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Structures of non-circular cross section: dynamic response due to vortex shedding

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STRUCTURES OF NON-CIRCULAR CROSS-SECTION: DYNAMIC RESPONSE DUE TO VORTEX SHEDDING

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STRUCTURES OF NON-CIRCULAR CROSS-SECTION: Dynamic Response Due to Vortex Shedding

1. NOTATION AND UNITS

		Units	
		SI	British
a, b	dimensions in along- and across-wind directions respectively (see Sketch 1.2)	m	ft
B	bandwidth parameter [= 0.06 for sharp-edged structures, see Equation (4.7)]	—	—
B_1	off-critical bandwidth parameter [see Equation (4.11)]	—	—
$C_{D\infty}$	section-shape mean drag coefficient appropriate to two-dimensional flow conditions; (mean drag force per unit length)/($\frac{1}{2}\rho V_c^2 b$)	—	—
$C_{L'}$	<i>rms</i> value of fluctuating lift* coefficient due to vortex shedding acting on finite-length structure; (<i>rms</i> lift force per unit length)/($\frac{1}{2}\rho V_c^2 a$)	—	—
$(C_{L'0})_\infty$	value of C_L , for stationary cylinder ($\eta = 0$) corresponding to two-dimensional flow conditions	—	—
$C_{L'j}$	modal value of C_L , referring to vibration in mode j	—	—
c_A	factor accounting for aspect ratio effect on correlation length scale [see Equation (4.5)]	—	—
c_T	factor accounting for effect of turbulence and section shape on correlation length scale [see Equation (4.5)]	—	—
f_{ar}	factor accounting for aspect ratio effect on fluctuating lift coefficient; $C_{L'0}/(C_{L'0})_\infty$	—	—
f_{Ls}	factor accounting for correlation length scale effect on $C_{L'}$	—	—
f_t	response modulation parameter [see Equation (4.8)]	—	—
f_η	factor accounting for amplitude effect on $C_{L'}$	—	—
H	height or length of structure	m	ft
h_R	reference height (see Sketch 1.1)	m	ft

For Footnotes, refer to end of Notation Section.

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I_u	intensity of along-wind component of turbulence at h_R	–	–
L_s	correlation length scale of vortex shedding	m	ft
^xL_u	longitudinal length scale of turbulence	m	ft
M_r	local value of structure mass in lumped-mass representation (see Section 4.2.1)	kg	[†] slug
M_{tot}	total mass of structure	kg	[†] slug
m_j	mode-generalised mass per unit length for mode j	kg/m	[†] slug/ft
N_1, N_2	parameters in mode shape representation for cantilever structures (see Figures 3 and 3)	–	–
n_j	natural frequency of mode j	Hz	Hz
n_s	vortex shedding frequency at reference point R	Hz	Hz
Q_j	mode-generalised across-flow deflection, equal to deflection where $\mu_j = 1$	m	ft
q	local across-flow deflection	m	ft
q_j	local mode-generalised, across-flow deflection; $\mu_j Q_j$	m	ft
R	reference point for flow properties (see Sketch 1.1)	–	–
Re	Reynolds number, Vb/ν	–	–
r	location along structure (see Sketch 1.1)	m	ft
St	Strouhal number (= $n_s b/V$) at reference point R	–	–
V	wind or flow speed at off-critical conditions (see Section 4.2.2)	m/s	ft/s
V_c	wind or flow speed, at reference point R, at which vortex-shedding response is a maximum	m/s	ft/s
$V_{D, T}$	design wind or flow speed averaged over specified period (see Section 4.2.1)	m/s	ft/s
$(\zeta_s)_j$	structural damping ratio in mode j	–	–
$(\zeta_{s0})_j$	value of model structural damping ratio at vary small amplitudes of vibration [see Equation (4.3)]	–	–
$(\zeta'_s)_j$	rate of change of modal structural damping ratio with amplitude [see Equation (4.3)]	–	–

For Footnotes, refer to end of Notation Section.

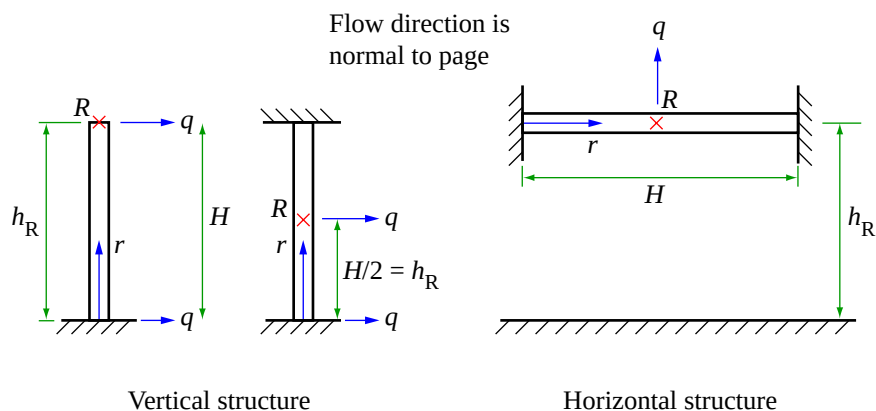
η	non-dimensional deflection; Q_j/b	—	—
μ_j	mode shape function in mode j	—	—
μ_{jr}	value of μ_j at location r along structure	—	—
ρ	air density or density of approaching flow	kg/m ³	†slug/ft ³
ν	kinematic viscosity (for air, $\nu = 1.45 \times 10^{-5}$ m ² /s)	m ² /s	ft ² /s

