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Pitching moment derivative due to rate of pitch for projectiles at supersonic speeds



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PITCHING MOMENT DERIVATIVE DUE TO RATE OF PITCH FOR PROJECTILES AT SUPERSONIC SPEEDS

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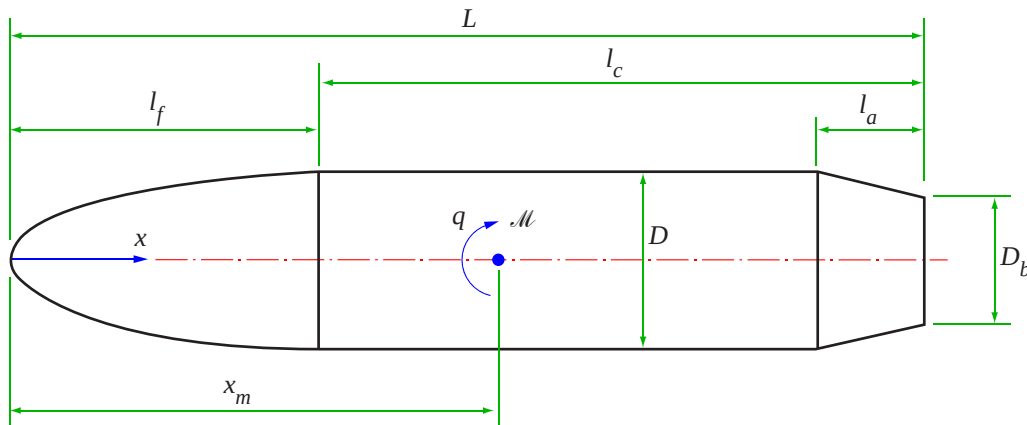
PITCHING MOMENT DERIVATIVE DUE TO RATE OF PITCH FOR PROJECTILES AT SUPERSONIC SPEEDS

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in ESDU 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin on the body centre-line and with the body diameter as the characteristic length. The derivatives M_q , $M_{\dot{w}}$ and Z_q are often written as C_{mq} , $C_{m\dot{\alpha}}$ and C_{zq} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{mq} , $C_{m\dot{\alpha}}$ and C_{zq} differentiation may be carried out with respect to $qD/2V$ and $\dot{\alpha}D/2V$ instead of qD/V and $\dot{\alpha}D/V$ as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

		SI	British
$C_{m\alpha}$	pitching-moment-curve slope of body at zero angle of attack including boundary-layer effects, $(\partial \mathcal{M}/\partial \alpha)/(\frac{1}{2}\rho_{\infty} V_{\infty}^2 S_{ref} D)$ see ESDU 89008 ²⁴ and ESDU 87033 ²³	rad ⁻¹	rad ⁻¹
$C_{N\alpha}$	normal-force-curve slope of body at zero angle of attack, $(\partial N/\partial \alpha)/(\frac{1}{2}\rho_{\infty} V_{\infty}^2 S_{ref})$	rad ⁻¹	rad ⁻¹
$C_{N\alpha l}$	normal-force-curve slope at zero angle of attack for long bodies with boundary-layer effects removed, see ESDU 89008 ²⁴	rad ⁻¹	rad ⁻¹
$\Delta C_{N\alpha}$	change in normal-force-curve slope at zero angle of attack due to conical boat-tail, see ESDU 87033 ²³	rad ⁻¹	rad ⁻¹
C_V	volume coefficient of body, $V/(S_{ref} L)$, see ESDU 77028 ²¹		
D	body maximum diameter	m	ft
D_b	diameter of body base	m	ft
F_1, F_2	empirical factors, see Section 3		
G	factor to allow for forebody shape, see ESDU 89008 ²⁴		
k	factor on $C_{N\alpha l}$ to account for length of body aft of forebody, see ESDU 89008 ²⁴		
L	body length	m	ft
l_a	boat-tail length	m	ft
l_c	body length aft of forebody	m	ft
l_f	forebody length	m	ft
$\mathcal{M}$	pitching moment	Nm	lbf ft

