

DELAMINATION OF TAPERED COMPOSITES

1. THEORY, NUMERICAL RESULTS, DESIGN GUIDELINES

1.1 Introduction

Laminated plates of the type shown in Sketch 1.1 are made from thin orthotropic layers of continuous fibre reinforced composite. The plates may be manufactured with variable thickness by terminating, or dropping-off, some of the plies along the length of the laminate as shown in Sketch 1.2. It is seen that discontinuities are introduced in the form of steps, where the plies are terminated, and this gives rise to high local stresses that may cause the dropped plies to delaminate from adjacent plies. ESDUpac A9103 provides a Fortran computer program for the approximate calculation of the axial load to cause delamination in tapered laminated plates, where the load is applied in the direction of the taper as shown in Sketch 1.2. The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable program for PCs. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

Delamination is predicted using linear elastic fracture mechanics combined with simple strength of materials theory. An equation is obtained, for the applied load to cause delamination, in terms of the local dropped ply geometry, the effective elastic moduli, and the critical strain energy release rate. This latter quantity may be obtained from experiment and a method for doing this is discussed in this Item. Also, some guidelines are given for good design practice.

Delamination in laminated plates has been studied extensively in recent years, but most of the work has been concerned with the well known free edge problem (see Data Item No. 95028*). In that case the three dimensional stresses at the free edge, and ply delamination, can be predicted using a closed form analytical method of solution. Although the free edge problem is similar to that being considered in this Item, the tapered plate was found not to be amenable to the analytical method because of the discontinuity in the stress boundary conditions at the corner of the step. This was why a solution to the present problem was sought using fracture mechanics. It must be emphasised that the solution given here is approximate, and although the theoretical predictions agree quite well with the experimental results given in this Item there may be other lay-up geometries and materials in which it will be found that the agreement between theory and experiment is not so good. This will apply more so to asymmetric tapered plates in compression where instability effects may lead to premature failure. **Thus the predicted delamination loads should only be used as a guide for initial design purposes.** Where possible, verification of final designs should be obtained from tests on specimens that are representative of the actual method of manufacture, materials, geometry and loading.

* Data Item No. 95028 "Delamination and free edge stresses in composite laminates subjected to uniform prescribed axial strain and temperature change".

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

1.2.1 Layer and Plate Properties, and Reference Axes

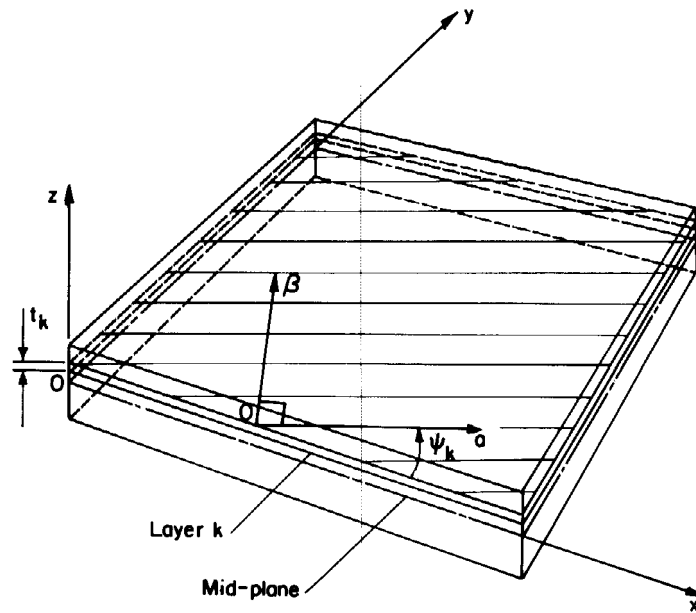
$\mathbf{A}$	in-plane stiffness matrix	N/m	lbf/in
A_{ij}	element of $\mathbf{A}$	N/m	lbf/in
b	plate width	m	in
$E_{\alpha k}, E_{\beta k}$	moduli of elasticity of k th layer in α_k - and β_k -directions respectively	N/m ²	lbf/in ²
$\bar{E}$	effective modulus	N/m ²	lbf/in ²
$\bar{E}_1, \bar{E}_2$	effective moduli for materials 1 and 2 respectively (see Sketch 1.3)	N/m ²	lbf/in ²
$\bar{E}_{12}$	effective modulus for combined materials 1 and 2	N/m ²	lbf/in ²
$G_{\alpha\beta k}$	shear modulus of k th layer in $(\alpha, \beta)_k$ direction	N/m ²	lbf/in ²
h	plate thickness (see Sketch 1.3)	m	in
h_1	thickness of material 1 (see Sketch 1.3)	m	in
h_2	total thickness of material 2 (see Sketch 1.3)	m	in
n	number of delaminating interfaces at same position along taper		
R	reference number for layer type (used in program, see Section 2)		
t_k	thickness of k th layer	m	in
ψ_k	angle of principal axis of orthotropy, α_k , of k th layer measured from plate reference axis, $0x$, as shown in Sketch 1.1	degree	degree
$\nu_{\alpha\beta k}$	Poisson's ratio for k th layer resulting from load applied in α_k -direction with induced strain in β_k -direction		
$0(\alpha, \beta, z)$	principal axes of orthotropy of a layer (see Sketch 1.1)		
$0(x, y, z)$	plate main reference axes (see Sketch 1.1)		