Subscripts

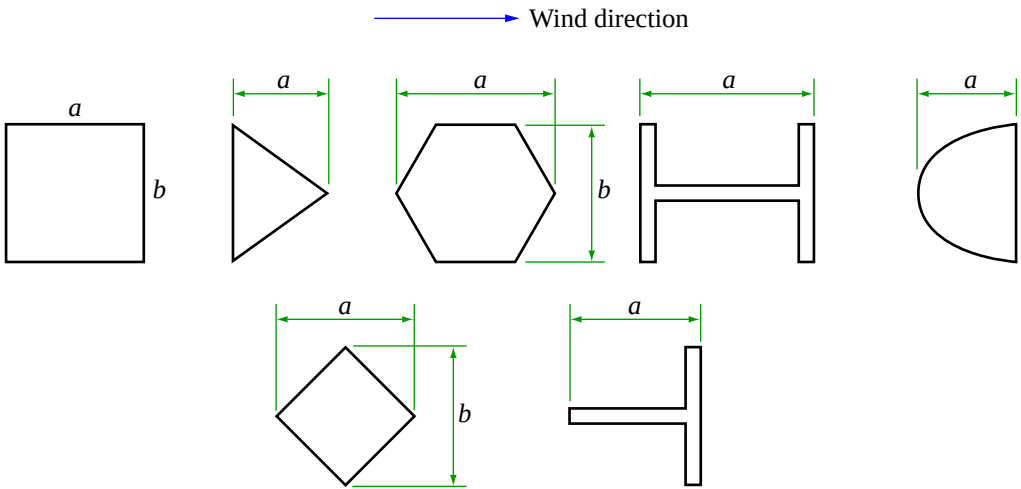
B	refers to broad-band, random-amplitude response
c	refers to value at $V = V_c$
j	refers to mode j
max	refers to expected-maximum value
N	refers to narrow-band, constant amplitude response
rms	refers to root-mean-square value
0	refers to value for stationary cylinder (i.e. $\eta = 0$)
∞	refers to two-dimensional flow conditions (i.e. for structures built-in at both ends or for structures where $H/b \rightarrow \infty$, see footnote to Section 4.2.2)

* Across-flow forces are referred to as lift forces (following the aeronautical convention) but in the context of wind flow their direction of action is normal to the mean wind direction. Therefore lift forces act in the horizontal direction for vertical structures, in the vertical direction for horizontal structures or in a combination of these directions for inclined structures.

† 1 slug = 32.174 lb (pounds mass)



Sketch 1.1 Physical dimensions and axes



Sketch 1.2 Dimension parameters of typical non-circular sections

2. BACKGROUND

2.1 Purpose and Scope

ESDU 90036 is one of a group of Data Items that provide procedures for the calculation of the across-wind response of structures arising from vortex shedding. For structures that are likely to respond dynamically to wind effects, oscillations in the across-wind direction can be particularly severe and are often much greater than those in the along-wind direction. The procedures of this Item are applicable to structures of non-circular cross-section which, within the wind environment, are assumed to be isolated from wind-interference effects due to other nearby structures.

Calculation procedures that are applicable to structures of circular cross-section are given in References 12 and 13. In particular Reference 12 contains background information, also applicable to the methods of this Item, on the nature of the vortex-induced forces and the general derivation of the calculation methods. Because for many sharp-edged structures the separation points of the flow around the structure are assumed to be fixed over the whole range of practical flow conditions, the assumption of Reynolds number independence can be made, leading to a considerable simplification over the methods for circular cylindrical structures.

Due to the nature of the interaction between the fluid and the structure, section shapes can be categorised into three groups, each of which exhibit vortex shedding characteristics to a similar degree. The provisional classification of non-circular section shapes is discussed in Section 3. The calculation procedure is presented in Section 4 together with guidance on the wind and structure parameters. A Calculation Sheet is provided which may be photocopied by the user to provide a permanent record of the calculations. The use of the calculation procedure is illustrated by the worked examples of Section 5 which further illustrate some design criteria relating to wind response calculations. Appendix A describes the derivation of the calculation methods and provides a comparison of measured *rms* deflections with predicted values.

