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Methods for improving damping Part 2: layered damping treatment for structures: analysis and examples

Associated software: ESDUpac A9113
ESDUpac B9113
ESDUpac C9113



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THE PREPARATION OF THIS DATA ITEM

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METHODS FOR IMPROVING DAMPING PART 2: LAYERED DAMPING TREATMENT FOR STRUCTURES: ANALYSIS AND EXAMPLES

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	2
3. MODELLING OF STRUCTURE FOR ANALYSIS OF DAMPING TREATMENTS	3
4. COMPUTATION OF STRUCTURAL LOSS FACTOR AND RESONANT FREQUENCY	3
5. FREE-LAYER DAMPING TREATMENTS (ESDUPAC A9113) (The current version number of ESDUpac A9113 is 1.1).	4
5.1 Data Input	4
5.1.1 General	4
5.1.2 Parameters for plate structure	4
5.1.3 Parameters for beam structure	5
5.1.4 Structure and damping material thicknesses and material properties	5
5.1.5 Damping material properties	5
5.1.6 Input repeat options	6
5.2 Data Output	6
6. METHODS OF ANALYSIS PROVIDED FOR CONSTRAINED-LAYER DAMPING TREATMENTS	6
7. CONSTRAINED-LAYER DAMPING OF STRUCTURES HAVING VIBRATION DISPLACEMENTS OF SINUSOIDAL FORM (ESDUPAC B9113) (The current version number of ESDUpac B9113 is 1.0).	7
7.1 Data Input	7
7.1.1 General	7
7.1.2 Parameters for plate structure	7
7.1.3 Parameters for beam structure	8
7.1.4 Structure material properties	8
7.1.5 Damping treatment parameters	8
7.1.6 Layer properties (all damping layers identical)	8
7.1.7 Layer properties (damping layers not identical)	8
7.1.8 Damping material properties	9
7.1.9 Input repeat options	9
7.2 Data Output	10

8.	SINGLE CONSTRAINED LAYER DAMPING OF BEAMS WITH VARIOUS EDGE CONDITIONS (ESDUPAC C9113)	
	(The current version number of ESDUpac C9113 is 1.0).	10
8.1	Data Input	10
8.1.1	General	10
8.1.2	Parameters for beam structure	10
8.1.3	Damping treatment parameters	11
8.1.4	Damping material properties	11
8.1.5	Input repeat options	11
8.2	Data Output	12
9.	DERIVATION	12
10.	EXAMPLES	12
10.1	Free-layer damping of beams	12
10.2	Free-layer damping of plates	17
10.3	Constrained-layer damping of plates (effective dimensions examples)	20
10.4	Constrained-layer damping of plates (multiple layer example)	25
10.5	Constrained-layer damping of beams	31

METHODS FOR IMPROVING DAMPING

Part 2: layered damping treatment for structures: analysis and examples

NOTE

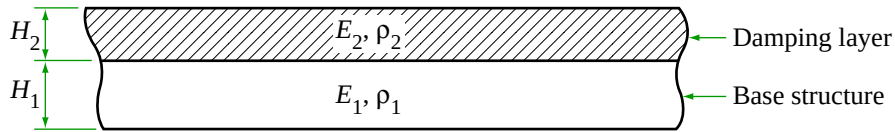
This is Part 2 of “Methods for Improving Damping” and should be used in conjunction with
Part 1: an introduction and guide to practical methods of increasing structural damping (ESDU 91012), and
Part 3: damping material data (ESDU 92001).

1. NOTATION

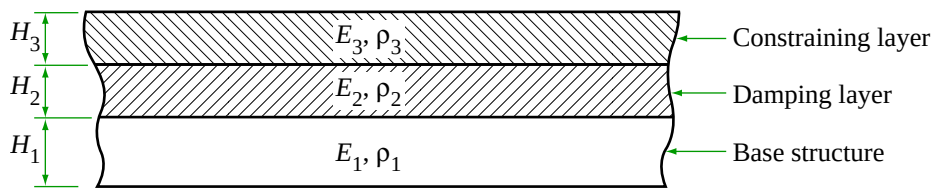
a, b	length of plate in x - and y -directions respectively	m	in
d	indicator of damping layer material in multiple damping material treatment		
E_1, E_2, E_3	real part of Young's modulus of individual layers (see Sketch 1.1)	N/m ²	lbf/in ²
f	frequency	Hz	Hz
f_{mn}	natural frequency of mn^{th} plate mode	Hz	Hz
G_2	real part of complex shear modulus of damping layer	N/m ²	lbf/in ²
H_1, H_2, H_3	thickness of individual layers (see Sketch 1.1)	m	in
L	length of beam	m	in
m, n	mode numbers in x - and y -directions respectively		
N_d	number of damping layers in constrained-layer damping treatment		
N_f	number of frequencies for which damping material properties are to be input		
N_m	number of damping materials in constrained-layer damping treatment		
T	temperature	°C	°C
β_2	shear loss factor of damping layer		
η_2	extensional loss factor of damping layer		
ν	Poisson's ratio of plate material		
ρ_1, ρ_2, ρ_3	density of individual layers (see Sketch 1.1)	kg/m ³	*lb/in ³

* See first footnote in Section 5.1.5.