1.2.2 Stresses, Strains and Delamination

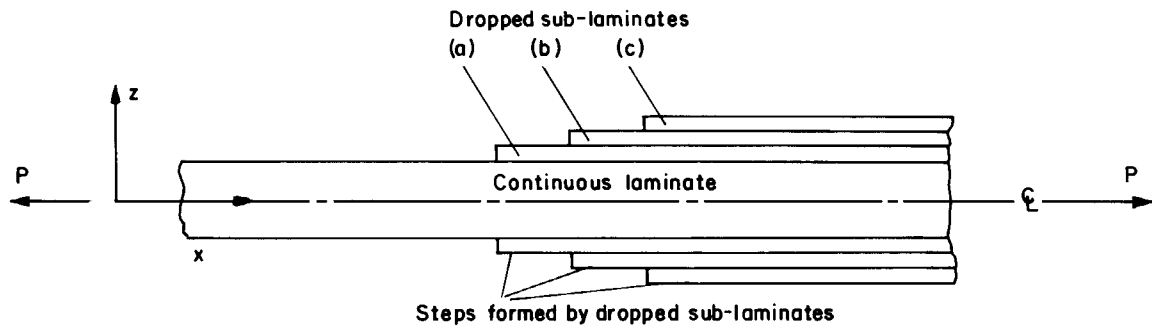
A_d	delamination area	m^2	in^2
a_d	delamination length	m	in
C	compliance	m/N	in/lbf
G_c	critical strain energy release rate	N/m	lbf/in
l	arbitrary length along taper	m	in
$\mathbf{N}$	column matrix of direct load intensities and shear load intensity referred to (x, y) co-ordinates $\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}$	N/m	lbf/in
P	applied load in direction of taper	N	lbf
u	overall displacement in direction of taper	m	in
γ	configuration factor, used in Equation (1.2)		
ϵ	column matrix of mid-plate direct strains and shear strain referred to (x, y) co-ordinates $\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix}$		
$\bar{\sigma}_1, \bar{\sigma}_2$	mean stresses in materials 1 and 2 respectively at thick section of plate (see Sketch 1.3)	N/m^2	lbf/in^2