2.2 Applicability, Limitations and Uncertainties

The methods of this Item are applicable to a wide range of structures such as buildings, towers, chimney stacks and structural elements. It is assumed that the structure is isolated, with no flow interference effects due to other structures in the vicinity. The cross-sectional shapes for which this Data Item is applicable are described in Section 3 and illustrated in Table 3.1. The methods are applicable to both uniform and shear flows. In practice, uniform flow conditions relate to horizontally-aligned structures such as beams. Shear flow conditions, in which the flow speed increases with increasing height above the ground, are applicable to cantilever-type structures such as tall buildings or towers.

The methods on which the calculation procedures are based are derived from measurements on parallel-sided structures, with regular non-circular sections aligned symmetrically with the flow. The methods are not directly applicable to sections aligned non-symmetrically to the flow although it is likely that the largest response occurs when the structure is aligned symmetrically. Guidance on the application of the methods to uniformly tapered or step-sided structures is given in Section 4.3. For polygonal shapes with more than about eight sides or shapes with rounded corners, Reynolds number effects can be important and the flow separation points are not fixed. Guidance on the procedure to use for these structures is given in Section 4.4.

As there is a paucity of data with which to fully validate the methods, there are large uncertainties associated with their application. The parameter giving rise to the largest uncertainty is the fluctuating lift coefficient, $(C_{L'}')_{\infty}$. Figure 2 contains data for the estimation of $(C_{L'}')_{\infty}$ and gives best-estimate curves superimposed upon scatter bands representing the available experimental data. To obtain an appreciation of the uncertainties in the methods it is necessary to consider the upper and lower bounds of the scatter

bands in $(C_{L'}^2)_\infty$. The uncertainties are likely to be least for structural shapes that resemble square sections, for which most data are available. The methods have been shown to model well the response of full-scale towers of square and rectangular section using the best-estimate curves. The uncertainty in the response at the critical wind speed is illustrated by the comparison of measured and predicted response for a number of cases in Section A5 of Appendix A.

The methods presented are applicable where the flow is a gas, for example the atmospheric wind.

2.3 Components of Overall Across-wind Response

The overall dynamic across-wind response of a structure includes components due to a number of flow phenomena. For buildings and structural elements of non-circular cross-section this response generally arises from the following mechanisms.

- (i) Vortex shedding, in which the process of shedding vortices alternately from the sides of a structure induces fluctuating across-wind forces on the structure.
- (ii) Galloping, in which above a critical value of flow velocity, self-excited vibrations arise due to structural motion and increase in amplitude with increasing flow velocity. This is an instability phenomenon and is potentially catastrophic, particularly in low turbulence flow.
- (iii) Turbulence buffeting, arising from fluctuations in wind speed and direction.

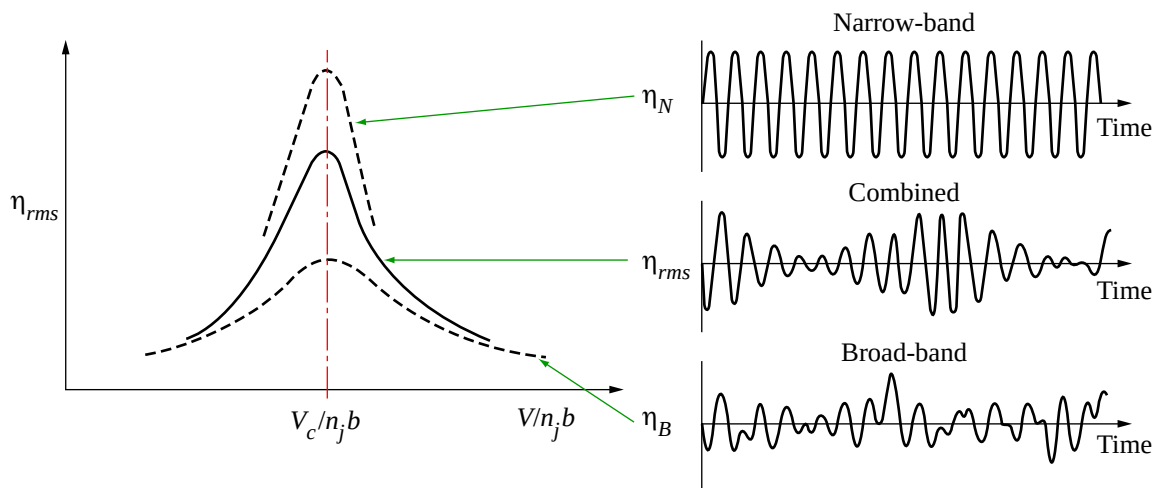
Additionally, bodies in which motion in two or more degrees of freedom can be coupled may be subject to flutter instability. Bodies such as bridge decks are particularly susceptible to this form of excitation if the critical wind speed for flutter instability is within the design wind speed range. Guidance on flutter excitation is given in Reference 2. The response in the across-wind direction due to turbulence buffeting can be estimated using the methods of ESDU 89049¹⁵ and Data Item 91010¹⁷ treats galloping instability of structures.

