}{L_F} - 0.1 \right) 13.5. \quad (6.6)$$

Equations (3.10) and (3.11) become

$$\theta_\beta = 180 + \tan^{-1}(\eta_F/\xi'_{\beta F}) = 180 + \tan^{-1}(0.136/\xi'_{\beta F}) \quad (6.7)$$

$$\text{and } \xi'_{\beta F} = \xi'_F/\beta = \xi'_F/0.6. \quad (6.8)$$

The necessary calculations to evaluate the summation in Equation (6.1) are summarised in Table 6.2.

From Table 6.2 later,

$$\sum_{i=1}^{22} I_j = 1.231 \times 10^{-4} + 0.538 \times 10^{-4} + 0.002 \times 10^{-4} = 1.771 \times 10^{-4},$$

$$\text{so that } \frac{\varepsilon_F}{\alpha_F} = 27.03 \times 1.771 \times 10^{-4} = 4.79 \times 10^{-3}.$$

$$\text{Therefore } \varepsilon_F = 4.79 \times 10^{-3} \times 5.03 = 0.024 \text{ degrees.}$$

The total airframe-induced upwash angle at the angle of attack vane is therefore

$$\varepsilon = \varepsilon_W + \varepsilon_F = 0.033 + 0.024 = 0.057 \text{ degrees.}$$

This is a small correction, but it is consistent with the relatively low angle of attack associated with the high subsonic flight condition considered here. It should also be borne in mind that the effect of the noseboom itself will generally increase the required correction.

TABLE 6.2 Evaluation of $\sum_{j=1}^{22} I_j$ in Equation (6.1)

j	$-x_F/L_F$ Table 6.1	$-\xi_F'$ Equ. (6.6)	$-\xi_{\beta F}'$ Equ. (6.8)	r_e/r_{emax} Table 6.1	θ_{β} (deg) Equ. (6.7)	I_j Equ. (6.2)
0	0	1.350	2.250	0	176.54	—
1	0.02	1.620	2.700	0.136	177.12	0.467×10^{-5}
2	0.04	1.890	3.150	0.232	177.53	1.168×10^{-5}
3	0.06	2.160	3.600	0.304	177.84	1.565×10^{-5}
4	0.10	2.700	4.500	0.384	178.27	2.997×10^{-5}
5	0.16	3.510	5.850	0.416	178.67	2.954×10^{-5}
6	0.20	4.050	6.750	0.416	178.85	1.163×10^{-5}
7	0.24	4.590	7.650	0.416	178.98	0.778×10^{-5}
8	0.28	5.130	8.550	0.440	179.09	0.576×10^{-5}
9	0.32	5.670	9.450	0.448	179.18	0.452×10^{-5}
10	0.34	5.940	9.900	0.456	179.21	0.188×10^{-5}
						$\sum_{j=1}^{10} I_j = 1.231 \times 10^{-4}$
10	0.34	5.940	9.900	0.960	179.21	—
11	0.35	6.075	10.125	1.000	179.23	0.398×10^{-5}
12	0.40	6.750	11.250	0.960	179.31	1.650×10^{-5}
13	0.46	7.560	12.600	0.784	179.38	1.151×10^{-5}
14	0.50	8.100	13.500	0.720	179.42	0.427×10^{-5}
15	0.60	9.450	15.750	0.720	179.51	0.698×10^{-5}
16	0.67	10.395	17.325	0.840	179.55	0.335×10^{-5}
17	0.72	11.070	18.450	0.840	179.58	0.222×10^{-5}
18	0.82	12.420	20.700	0.840	179.62	0.394×10^{-5}
19	0.86	12.960	21.600	0.680	179.64	0.103×10^{-5}
						$\sum_{j=11}^{19} I_j = 0.538 \times 10^{-4}$
19	0.86	12.960	21.600	0.320	179.64	—
20	0.90	13.500	22.500	0.240	179.65	12.502×10^{-8}
21	0.96	14.310	23.850	0.080	179.67	6.485×10^{-8}
22	1	14.850	24.750	0	179.69	0.383×10^{-8}
						$\sum_{j=20}^{22} I_j = 0.002 \times 10^{-4}$