1.2.3 Suffixes

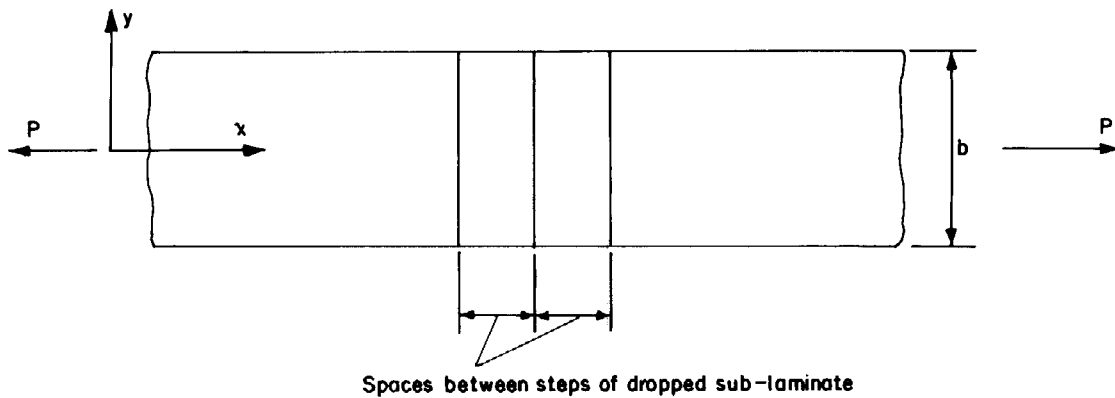
i, j	numbers signifying position
k	layer identification number
x, y	relate to x - and y -directions respectively
α, β	relate to α - and β -directions respectively
$1, 2$	relate to materials 1 or 2 respectively



Sketch 1.1 Laminated plate geometry



(a) Section through tapered plate



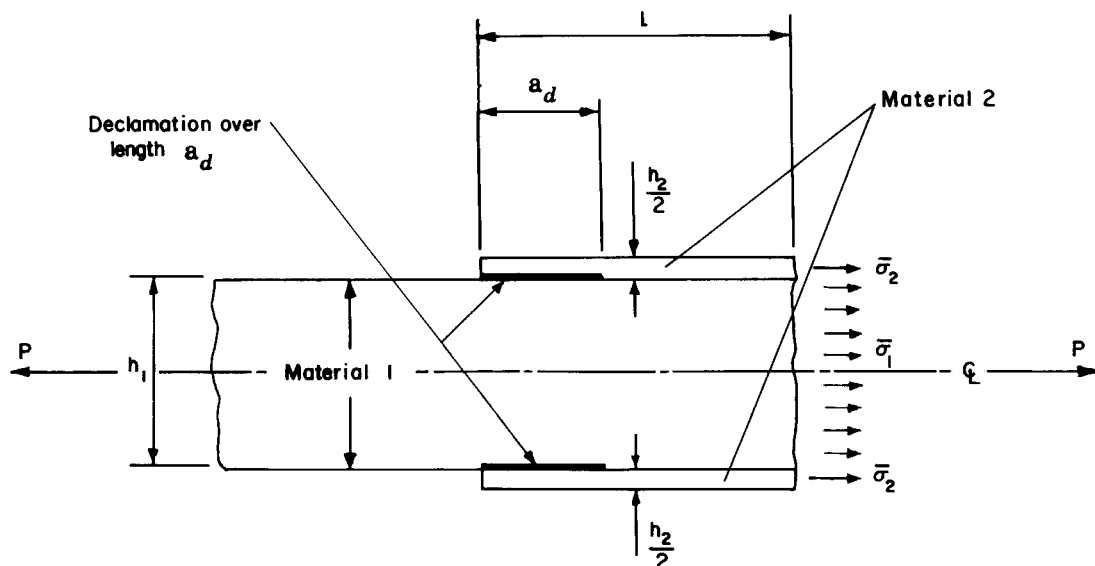
(b) Plan view (on top surface)

Sketch 1.2 Tapered plate with axial load (Configuration No. 1, $n = 2$, $\gamma = 1$, see Section 1.4)

1.3 Theory

A tapered plate subjected to an axial load P , with a number of dropped sub-laminates (a), (b) and (c), is shown in Sketch 1.2. The sub-laminates and continuous laminate may contain any combination of plies with different fibre orientations and material properties. The dropped sub-laminates are assumed to be far enough apart to enable them to be considered in isolation. From finite element analysis studies the steps may be taken to be isolated if they are more than five times the step thickness apart.

Consider delamination at the interface between sub-laminates (a) and (b). Sub-laminate (a) can be combined with the continuous laminate to give one effective material, termed material 1, and the dropped sub-laminate (b) will be called material 2. The problem can now be formulated with the two different effective materials 1 and 2 as shown in Sketch 1.3.



