

ENERGY HEIGHT METHOD FOR FLIGHT PATH OPTIMISATION

Addendum A

Numerical method suitable for rapid use on IBM PC compatible computers

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
C_L	lift coefficient (= L/qS)		
$C_{L_{buffet}}$	lift coefficient at buffet limit		
$C_{L_{max}}$	maximum lift coefficient		
f	fuel used	kg	slug
g	acceleration due to gravity	m/s ²	ft/s ²
g_0	standard sea-level value* of g	m/s ²	ft/s ²
H	geopotential height*	m	ft
H_e	energy height (= $H + V^2/2g_0$)	m	ft
L	aerodynamic lift	N	lbf
q	kinetic pressure (= $\rho V^2/2$)	N/m ²	lbf/ft ²
S	wing reference area	m ²	ft ²
t	time	s	s
V	true airspeed (air-path velocity or resultant velocity relative to air)	m/s	ft/s
γ	angle of climb (flight-path inclination angle)		
λ_f	constant (Lagrangian) multiplier used in solution of problems with distance constraint (see Equation (4.17) of Data Item No. 90012)	kg/m	slug/ft
λ_t	constant (Lagrangian) multiplier used in solution of problems with distance constraint (see Equation (4.16) of Data Item No. 90012)	s/m	s/ft
ρ	air density	kg/m ³	slug/ft ³
Subscript			
<i>std</i>	denotes conditions in International Standard Atmosphere, ISA		

* See Data Item No. 77022 (Reference 1) for definitions.

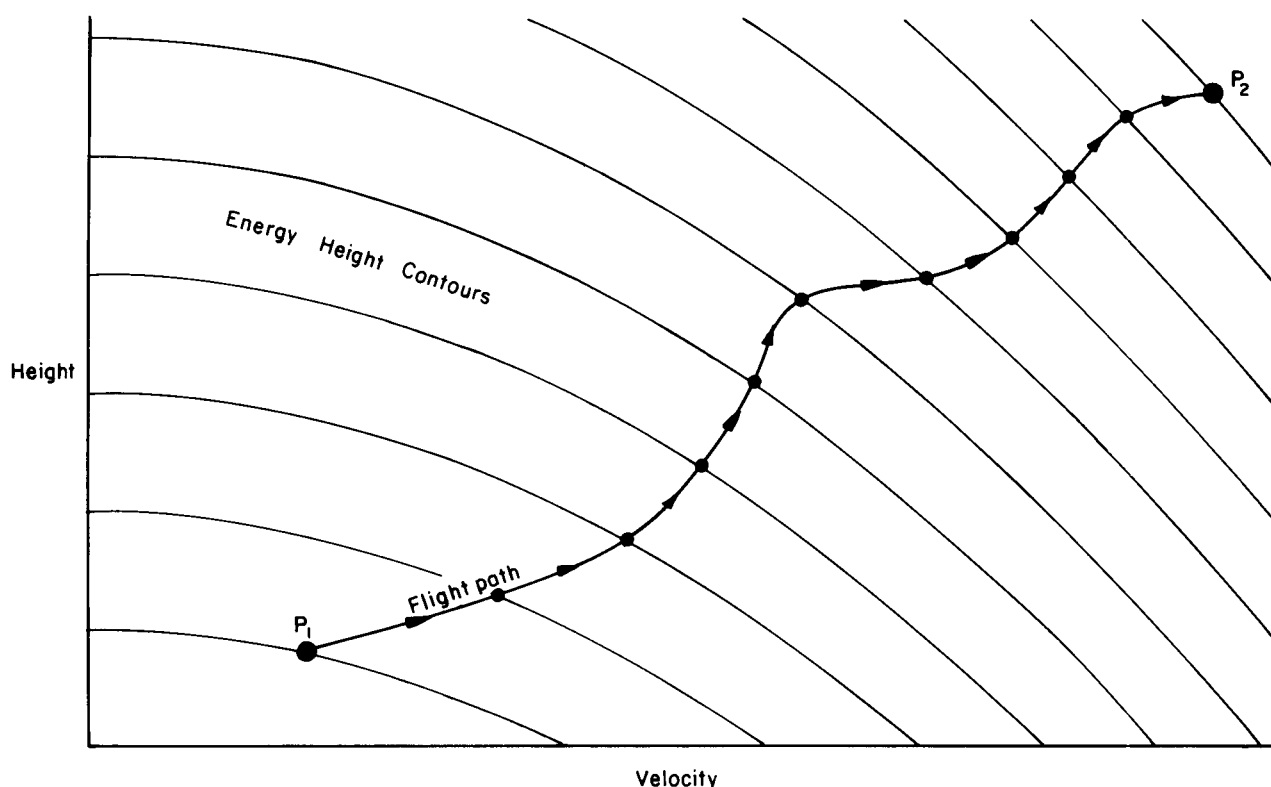
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2. INTRODUCTION