Vortex shedding and galloping often occur coincidentally as the characteristics of both types of vibration are similarly influenced by the size and shape of the structural section. In particular, significant oscillations due to both of these mechanisms are associated with structural sections which have appreciable afterbodies, relative to the separation points of the shed vortices, but have no flow reattachment (see Section 3). Although interaction between these mechanisms can occur it is very difficult to quantify these effects. It is usual to calculate separately the magnitudes of the responses due to these components (and due to turbulence buffeting) and sum the variances of these values as an approximation. However this procedure is not necessarily conservative and should be treated with caution. Guidance on the interaction between vortex shedding and galloping is given in Derivation 57.

2.4 Vortex Shedding Response and Design Criteria

The process of shedding vortices alternately from the sides of a slender structure causes time-varying* side forces to be exerted on the structure. These forces can lead to significant oscillations (normal to the wind direction) when the frequency of vortex shedding is close to one of the natural frequencies of the structure. The wind speed at which the maximum response occurs is known as the critical wind speed, V_c .

Two distinct types of response are possible, narrow-band and (by comparison) broad-band, as illustrated in Sketch 2.1. At small amplitudes the response is usually random and relatively broad-band. When the oscillation amplitudes are relatively large, the shedding frequency locks into the natural frequency over a range of wind speeds, and the response tends to be constant-amplitude, taking a sinusoidal (narrow-band) form. However, under some circumstances the conditions can be such that a combination of both forms of response can occur. The amplitude may then change (over irregular time periods) from being random and generally small to being sinusoidal and large. The calculation procedure models this behaviour and the predicted response is a mixture of narrow- and broad-band components, combined using a response modulation factor. Guidance on the derivation of this factor (and other parameters in the calculation procedure) is given in Section A2.3 of Appendix A. The nature of vortex shedding is described in detail in Section 4 of Derivation 48.



Sketch 2.1 Illustration of constant amplitude (narrow-band), random amplitude (broad-band) and combined response

When the response is predominantly of the relatively broad-band†, random-amplitude type the oscillation amplitudes are relatively small and usually do not give rise to problems such as fatigue cracking or failure (although fatigue endurance estimates may still be required). However, serviceability criteria may be exceeded in buildings.

* Vortex shedding also causes time-varying forces to be exerted on the structure in the along-wind direction. However, these forces are rarely significant in practical wind loading applications as they are much smaller than, and at twice the frequency of, the across-wind forces.

† The term “broad-band” is used throughout this Item as a convenient reference to the random-amplitude type of response which, although of nearly constant frequency, is more broad-band compared with the constant amplitude, sinusoidal type of response.

When the response is predominantly of the more narrow-band (constant amplitude) type the maximum oscillation amplitudes are usually much larger and in many instances damage, and sometimes failure, occurs through fatigue over a relatively short period of time. Unfortunately full-scale observations of deflection amplitudes in this situation tend to be subjective and unreliable due to limited observation times and difficulties in making accurate assessments from ground level. As a result, comparisons of predicted and observed deflections exhibit considerable scatter (see for example Section 9 of Derivation 48). The problem is made more uncertain since relatively small changes in the assumed parameters and uncertainties in others (e.g. damping) give rise to predictions that can vary by $\pm 100\%$.

For the preceding reasons a practical criterion is to consider a structure design unsatisfactory if a predominantly narrow-band type of response is predicted.

The prediction of a predominantly random-amplitude response does not necessarily mean that the design is satisfactory; there is still a need to calculate fatigue endurance and check serviceability criteria. In order to provide data for fatigue analysis a simple method for estimating the variation of the response at off-critical wind speeds ($V \neq V_c$) is included.

2.5 Methods of Reducing Response

If the calculated response values due to vortex shedding (or any other excitation mechanism) exceed the design or serviceability limits, it is necessary to consider means of reducing these values to acceptable levels. However this is often difficult and expensive in practice, particularly for tall buildings. One of the following remedial measures may be suitable for the purpose of reducing the across-wind response due to vortex shedding.