Sketch 1.3 Geometry of a typical dropped sub-laminate

A delaminated length equal to a_d is assumed to be present, and the following standard fracture mechanics equation (see, for example, Derivation 1) may be used to predict the load at which the delamination will spread further.

$$P = \left(\frac{2G_c}{dC/dA_d} \right)^{1/2} \quad (1.1)$$

Using simple strength of materials theory, the rate of change of the compliance C with A_d , namely dC/dA_d , can be obtained in terms of the step geometry and effective elastic modulus for materials 1 and 2. Details of this analysis are given in Appendix A where it is also shown that the mean stress in material 1 required for delamination is given by

$$\bar{\sigma}_1 = \gamma 2\bar{E}_1 \{ G_c h_1 / [\bar{E}_2 h_2 (h_1 + (\bar{E}_2 h_2 / \bar{E}_1))] \}^{1/2} \quad (1.2)$$

and that $N_x = P/b = \bar{\sigma}_1 [h_1 + (h_2 \bar{E}_2 / \bar{E}_1)] \quad (1.3)$

It should be noted that in Equation (1.2) γ is a factor that is dependent on the type of configuration being analysed, and that other configurations are considered in Section 1.4.

The effective moduli $\bar{E}_1$ and $\bar{E}_2$ are for materials 1 and 2, respectively. They are obtained from classical laminated plate theory using the standard load/strain matrix equation in the form (see Data Item No. 94003*)

$$[N]_{3 \times 1} = [A]_{3 \times 3} [\epsilon]_{3 \times 1} \quad (1.4)$$

However, the above equation may be used in two ways depending on the assumptions that are made regarding the applied loads and strains. If the analysis is being carried out for a wide plate with edge support

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

members then it may be assumed that $\varepsilon_y = \varepsilon_{xy} = 0$ giving

$$\bar{E} = A_{11}/h, \quad (1.5)$$

where $\bar{E} = \bar{E}_1$ or $\bar{E}_2$ and $h = h_1$ or h_2 .

This case will be referred to as zero transverse strain.

If a thin strip with free longitudinal edges is being analysed then it may be assumed that $N_y = N_{xy} = 0$ giving

$$\bar{E} = (1/h)[A_{11} + (2A_{12}A_{23}A_{13} - A_{12}^2A_{33} - A_{13}^2A_{22})/(A_{22}A_{33} - A_{23}^2)], \quad (1.6)$$

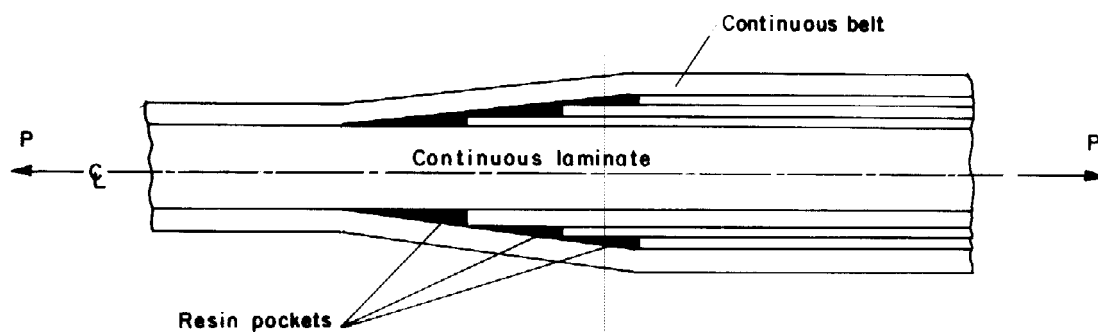
where $\bar{E} = \bar{E}_1$ or $\bar{E}_2$ and $h = h_1$ or h_2 .

This case will be referred to as zero transverse load.

In many cases there is likely to be little difference between the two approaches. The computer program given in Section 2 provides results for both conditions.