M_∞	free-stream Mach number		
M_q	pitching moment derivative due to rate of pitch, ($\partial \mathcal{M} / \partial q$) / ($\frac{1}{2} \rho_\infty V_\infty S_{ref} D^2$)		
$M_{\dot{w}}$	pitching moment derivative due to acceleration in heave, ($\partial \mathcal{M} / \partial \dot{w}$) / ($\frac{1}{2} \rho_\infty S_{ref} D^2$)		
N	normal force	N	lbf
q	pitch rate	rad/s	rad/s
R_L	Reynolds number based on body length		
$S(L)$	cross-sectional area of body base	m ²	ft ²
$S(x)$	local body cross-sectional area	m ²	ft ²
S_{ref}	body maximum cross-sectional area, $\pi D^2/4$	m ²	ft ²
V	volume of body	m ³	ft ³
V_∞	free-stream velocity	m/s	ft/s
w	heave velocity	m/s	ft/s
$\dot{w}$	acceleration in heave	m/s ²	ft/s ²
x	axial distance from nose tip	m	ft
x_c	axial distance from nose to centroid of body volume, ($\int_0^L S(x) x dx$) / V	m	ft
x_m	axial distance from nose to pitching axis and moment reference point	m	ft
α	angle of attack	rad	rad
β	boat-tail angle	deg	deg
ρ	tangent or secant ogive radius of curvature	m	ft
ρ'	radius of curvature of tangent ogive having same length and maximum diameter as a given secant ogive or other forebody, see sketch on Figure 2a	m	ft
ρ_∞	free-stream density	kg/m ³	slug/ft ³



Sketch 1.1 Body geometry

2. INTRODUCTION

This Item gives a semi-empirical method for estimating the pitch-rate derivative M_q for projectiles at supersonic speeds and for low angles of attack. The method in the main text applies to axisymmetric, forebody-cylinder combinations with or without conical boat-tails. It is based on an adaptation of the slender-body-theory prediction for the sum of the pitch-rate derivative and the acceleration-in-heave derivative, $M_q + M_{\dot{w}}$. To isolate M_q , a modified form of the slender-body-theory prediction for $M_{\dot{w}}$ is used.

The slender-body-theory equations that underlie the method are given in Appendix A (or see the work in Derivation 1, for example).

A simple extension to the method to include projectiles with rear fins is given in Appendix B.

The aerodynamic derivatives are expressed in aero-normalised form and for a geometric-body axis system, see ESDU 86021 (Derivation 22).

The other ESDU Items that are required for use with this Item are listed in Section 5.1.2 of the Derivation.

3. METHOD

Slender-body theory applied to a body of revolution gives a result that can be written

$$M_q + M_{\dot{w}} = -C_{N\alpha} \left(1 - \frac{x_m}{L}\right)^2 \left(\frac{L}{D}\right)^2, \quad (3.1)$$

see Appendix A. The present method represents $C_{N\alpha}$ by the sum of the forebody and boat-tail contributions to the normal-force-curve slope of the body, as predicted by the methods of ESDU 89008 (Derivation 24) and ESDU 87033 (Derivation 23), respectively. In ESDU 89008 the forebody-generated contribution is expressed as $GkC_{N\alpha l}$. The individual parameters G , k and $C_{N\alpha l}$ depend on body geometry via l_f/D , l_c/D and ρ'/ρ , and on free-stream Mach number, M_∞ , and they are fully explained in ESDU 89008. In ESDU 87033 the boat-tail contribution is expressed as $\Delta C_{N\alpha}$, which depends on body geometry via l_a/D and D_b/D , and on M_∞ .

Examination of experimental data for $M_q + M_{\dot{w}}$ has demonstrated that a further modification to Equation (3.1) is necessary in order to improve prediction of the damping. This is accomplished by introducing the empirical factors F_1 and F_2 to change the effective moment arm in Equation (3.1) from $L - x_m$ to $F_1 F_2 L - x_m$. The factor F_1 was constructed for bodies with $L/D = 5$ and is given in Figure 1a as a function of M_∞ and x_m/L . The factor F_2 in Figure 1b provides a correction for body fineness ratio.