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(a) Free-layer damping treatment



(b) Constrained-layer treatment

Sketch 1.1 Thickness and properties of individual layers

2. INTRODUCTION

This Item is one of three related Items providing information on, and design data for, improving damping in structures. Data are provided in the form of computer programs on diskettes, referred to as ESDUpacs A9113*, B9113 and C9113. The first ESDUpac is for the analysis of free-layer damping treatments and the latter ESDUpacs are for the analysis of constrained-layer damping treatments. In this Item, examples are given of the use of the prediction procedures. These examples serve to illustrate the levels of damping that may be achieved by the use of layered damping treatments on beam and plate like structures. The basic equations used for the estimation of free-layer damping are given in companion ESDU 91012 and those for constrained-layer damping are given in Derivations 1 and 2. ESDU 91012 provides an introductory guide to practical methods for improving damping. The third Item in this series, ESDU 92001, provides data on damping materials that are an essential input to the analysis procedures in this Item.

It is essential that this Item be used in conjunction with ESDU 91012 since it is in that latter Item that the assumptions used in deriving the equations for the analysis of layer damping treatments are given. Also information on the recommended approach to the design of damping treatments should be followed together with the guidelines on good practice in damping treatment designs.

* The full ESDUpac number has an appended V followed by two digits. The digits indicate the ESDUpac version number. For example, the full ESDUpac number for version 1.1 of ESDUpac A9113 is A9113V11. The version appendage will not be used when referring to ESDUpacs in the text of this Item. See introduction to ESDUpacs for a full description of the ESDUpac numbering system. The current version of each ESDUpac is given under the heading to the related Section 5, 7 or 8 and on the example program outputs in Section 10.

3. MODELLING OF STRUCTURE FOR ANALYSIS OF DAMPING TREATMENTS

The computation procedures in this Item are developed for idealised beam and plate structures. Practical structures are more complex and care should be taken to ensure that the response of the idealised structure analysed represents most closely the dynamic response of the actual untreated structure. The recommended approach is to represent the structure by a model with geometric and other parameters chosen such that the idealised model structure has a natural mode of vibration of the same type, mode order and approximately the same natural frequency, as the mode under investigation of the actual structure.

As an example of modelling, consider a cantilever beam tapering in thickness and vibrating in its natural modes with predominantly flexural displacements*. The thickness of the beam can be chosen such that the investigated resonant frequency of the actual structure is matched by the appropriate natural frequency of the model beam. Using the estimated beam thickness and actual mean width and length, the density of the model can be adjusted to reproduce the mass of the actual structure. The modified beam dimensions and density should then be used in the appropriate computer program to investigate the effects of damping treatments.

In the analysis of stiffened plate structures, it is usually most convenient to adjust the model plate dimensions a and b to give a good representation of the actual panel response. This is particularly so when constrained-layer damping treatments are considered and the model structure is assumed to be simply-supported. An illustration of the adjustment of the dimensions of a plate structure to form an idealised model suitable for constrained-layer damping treatment analysis is given in Example 10.3 in Section 10. For plate structures it is most important that the structure analysed has approximately the same mode shape as the real structure so that the strain energy distribution closely represents that of the real structure.

4. COMPUTATION OF STRUCTURAL LOSS FACTOR AND RESONANT FREQUENCY

The computation of the structural loss factor and resonant frequency[†] for a particular mode of vibration, and given damping treatment, is an iterative procedure. This is because the properties of the damping material are a function of frequency (see ESDU 91012, Section 6.2), and the addition of damping material will change the resonant frequency. Consequently, an input to the programs is the damping material properties, modulus and loss factor, over a range of frequencies[‡]. The starting resonant frequency, in many cases, is estimated within the program**. The necessary damping material properties are readily obtained from a reduced-temperature nomogram for the material considered. Material properties for a number of the commonly used damping materials are presented in ESDU 92001.

The computations for free-layer damping treatments and constrained-layer damping treatments are detailed in Sections 5 and 6 respectively.