Data Item No. 90012 deals with the theory of the Energy Height Method of flight path optimisation and gives some graphical methods for derivation of solutions. Although, as is stated in the Data Item, the use of computers enables much greater detail to be obtained in these graphical methods, they are not rapid in operation. However, by suitable reorganisation of the calculation routines and by judicious selection of the grids for the calculations, numerical solutions can be obtained very rapidly with modest computing equipment. This Addendum explains how such procedures may be implemented.

3. ENERGY HEIGHT METHOD SOLUTION BY COMPUTER

The Energy Height Method determines the optimum path as that which minimises dt/dH_e (or df/dH_e) with respect to V at each value of H_e . Method (a) in Appendices A and B of Data Item No. 90012 derives this path as the envelope of curves of dt/dH_e versus H_e at constant H (i.e. the lowest point of all the curves at constant H_e), whilst method (b) derives it as the locus of points where contours of constant energy height touch contours of constant dt/dH_e . Consideration of these two methods shows that the optimum path may also be found by means of a simple "linear search" for the location of the minimum value of dt/dH_e (in the case of optimisation for time) along each energy height contour (Sketch 3.1). This procedure can form the basis of a simple computer program.



Sketch 3.1 Velocity-height diagram

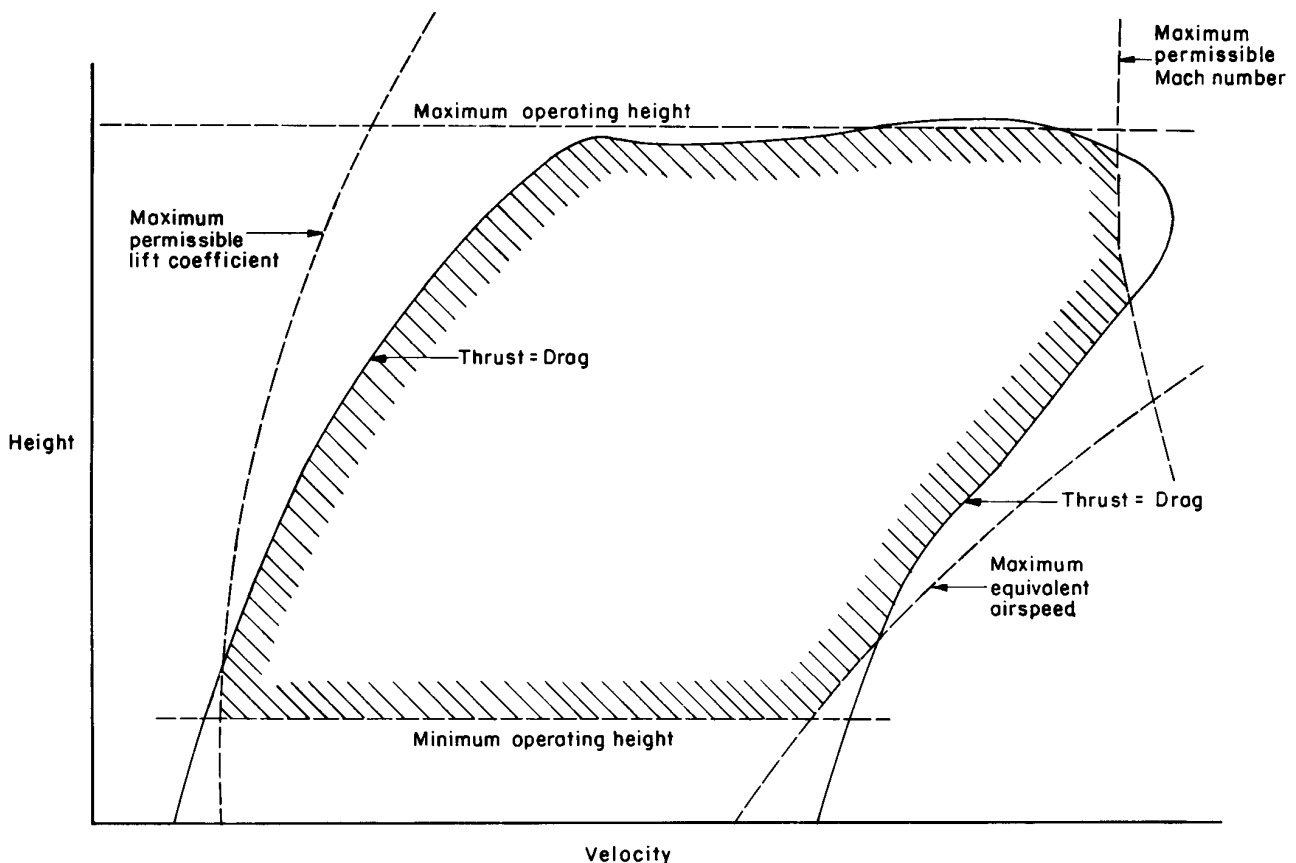
The main problem with the procedure is that of achieving sufficient accuracy without incurring excessive run time. To achieve a solution correct within (say) 1 knot by direct means requires calculation of dt/dH_e at a large number of separate values of velocity at every energy height. The method presented here uses an iterative approach to define the location of the optimum with progressively increasing accuracy.

4. FEASIBLE REGION FOR SOLUTIONS

Operation of the aircraft is restricted by considerations such as the following.

- (a) Design constraints; for example,
maximum permissible design speed and altitude.
- (b) Performance constraints; for example,
 $C_L < \text{maximum permissible value of } C_L$,
i.e. $C_{L_{max}}$, $C_{L_{buffet}}$, etc., factored by airworthiness regulations,
thrust $\geq$ drag.
- (c) Operational constraints; for example,
terrain limit (such as height > 0).

Application of these limits produces a “feasible region”, within which the solution is sought. Sketch 4.1 shows a typical example.