Thus the predicted value of $M_q + M_{\dot{w}}$ is

$$M_q + M_{\dot{w}} = - (GkC_{N\alpha l} + \Delta C_{N\alpha}) \left(F_1 F_2 - \frac{x_m}{L}\right)^2 \left(\frac{L}{D}\right)^2. \quad (3.2)$$

To estimate $M_{\dot{w}}$ and so isolate M_q , the slender-body-theory prediction of the ratio $(M_{\dot{w}}/C_{m\alpha})$, see Appendix A, is multiplied by the value of $C_{m\alpha}$ that is obtained as the sum of the prediction for a forebody-cylinder combination provided by the method of ESDU 89008 and the contribution from any boat-tail as predicted by the method of ESDU 87033.

Then

$$M_{\dot{w}} = C_{m\alpha} \left\{ \frac{C_V \left(\frac{x_c}{L} - \frac{x_m}{L} \right) \left(\frac{L}{D} \right)}{\left(\left(1 - \frac{x_m}{L} \right) \left(\frac{D_b}{D} \right)^2 - C_V \right)} \right\}, \quad (3.3)$$

where C_V is the volume coefficient of the body which may be calculated for commonly used shapes from the data in ESDU 77028 (Derivation 21), and x_c is the distance of the centroid of body volume from the nose, and can be established by integration along the body. As an aid, Figures 2a and 2b give x_c/l_f for a secant ogive nose, and $\{x_c - (L - l_a)\}/l_a$ for a conical frustum boat-tail.

The derivative M_q can then be found by subtracting the prediction of Equation (3.3) from the prediction of Equation (3.2). From the form of Equation (3.3) it is clear that the predicted body contribution to $M_{\dot{w}}$ will be small when $x_c \approx x_m$.

4. ACCURACY AND APPLICABILITY

Experimental data are available only for a limited range of body geometries but the method applies generally to forebody-cylinder combinations, with or without conical boat-tails. Reliance has been placed mainly on the wind-tunnel data of Derivations 7 and 8. Supporting data from other tunnel tests in Derivations 4, 12, 13, 16 and 18, and from free-flight range tests on spin-stabilised bodies in Derivations 2, 3, 6, 9, 10, 11, 16, 17 and 18 have been used to extend the data base. All of the test data were for the sum $M_q + M_{\dot{w}}$ and so had to be compared with the predictions of Equation (3.2).

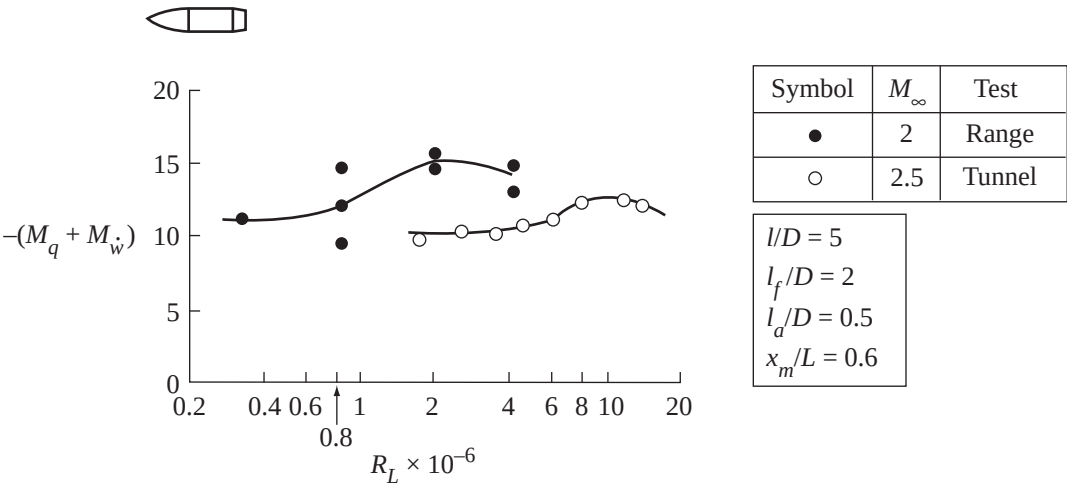
As discussed in Reference 26 it is well established that boundary-layer transition effects can significantly influence dynamic damping and this complicates the analysis of experimental data. Sketch 4.1 from Derivation 16 illustrates the problem for both range and tunnel tests. The rise in $-(M_q + M_{\dot{w}})$ that occurs as Reynolds number increases is associated with the first appearance of transition near the base of the body. As the transition region subsequently moves forward towards the pitch axis its influence lessens and the damping returns towards its original level. An explanation of this behaviour is that the body motion promotes a fluctuating and asymmetrical transition and as a consequence, for transition aft of the pitch axis, there is an increase in $-M_{\dot{w}}$. The change in $M_{\dot{w}}$ and hence in $M_q + M_{\dot{w}}$ can be large as evidenced in Sketch 4.1. Where data are available over a range of Reynolds number it is possible to determine when the effects of transition are small and the method of this Item has concentrated on those data. Thus Equation (3.2) will provide conservative estimates of the dynamic damping $M_q + M_{\dot{w}}$ when transition effects are present, but the accuracy of estimates of M_q made by combined use of Equations (3.2) and (3.3) will be unaffected.