- (1) Increase the natural frequency of the structure, typically by increasing the stiffness, such that the critical wind speed increases to a value significantly higher than the design wind speed.
- (2) Increase the damping of the structure. Guidance on methods of increasing $(\zeta_s)_j$ of tall buildings is given in ESDU 83009⁹.
- (3) Reduce the aerodynamic excitation by, for example, changing the shape of the building. For tall buildings rounding the corners (even by relatively small amounts) can significantly reduce the across-wind response. Guidance on the calculation of the response for structures with rounded corners is given in Section 4.4. Aerodynamic spoilers such as porous shrouds have been used successfully to reduce the aerodynamic excitation of tall non-circular stacks but these devices are not suitable for buildings. Note also that these devices increase the drag of the structure which will lead to increases in the along-wind response.

3. CLASSIFICATION OF NON-CIRCULAR SECTION SHAPES

The methods of this Item are based upon data obtained for structures with uniform non-circular cross-section along their length. It is assumed that the flow separation points are fixed over the whole range of flow conditions. The alternate shedding of vortices induces fluctuating pressures downstream of the flow separation points, which in turn induce upstream pressure fluctuations which are of smaller magnitude. When the structural section has an appreciable afterbody relative to the flow separation points, the pressure fluctuations act on the afterbody to induce significant across-flow forces on the structure. However, in the absence of such an afterbody, much smaller across-flow forces are generated by the pressure fluctuations upstream of the flow separation points.

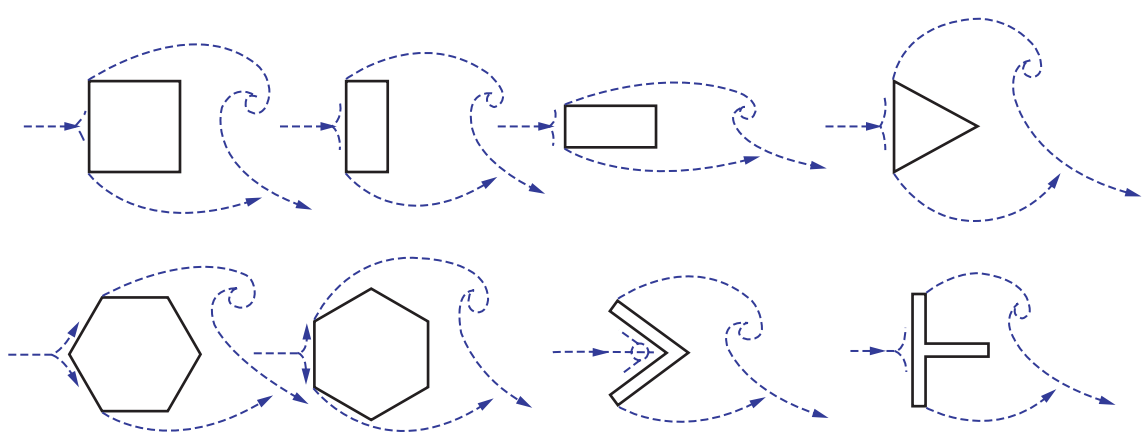
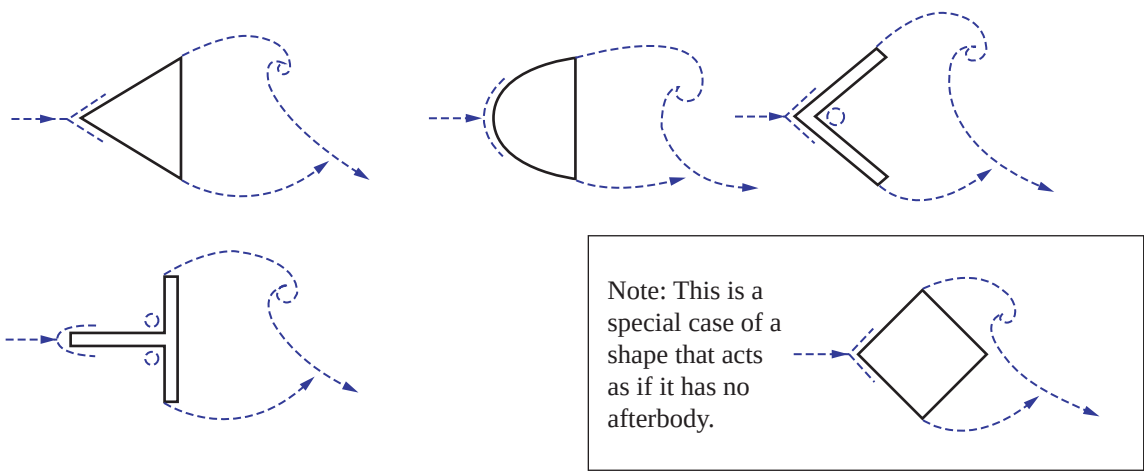
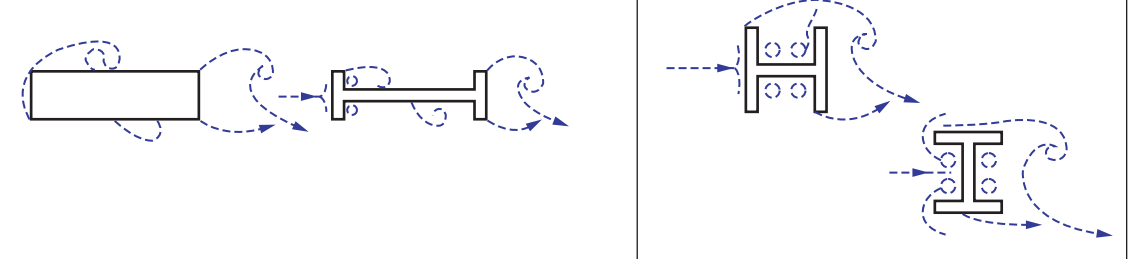
This phenomenon allows section shapes to be classified into three shape-dependent groups as shown in Table 3.1. The classification of a particular section shape into one of those groups depends on the presence or otherwise of an appreciable afterbody within the wake of the flow.