1.4 Other Configurations

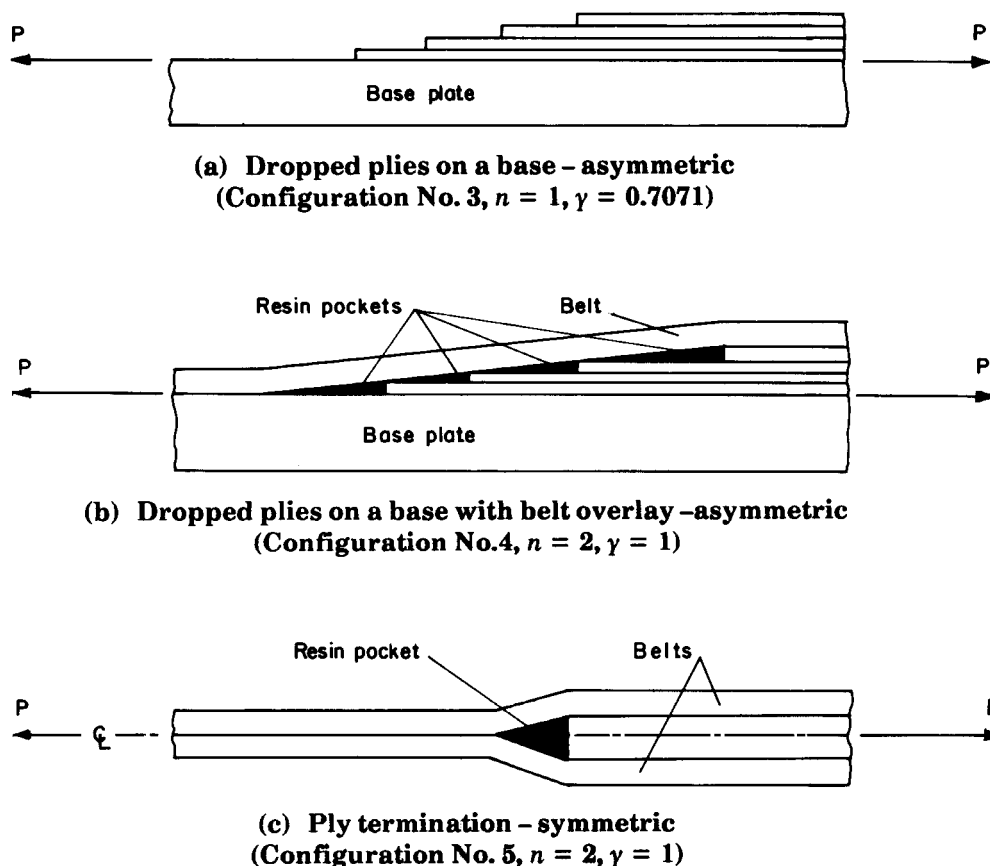
The dropped ply configuration shown in Sketch 1.2 (configuration no. 1) is symmetric about the centre-line of the core. Delamination occurs on each side of the centre-line and the value of γ in Equation (1.2) is unity. If, however, the dropped sub-laminates are covered by an outer continuous laminate called the "belt", then the configuration shown in Sketch 1.4 is generated.



Sketch 1.4 Tapered plate with belt (Configuration No. 2, $n = 4$, $\gamma = 1.414$)

Delamination will again occur above and below the centre-line but this time each dropped sub-laminate is adhered to two surfaces and the conditions for delamination are assumed to be the same at each surface. This results in a stronger bond, and the value for γ is found to be 1.414. Studies of the stress fields in the resin layers above and below the dropped sub-laminates (see Derivation 2) show that the above assumption is not strictly true. However, that analysis uses linear elastic finite elements, and it is expected that in practice the predicted high stress concentrations will be decreased due to plasticity effects in the resin and become more evenly distributed between the top and bottom surfaces, thereby giving similar delamination conditions.

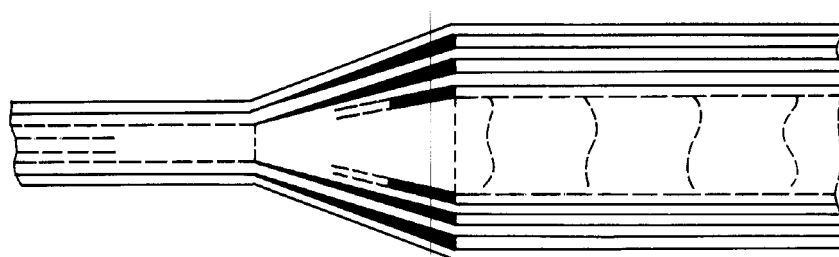
Three other configurations are shown in Sketch 1.5. They are for two asymmetric cases, where the dropped plies and belt are laid upon a base material, and for the special symmetric termination case. The values required for γ are 0.7071, 1 and 1 respectively. In the asymmetric cases no account is taken of the possibility of induced bending and so the analysis is expected to be less accurate than that of the symmetric cases.