The construction of F_1 was principally derived from the tunnel data in Derivations 7 and 8 for bodies with $L/D = 5$. The value of F_2 at $L/D = 3$ was derived from further tests in those Derivations. Values of F_2 for $L/D > 5$ were deduced from range and tunnel data on longer bodies. However, data were available only for $L/D \leq 10$ and the method could not be verified for higher fineness ratios; the method must therefore be used with caution in such cases. Sketch 4.2 demonstrates the agreement between prediction and some of the data of Derivations 7 and 8. Also illustrated is the effect of not introducing the factors F_1 and F_2 and relying solely on the substitution of an accurate estimate for $C_{N\alpha}$ in Equation (3.1). Incorporation of the factors produces a better modelling of the test data. Overall, the method reproduces the data in Derivations 7 and 8 to within $\pm 20\%$.

Data inferred from some range tests depart more significantly from their predicted values. Sketch 4.3 from Derivation 7 illustrates the problem with a comparison of tunnel and range data for similarly shaped bodies. There is good agreement for the static pitching moment, but at low supersonic speeds the range dynamic damping data show a considerable bias away from the satisfactorily predicted tunnel data. That bias may be associated mostly with the behaviour shown in Sketch 4.1. Some uncertainty may also be present because, as shown in the general analyses of Derivation 5, if the Magnus moment coefficient of the spinning body has a non-linear variation with angular displacement, range values of the damping derivatives display a related dependence on precessional and nutational motion, and a correction must be made to derive the damping derivatives that correspond to small angle oscillations in the pitch plane. Thus uncertainties in the estimated Magnus moment can reflect on the values ascribed to the damping derivatives.

Overall, the experimental data show most scatter through the transonic range, and use of the method should be limited to $1.2 \leq M_\infty \leq 5$. The tunnel data in Derivations 7 and 8 show that $M_q + M_{\dot{w}}$ usually varies only slightly as angle of attack increases up to about 10° , at least for the low mean oscillation angles (1.8°) of those tests. However, this result is for planar motion. For the reasons noted above in relation to the Magnus moment, in unconstrained flight the uncorrected range values of the damping derivatives would still show a dependence on the amplitude of angular motion.

For the configurations studied, forebody lengths were in the range $2 \leq l_f/D \leq 3$, with overall fineness ratios mainly in the range $3 \leq L/D \leq 6$, though there were some data available for $6 \leq L/D \leq 10$. Forebody shapes were cones or secant ogives. Most of the configurations with tapering conical afterbodies had a boat-tail length $l_a \approx 0.5D$ and a boat-tail angle β of about 8° . The method is not limited by boat-tail geometry except through the restrictions imposed by ESDU 87033 but, in the absence of a wider range of experimental data for comparisons, predictions must be regarded as tentative for afterbody geometries that depart significantly from the range of test data.



Sketch 4.1 Example of Reynolds number effect on dynamic damping

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