Strong vortex excitation forces are associated with the section shapes typical of Group A, in which an afterbody clearly exists downstream of the separation points and there is no flow reattachment. These shapes are also particularly susceptible to galloping excitation. Weak excitation forces are associated with the section shapes typical of Group B, where there is no appreciable afterbody in the wake. Relatively small-scale forces are induced on the body by pressure fluctuations upstream of the separation points in this group. Group C relates to structures for which the flow separates at the front of the section, reattaches along the sides, and re-separates at the rear of the section, generating shed vortices. However, if the section shape protrudes into the wake behind the points of re-separation it is more likely to lie in Group A (although torsional vibrations may be more significant than lateral vibrations in such a case). Group C also includes some special cases of complex section shapes such as H-sections. For these shapes the regularity of the pressure fluctuations within the separated shear layer is reduced to some degree by the irregular nature of the flow trapped within the section. Examples of shapes that lie in Group C are rectangular sections in which the flow separates at the front corners but reattaches along the sides (typically where a/b is greater than about 2.8). The vortex-induced forces associated with Group C generally lie in magnitude somewhere between those of Groups A and B.

Changes in Reynolds number and turbulence in the approaching flow can lead to flow reattachment for certain shapes, which would lead to a re-classification from Group A to Group C. The definitions described above refer only to translational oscillations due to vortex shedding, and torsional vibration is not considered.

For structural shapes with rounded corners or for polygonal shapes with numerous sides (typically eight or more), the flow separation points are dependent on the flow conditions and the vortex shedding parameters will be Reynolds number dependent. A modified form of the method for circular-cylindrical structures will be applicable in these cases (see Section 4.4).

TABLE 3.1 *Classification of Structural Shapes

TYPICAL SHAPES AND SUGGESTED RANGES OF APPLICABILITY	
GROUP A	$a/b < 2.8^\dagger$ 
GROUP B	 Note: This is a special case of a shape that acts as if it has no afterbody.
GROUP C	$a/b > 2.8^\dagger$  Note: These shapes are special cases due to the breakdown of flow regularity.

* Vortex spacing is not drawn to scale

† These values assume low turbulence intensity. High values of turbulence can lead to flow reattachment, and section shapes can move from Group A to Group C.

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4. CALCULATION OF RESPONSE

4.1 General

The calculation procedure for sharp-edged, parallel-sided structures is given in Section 4.2, and is similar to that of Derivation 48 for circular, cylindrical structures. For most sharp-edged structures the flow separation points are assumed to be fixed over the whole range of practical flow velocities or wind speeds. This allows the assumptions of Reynolds number independence and leads to a considerable simplification of the method.

The calculation procedure is in six main stages as illustrated in Sketch 4.1. The first stage requires the user to estimate the physical dimensions and certain modal properties of the structure, and the properties of the incident flow. The second stage calculates the vortex shedding parameters that are independent of amplitude. The third and fourth stages calculate estimates of the response assuming (i) a narrow-band process and (ii) a broad-band process, taking account of amplitude effects where appropriate. The rms response, η_{rms} , is calculated in the fifth stage and may be either predominately narrow-band, all broad-band or a combination of the two. The expected maximum response value is also calculated in the fifth stage. The off-critical response is calculated in the sixth stage.

Guidance on the estimation of the various parameters of the calculation procedure is provided where appropriate. A calculation sheet on which the results for the procedure can be recorded is given in Section A6. Worked examples of the application of the calculation procedure are given in Section 5. Guidance on response calculations for tapered and step-sided structures is given in Section 4.3, and similar guidance is given for multi-sided polygonal structures and structures with rounded corners in Section 4.4. Guidance on the derivation of the method and associated parameters is given in Appendix A.