* The methods of analysis considered in this Item are suitable for bending modes of vibration. The results of the analysis must be treated with caution if the natural mode of response is a combined bending and twisting mode.

† The resonant frequency is a frequency at which the response of the system to a given input is a maximum, *i.e.* the system is at resonance. For lightly damped structures, such as those generally considered in this Item, the natural and resonant frequencies are approximately equal.

‡ It should be noted that the damping material properties are both frequency and temperature dependent, the variations of these parameters being greatest for changes in temperature (see Section 6 of ESDU 91012).

** For plate structures with free-layer damping treatments, the natural frequency for the mode of vibration considered is input except for the case of a plate with all edges simply-supported.

5. FREE-LAYER DAMPING TREATMENTS (ESDUpac A9113)

(The current version number of ESDUpac A9113 is 1.1).

The computer program referred to as ESDUpac A9113 provides a means for estimating the resonant frequency and loss factor of a beam or plate structure with a free-layer damping treatment. The damping treatment is assumed to be applied over the whole of one surface, or symmetrically applied over the whole of both surfaces, of the structure. The program is applicable to any mode of vibration and to beam and plate structures having any combination of clamped, simply-supported and free boundary conditions. The method of analysis and assumptions made in the theory used are described in Section 7.2 of ESDU 91012.

5.1 Data Input

The input data may be entered either from the keyboard or from a file with directed input. Using a file input allows data correction, or case data changes, to be made using file editing facilities. Entering case data from the keyboard provides the opportunity for the program to give visual requests for each item of data as it is required.

The form of the input file is illustrated by the input files listed for the examples in Sections 10.1 and 10.2.

Data are input in free format in the following order and any coherent system of units may be used.

5.1.1 General

Run title

Three lines are allocated for the run title (maximum of 75 characters per line). A blank line must be inserted in the input file for any of the title lines not used.

File name for output (maximum 12 characters allowed).

Is a plate structure to be considered?

Answer 1 (YES) or 0 (NO)

Is damping treatment on one side only?

Answer 1 (YES) or 0 (NO)

If a beam structure is to be considered, go to Section 5.1.3 for the next input; otherwise continue.

5.1.2 Parameters for plate structure

Is the plate simply-supported on all edges?

Answer 1 (YES) or 0 (NO).

If the answer to the above question is yes, input the following.

Plate dimensions.

a, b

Mode numbers.

m, n

If the answer to that question was no, input the following.

Plate natural frequency.

f_{mn}

Go to Section 5.1.4 for the next input.

5.1.3 Parameters for beam structure

Boundary conditions code.

integer number

Input 1 for beam pinned at both ends.

Input 2 for beam clamped at both ends.

Input 3 for beam free at both ends.

Input 4 for beam clamped at one end and pinned at the other.

Input 5 for beam clamped at one end and free at the other.

Beam length.

L

Mode number.

m

5.1.4 Structure and damping material thicknesses and material properties

Untreated structure thickness.

H_1

Thickness of damping layer.

H_2

Structural material properties*.

E_1, ρ_1

If the structure is a simply-supported plate input the following.

Poisson's ratio of plate material.

ν

5.1.5 Damping material properties[†]

Density of damping material*.

ρ_2

Number of frequencies for which damping material properties are to be input[‡] (maximum 10).

N_f

* A density value expressed in British units as lb/in^3 has to be divided by 386.4 before being input into the computer program. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2 .) For example, if a material density is 0.097 lb/in^3 , the value input should be $2.510 \times 10^{-4} \text{ lb s}^2/\text{in}^4$.

[†] It is important that the damping material properties used are those applicable at the temperature at which the resonant frequency to be damped occurs.

Damping material properties at frequency f .
 f, E_2, η_2

The above material properties are input in ascending order of frequency, f , for each of the N_f frequencies.

5.1.6 Input repeat options

Are other damping layer thicknesses to be considered?
 Answer 1 (YES) or 0 (NO).

If the answer to the above question is 'YES' input H_2 , otherwise continue.

Are other damping material properties to be considered?
 Answer 1 (YES) or 0 (NO).

If the answer to the above question is 'YES' repeat input from Section 5.1.5. If the answer to this final question is 'NO' the input is complete and, if keyboard input is used, the computation is terminated.

5.2 Data Output

The output is written to the file named in the input data (see Section 5.1.1). On running the program the input data, structure geometry and material properties, and the damping layer thickness and material properties are listed on the output file to provide a permanent record of the data and to allow checks to be made for data input errors. If the untreated structure natural frequency is calculated this is output followed by the treated structure resonant frequency, loss factor and the ratio of the masses of the treated and untreated structure for each case considered.