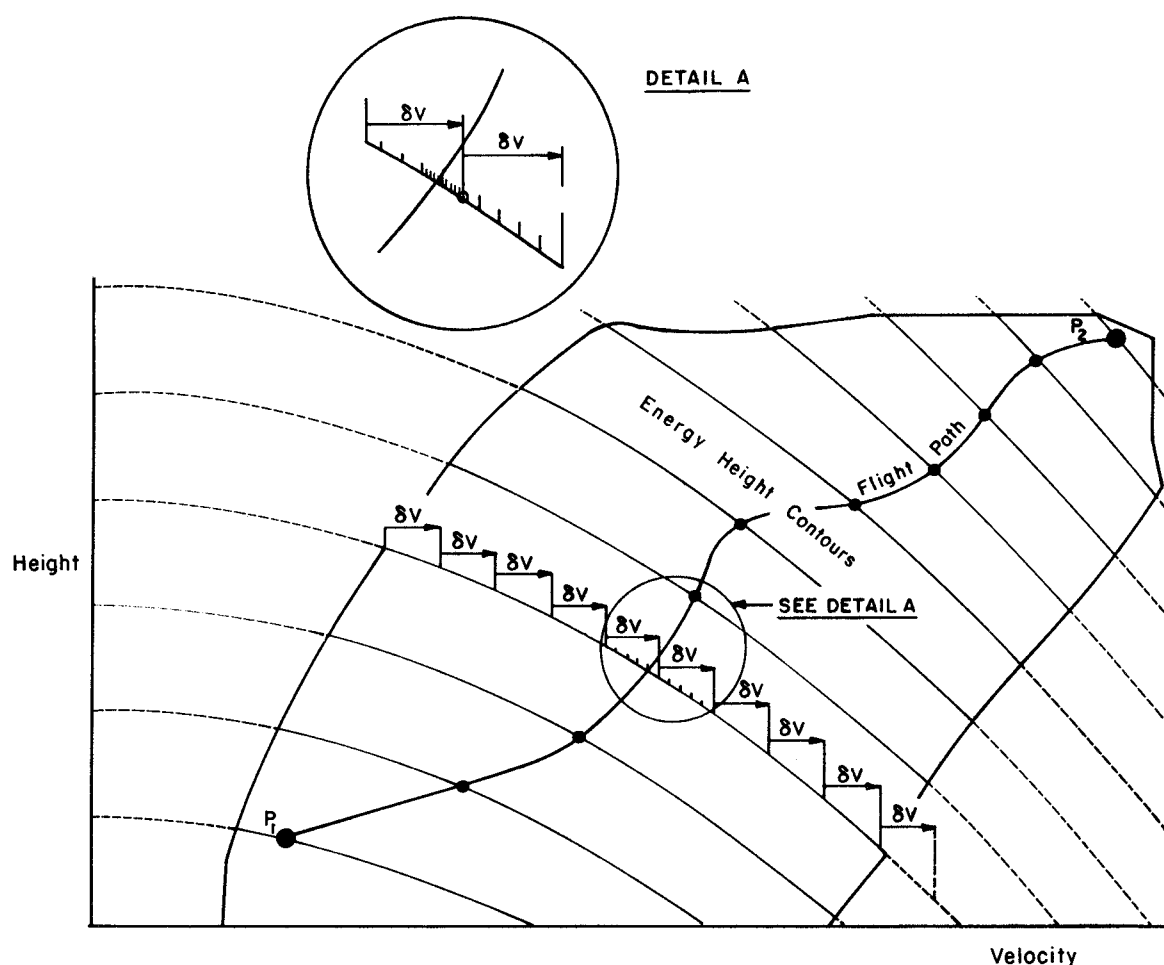
Sketch 4.1 Typical feasible region

The above list of constraints is intended only to indicate some types of limitations that might apply. It is not exhaustive and complete consideration of those relevant to each individual problem must be given prior to calculation of the feasible region for that problem.

5. ALGORITHM

For the purposes of the algorithm, the velocity-height plane is defined in terms of energy height and velocity, as indicated in Sketch 3.1.

The algorithm consists of a simple linear search for the location of the minimum value of dt/dH_e (or df/dH_e) along each selected energy height contour (see Sketch 5.1), starting at the lowest velocity within the limits of the feasible region and continuing to the highest. These limits have to be calculated before the numerical solution of the problem can begin.



Sketch 5.1 Calculation grid within feasible region

Choice of the best step size both in energy and velocity is important. Using velocity increments equal to the desired accuracy* is possible but reduced run times may be achieved by resorting to an iterative procedure, starting with a coarse velocity increment to locate the region in which the optimum path lies and successively using smaller steps within this restricted region to determine closer approximations to the precise path. The initial iteration, using velocity step size δV , determines the point along each energy height contour for which the value of dt/dH_e (or df/dH_e) has the lowest value. For the next iteration, a band the width of one velocity increment either side of this best value (*i.e.* a total band width of $2 \times \delta V$) is examined at a reduced velocity increment to refine the position of the optimum (see inset in Sketch 5.1). The process may be repeated as many times as is necessary to achieve a desired level of accuracy. The factor reducing the velocity increment at each iteration is related to the initial velocity increment, the required accuracy,

* For the purpose of the examples which follow, the desired accuracy has been arbitrarily selected as ± 1 knot in order to ensure a reasonably close approximation to the true solution.