For some structures, contributions to the across-wind response due to vortex shedding may originate from more than one mode of vibration. In these cases the value of η_{rms} associated with each mode for which $V_c < V_{D,T}$ should be calculated separately.

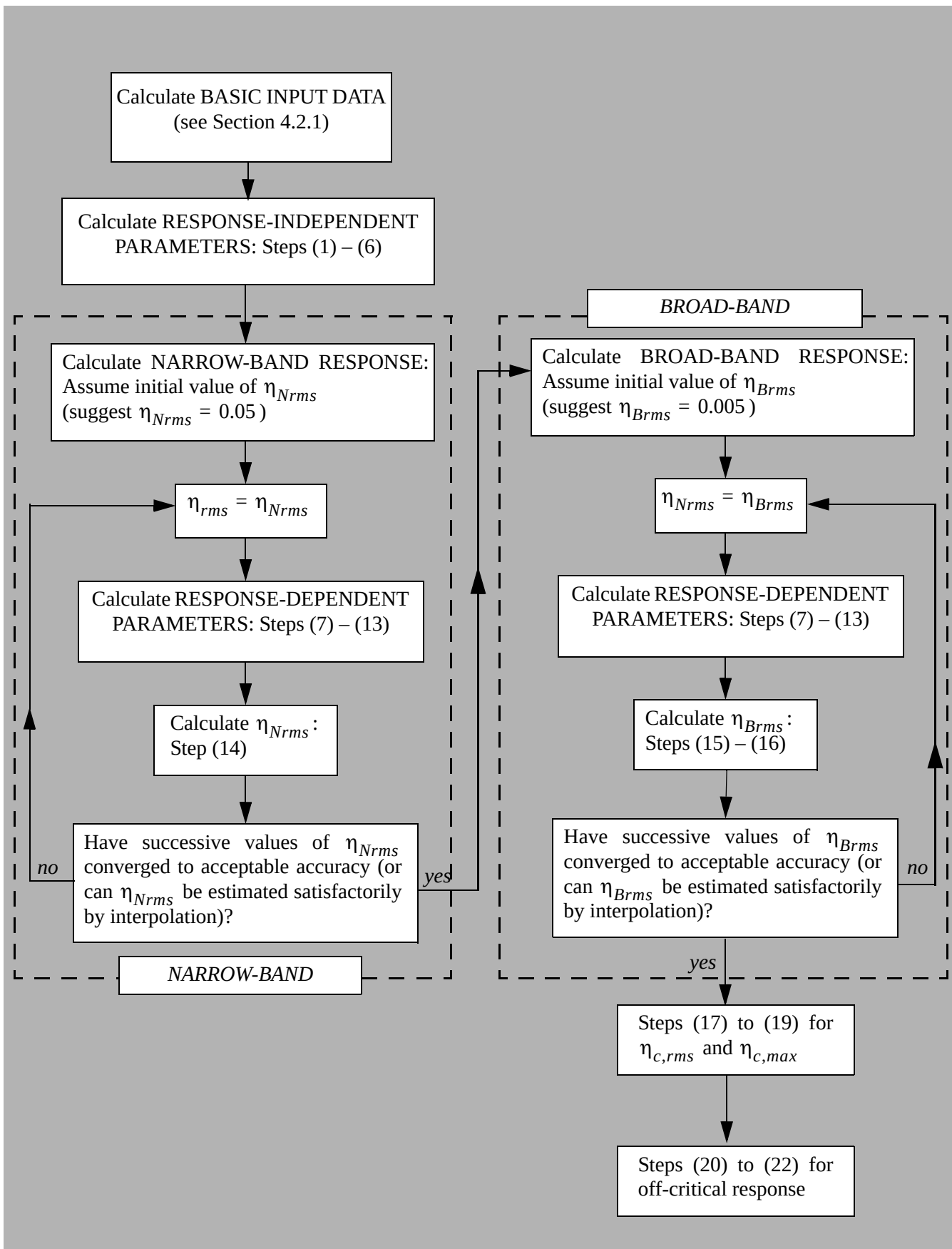
4.2 Response Calculation for Parallel-sided Structures

Response estimates may be recorded on copies of the Calculation Sheet presented in Section A6. Similarity in layout between Sections 4.2.1 and 4.2.2 and the Calculation Sheet allows the user to refer quickly to the appropriate parts of those Sections for guidance.

4.2.1 Basic input data

The input data that are required to characterise the structure and the nature of the wind are listed.

H	Height or length of structure or element (see Sketch 1.1).
a	Section dimension in along-wind direction (see Sketch 1.2).
b	Section dimension in across-wind direction (See Sketch 1.2).



Sketch 4.1 Flow chart for calculation procedure

n_j Undamped natural frequency of the structure in mode j .