If the frequency range of the input damping material properties does not cover the frequencies required, a warning is output and an indication is given of whether additional higher or lower frequency data are required.

6. METHODS OF ANALYSIS PROVIDED FOR CONSTRAINED-LAYER DAMPING TREATMENTS

Two computer programs are provided for the estimation of resonant frequencies and loss factors for plate-like structures having constrained-layer damping treatments. The first of these, referred to as ESDUpac B9113, is applicable to plates and beams vibrating in modes in which the displacement forms are sinusoidal. That program may be used for single or multiple-layered damping treatments. The second program, referred to as ESDUpac C9113, provides a means of estimating loss factors and resonant frequencies for beams having a single constrained-layer damping treatment without imposing the restriction of sinusoidal displacement forms. It is intended that ESDUpac C9113 be used to investigate the effect of different boundary conditions since it is applicable to beams having any combination of clamped, pinned and free end supports (other than pinned-free).

In both programs for constrained-layer damping treatments it is assumed that all damping occurs in the damping layer; consequently the extensional loss factors within the base structure and constraining layer are considered to be zero. Also, for both programs, the damping treatment is assumed to be applied over

‡ In the iteration procedure for the estimation of the resonant frequency, the starting frequency is the untreated structure natural frequency of the mode under consideration. Damping material data are required at a number of discrete frequencies (minimum of two frequencies) covering a frequency range including the considered mode natural frequency and the unknown resonant frequency. Some indication of the frequency range to be covered can be obtained from an examination of the Examples in Section 10. Within the computer program the damping material properties at intermediate frequencies are found by linear interpolation using the logarithm of frequency.

the whole of one surface of the structure. The methods of analysis and assumptions made in the theories used for constrained-layer damping are detailed in Derivation 2.

Specific information relating to ESDUpacs B9113 and C9113, together with details of input and output, are given in Sections 7 and 8 respectively.

7. CONSTRAINED-LAYER DAMPING OF STRUCTURES HAVING VIBRATION DISPLACEMENTS OF SINUSOIDAL FORM (ESDUpac B9113)

(The current version number of ESDUpac B9113 is 1.0).

The assumption that vibration displacements are sinusoidal makes ESDUpac B9113 applicable to plates having all edges simply-supported and beams having both ends pinned. The program may be used for single or multiple-layered damping treatments and for the latter, the damping material need not be identical in all layers.

For structures that do not have all edges simply-supported, the program may be used to give approximate loss factors by inputting effective panel dimensions. The natural frequency, for the mode of vibration under consideration, should first be estimated for the untreated structure and actual edge conditions. The effective dimensions are those of the structure with simply-supported edges and which has the same natural frequency for the same order of mode as the actual structure. Examples of the use of effective plate dimensions are given in Section 10.3.

7.1 Data Input

As described in Section 5.1 for the free-layer damping program, the input to this program may be either from keyboard or with directed input from a file. The form of the input file is illustrated by the input files listed for the examples in Sections 10.3 and 10.4.

Data are input in free format in the following order and any coherent system of units may be used.

7.1.1 General

Run title

Three lines are allocated for the run title (maximum of 75 characters per line). A blank line must be inserted in the input file for any of the title lines not used.

File name for output (maximum 12 characters allowed).

Is a plate structure to be considered?
Answer 1 (YES) or 0 (NO).

If a beam structure is to be considered, go to Section 7.1.3 for the next input; otherwise continue.

7.1.2 Parameters for plate structure

Plate dimensions.
 a, b, H_1

Mode numbers.
 m, n

Go to Section 7.1.4 for the next input.

7.1.3 Parameters for beam structure

Beam dimensions.

L, H_1

Mode number.

m

7.1.4 Structure material properties

Structure material properties*.

E_1, ρ_1

If the structure is a plate input the following.

Poisson's ratio of plate material.

ν

7.1.5 Damping treatment parameters

Number of damping layers (equal to the number of constraining layers – maximum 15).

N_d

If N_d is greater than one answer the following question.

Are all damping layers identical?

Answer 1 (YES) or 0 (NO).

If N_d is not equal to one and the answer to the above question is 'NO', go to Section 7.1.7 for the next input; otherwise continue.

7.1.6 Layer properties (all damping layers identical)

Constraining layer properties[†].

E_3, H_3, ρ_3

Damping layer properties*.

H_2, ρ_2

Go to Section 7.1.8 for the next input.

7.1.7 Layer properties (damping layers not identical)

Number of damping materials used (maximum 5).