Sketch 1.5 Configurations Nos 3, 4 and 5

All of the configurations considered so far have, at the most, two dropped sub-laminates at the same position along the taper, giving a maximum of four delaminating interfaces (see configuration no. 2). However, a much larger number of sub-laminates could be dropped-off at the same position and in this general case (see Sketch 1.6) there can be n delaminating interfaces. In this case, assuming that the surfaces delaminate at the same time and have the same value for G_c , the value of γ in Equation (1.2) can be calculated from the equation

$$\gamma = (n/2)^{1/2}. \quad (1.7)$$



Sketch 1.6 General Case. Configuration No. 0, γ calculated from Equation (1.7)

For a given value of h_2 , and with all other quantities held constant, the value of σ_1 in Equation (1.2) then becomes directly proportional to $n^{1/2}$. This shows that it would be advantageous to use a number of thin dropped sub-laminates interleaved between continuous plies rather than a relatively thick single dropped sub-laminate.

Equation (1.7) should not be used indiscriminately, as the assumption of similar G_c values at each of the interfaces of the dropped sub-laminates would be difficult to justify in cases where the dropped sub-laminates, at the same station, had very different axial stiffnesses. For example, when single dropped 0° and 90° plies are interleaved between continuous plies, the 0° layers would carry the greatest loads and would therefore be expected to delaminate before the 90° layers. Thus material 2 would consist solely of these 0° layers and material 1 would consist of the continuous layers *and* the dropped 90° layers that had not delaminated and that are still considered to be under load. The number of delaminating interfaces, n , would then be equal to only those on each side of the embedded 0° layers and would not include the dropped 90° ply surfaces.

1.5 Determination of the Critical Strain Energy Release Rate, G_c

Equation (1.2) shows that the mean stress, $\bar{\sigma}_1$, for delamination is dependent on the square root of the critical strain energy release rate, G_c . All other terms can be obtained from a knowledge of the taper geometry and the basic material mechanical properties. The most appropriate value to use for G_c is more difficult to obtain. Various investigations have been carried out for different configurations using the finite element method of analysis. These complex studies have shown that both opening and shear modes of failure contribute and that both vary with the length of delamination present. It is suggested that the value of G_c is determined from experiment. This can be done by carrying out a simple axial load/overall deformation test on a specimen that has the configuration shown in Sketch 1.3. If a mechanical extensometer is fixed to the specimen such that the attachment points always encompass the delaminating area as shown in Sketch 1.7(a), then it will be possible to obtain a load/displacement plot as shown by the line OA in Sketch 1.7(b).

During this initial phase, the delamination area will not increase until point A is reached. At this point the magnitude of the load is sufficient to cause the delamination area to increase by a small amount, causing a small increase in the overall deflection u , to point B. If the load is then removed to zero it will follow the line BO and the area OAB will be equal to the energy released by the specimen in causing the delamination to grow. A value for G_c is then obtained by dividing this energy by the increase in delamination area to give the energy released per unit area, or the energy release rate. In practice, it is found that at point A the load usually decreases slightly during delamination, so that the line AB is not horizontal as shown in Sketch 1.7(b). Also, the amount of delamination is sometimes difficult to assess. If glass fibre specimens are being tested, then the area can easily be seen with the naked eye. However, with materials that are opaque, the delamination area will have to be estimated from the amount of crack movement along the edges of the specimen.

The above experimental method will give a direct fundamental measure for G_c . If this measured value is then used in Equations (1.2) and (1.3) to find P for the test specimen, then this particular calculated value for P should ideally correspond with that at point A in Sketch 1.7(b). However, because of the approximations in the theory and experimental errors the two values for P will not usually coincide.

When this arises it is recommended that, in Equation (1.2), a value for G_c is used that will give the lowest of the two values for P .

Some materials may be prone to delaminate catastrophically to beyond the attachment points for the extensometer. In these cases it would not be possible to use the area method to find G_c , and so it would be found by using Equations (1.2) and (1.3).