Estimates of the translational natural frequencies of shear-, core- and frame-type buildings can be made using the graphical and computer methods of ESDU 79005³, 81036⁶ and 82019⁸ respectively. General guidance on methods for estimating natural frequencies (and mode shapes) of buildings and structural elements is provided in ESDU 91001¹⁷.

μ_j Mode shape function.

The mode shape function μ_j defines the deflected shape of the structure when the structure is oscillating in mode j . This parameter is expressed in a non-dimensional form such that the maximum value of μ_j is unity.

A range of approximate mode shape functions, of the form required by the calculation procedure, are given in ESDU 91001. Mode shapes estimated by using methods such as those provided in ESDU 79005, 81036 and 82019 for buildings should be compared with those of ESDU 91001, in order to determine equivalent values of the parameters N_1 and N_2 (see Figures 3 and 3) to use in the calculation procedure (for first and second modes of vibration only).

m_j Mode-generalised mass per unit length of structure in mode j

The mode-generalised mass per unit length of a structure can be written in several forms. For the calculation of the natural frequencies and associated mode-generalised parameters of building structures it is common to use a lumped-parameter technique in which the mass of the building is assumed to be concentrated at s locations.

For structures with s lumped masses the generalised mass per unit length in mode j is given by

$$m_j = \frac{1}{H} \sum_{r=1}^s M_r \mu_{jr}^2, \quad (4.1)$$

where M_r is the mass of the r th lump and μ_{jr} is the local value of the mode shape parameter. If the mass distribution is uniform along the structure, m_j is given by

$$m_j = \frac{M_{tot}}{H} \int_0^1 \mu_{jr}^2 d(r/H), \quad (4.2)$$

where M_{tot} is the total mass of the structure.

Values of the integral of Equation (4.2) for a number of standard mode shapes are given in ESDU 91001.

$(\zeta_s)_j$ Structural damping ratio in mode j .

Damping of structures depends on many parameters such as the amplitude and frequency of vibration, the method of fabrication and, in buildings, the internal features. For complex structures, methods of predicting the modal structural damping are usually based on full-scale measurements, although measured damping values exhibit considerable scatter and few are reliable. A simple expression for estimating the modal

structural damping ratio, $(\zeta_s)_j$, in the first translational modes of vibration of tall buildings is presented in ESDU 83009⁹, based upon full-scale measurements in which it is possible to identify the effects of parameters such as amplitude. This expression is^{*}

$$(\zeta_s)_j = (\zeta_{so})_j + (\zeta'_s)_j \frac{b\eta_{max}}{H}, \quad (4.3)$$

where $(\zeta_{so})_j$ and $(\zeta'_s)_j$ are the structural damping ratio at very small amplitudes of vibration and the rate of change of structural damping ratio with amplitude, respectively. The appropriate amplitude η_{max} is given by $\eta_{max} = \sqrt{2}\eta_{Nrms}$ for narrow-band excitation and $\eta_{max} = 4\eta_{Brms}$ for broad-band excitation (see Section 4.2.2).

Guidance on the appropriate values of $(\zeta_{so})_j$ and $(\zeta'_s)_j$ for buildings is given in ESDU 83009. However, for uniform prismatic buildings without attached structures at ground level estimates of the values of these two parameters can be made from the expressions tabulated below.

	"Most probable" estimate	"Expected lower limit" estimate
$(\zeta_{so})_j$	$\eta_j/100$	$\eta_j/250$
$(\zeta'_s)_j$	$10^{(0.5\sqrt{a})}$	$10^{(0.4\sqrt{a})}$
Note: dimension a is expressed in metres		

The structural damping of structural members, such as steel beams and columns, and other structures of relatively simple construction tends to be lower and have less amplitude dependency than complex structures such as buildings and bridges. This is because there are fewer connections and building elements at which vibrational energy is dissipated (due to bolt or joint slip or local yielding for example). Single values of $(\zeta_s)_j$, assumed to apply over the whole amplitude range, are usually adequate for structures of this type. Typical values of $(\zeta_s)_j$ for structural members and for other structures of relatively simple construction are given in ESDU 91001.

Because the calculated response is sensitive to the structural damping, $(\zeta_s)_j$, and because this parameter is not known accurately, the calculation procedure should be repeated for a range of values of $(\zeta_s)_j$.

ρ Air (or gas) density.

For air at sea level and 15°C, $\rho = 1.23 \text{ kg/m}^3$.

h_R Height of reference point R.

For a building $h_R = H$ and for a horizontal structure h_R is the height of the structure above the ground (see Sketch 1.1).

I_u Intensity of turbulence at reference point R.

^{*} In ESDU 83009 the equivalent expression $(\zeta_s)_j = (\zeta_{so})_j + (\zeta'_s)_j (x_H/H)$ is used, where x_H is the peak resonant component of the fluctuating displacement and is equivalent to $b\eta_{max}$ at the top of the building.