N_m

* A density value expressed in British units as lb/in³ has to be divided by 386.4 before being input into the computer program. (A force of 1 lbf acting on a mass of 1lb produces an acceleration of 386.4 in/s².) For example, if a material density is 0.097 lb/in³, the value input should be 2.510×10^{-4} lb s²/in⁴.

† A density value expressed in British units as lb/in³ has to be divided by 386.4 before being input into the computer program. (A force of 1 lbf acting on a mass of 1lb produces an acceleration of 386.4 in/s².) For example, if a material density is 0.097 lb/in³, the value input should be 2.510×10^{-4} lb s²/in⁴.

The following inputs in this Section must be given for each damping layer in turn. Layers are numbered from the baseplate (*i.e.* layer 1 is closest to the base structure).

Constraining layer properties* .

E_3, H_3, ρ_3 (layer 1)

E_3, H_3, ρ_3 (layer 2)

: : :

E_3, H_3, ρ_3 (layer N_d)

Damping layer properties*

H_2, ρ_2 (layer 1)

H_2, ρ_2 (layer 2)

: :

H_2, ρ_2 (layer N_d)

For the following input the damping material is indicated by an integer number in the range 1 to N_m . This number is defined by the order in which damping layer material properties are input in Section 7.1.8. For example, if the damping material in layer 1 is the second material for which properties are input in Section 7.1.8, the required value of d for layer 1 is 2.

Damping layer material.

d (layer 1)

d (layer 2)

:

d (layer N_d)

7.1.8 Damping material properties

Number of frequencies for which damping material properties are to be input* (maximum 10).

N_f

Damping material properties at frequency f .

f, G_2, β_2

The above material properties are input in ascending order of frequency, f , for each of the N_f frequencies.

If more than one damping material is used, the whole of the input in this Section is input for each material in turn.

7.1.9 Input repeat options

Are other damping material properties to be considered?

Answer 1 (YES) or 0 (NO).

If the answer to the above question is 'YES' repeat input from Section 7.1.8. If the answer to this final question is 'NO' the input is complete and, if keyboard is used, the computation is terminated.

* In the iteration procedure for the estimation of the resonant frequency, the starting frequency is the untreated structure natural frequency of the mode under consideration. Damping material data are required at a number of discrete frequencies (minimum of two frequencies) covering a frequency range including the considered mode natural frequency and the unknown resonant frequency. Some indication of the frequency range to be covered can be obtained from an examination of the Examples in Section 10. Within the computer program the damping material properties at intermediate frequencies are found by linear interpolation using the logarithm of frequency.

7.2 Data Output

The output is written to a file named in the input data (see Section 7.1.1). On running the program the input data, structural geometry and material properties, and the damping and constraining layer properties are listed on the output file to provide a permanent record of the data and to allow checks to be made for data input errors. For each case considered, the input damping material properties are output followed by the resonant frequency and loss factor of the treated structure, and the ratio of the masses of the treated and untreated structure.

If the frequency range of the input damping material properties does not cover the frequency range required, a warning is output together with advice as to whether additional higher or lower frequency data are required.

8. SINGLE CONSTRAINED LAYER DAMPING OF BEAMS WITH VARIOUS EDGE CONDITIONS (ESDUpac C9113)

(The current version number of ESDUpac C9113 is 1.0).

For the analysis in ESDUpac C9113 the constraining layer is free at the beam ends. This condition represents an “add-on” damping treatment which is the usual form of treatment when increased damping is found to be necessary in a service condition. The effect on the loss factor of beams having the ends of the constraining layer either constrained or unconstrained is illustrated in Section 7.3.2 of ESDU 91012.

When using ESDUpac C9113 for higher mode beam vibrations it is possible that, in some cases, the iteration procedure will not converge so that no solution will be found. Also it is possible for an apparent change of mode to occur during the iteration procedure. During testing of the program, using realistic damping treatments, solutions were found for all cases but the change of mode was identified in a small number of cases. The changes of mode were in each case associated with unsymmetric boundary conditions and high values of damping layer shear modulus.

8.1 Data Input

As described in Section 5.1 for the free-layer damping program, the input to this program may be either from the keyboard or with directed input from a file. The form of the input file is illustrated by the input file listed for the example in Section 10.5.

Data are input in free format in the following order and any coherent system of units may be used.

8.1.1 General

Run title

Three lines are allocated for the run title (maximum of 75 characters per line). A blank line must be inserted in the input file for any of the title lines not used.

Filename for output (maximum 12 characters allowed).

8.1.2 Parameters for beam structure

Boundary conditions code.
integer number

Input 1 for beam pinned at both ends.