

EXAMPLE OF STATISTICAL TECHNIQUES APPLIED TO CRUISE PERFORMANCE GUARANTEES

(Student's "t" Distribution)

1. NOTATION

F	F Variance Ratio test statistic, see Section 5.2 of Reference 1
n	size of sample
t	Student's "t" statistic, see Section 4 of Reference 1
s	standard deviation of sample
$s_{\Delta SAR}$	standard deviation of ΔSAR
ΔSAR	measured value minus scheduled value of specific air range, expressed as percentage of scheduled value
$\overline{\Delta SAR}$	mean value of ΔSAR
$\sigma_{\overline{\Delta SAR}}$	standard error of the mean of ΔSAR

Subscripts

p	denotes pooled value
1	denotes sample number 1
2	denotes sample number 2

2. INTRODUCTION

This Data Item presents an analysis of measured cruise performance data to illustrate the use of Student's "t" distribution in setting up confidence intervals, for differences between a sample mean and the population mean. In addition the use of the "F" test and the technique of "pooling" samples are illustrated by example. An introduction to statistical techniques for use in analysis of aircraft performance data is given in Data Item No. 91017 (Reference 1).

3. DATA AND ANALYSIS METHOD

Before issuing cruise specific air range, SAR , guarantees to a customer, in association with significant penalty clauses, it was decided to carry out flight tests at the cruise guarantee point to verify the level of specific air range that could be claimed without significant risk of triggering those penalties. In addition, to be competitive, the guarantees would need to be no worse than the current design estimate of specific air range less 2%. However, only a limited amount of flight test time was available so there was a need to keep the number of test points to a practicable minimum.

Because the planned small test sample would be the only source of information on both the mean and standard deviation of specific air range, the use of the Normal Distribution would not be appropriate and the analysis would need to be based on Student's "t" distribution.

The curves of Figure 2 of Reference 1 are based on Student's "t" and show the margins needed between sample and population means for three confidence levels (90%, 95% and 99%). It is clear from the shape of these curves that, for 90% and 95% confidence, most of the benefit to be had from increasing sample size has been achieved by the time the sample size reaches six – but to achieve the same margin to 99% confidence, some 12 to 14 test points would be needed. In this case, although 99% confidence would have been desirable, due to the size of the penalties, it was decided to go ahead on the basis of six test points and the results of the tests are summarised below, the measured specific air range values being quoted as the difference, ΔSAR , between measured and design-estimated values.

Test number	ΔSAR %
1	–3.21
2	–1.4
3	–3.96
4	–1.27
5	–1.66
6	–0.50

These six data points yielded a mean value (Equation (2.1b) in Reference 1)

$$\overline{\Delta SAR}_1 = -2\%$$

with a standard deviation (Equation (2.3c) in Reference 1)

$$s_{\Delta SAR_1} = 1.31\% .$$

With a sample size of 6 test points, Figure 1 of Reference 1 may be entered at 5 degrees of freedom to read off values of Student's "t" appropriate to 90%, 95% and 99% confidence as follows.

Confidence level	Student's "t"
90%	1.48
95%	2.00
99%	3.36

With a sample ΔSAR standard deviation of 1.31%, the standard error of the mean may be calculated using Equation (3.1) of Reference 1 to give

$$\sigma_{\Delta SAR} = \frac{s_{\Delta SAR}}{(n)^{1/2}} = \frac{1.31}{(6)^{1/2}} = 0.53\% .$$

Taking this result in conjunction with the sample mean ΔSAR of –2% and the values of Student's "t" from the table above, the ΔSAR guarantees may be calculated as follows.

Confidence level	ΔSAR Guarantee %
90%	$-2.00 - (1.48 \times 0.53) = -2.78$
95%	$-2.00 - (2.00 \times 0.53) = -3.06$
99%	$-2.00 - (3.36 \times 0.53) = -3.78$

Because of the severity of the penalty clauses, it was felt necessary, still, to maintain a 99% confidence that the guarantees did not exceed the cruise specific air range population mean, but the corresponding performance decrement of 3.78% was too large to make the aircraft competitive. Consequently, despite the pressure on flight-test time, a further six test points were flown, yielding the following results.

Test number	ΔSAR %
7	-5.07
8	-3.75
9	-2.86
10	-3.37
11	-2.76
12	-2.86

For these results the mean value was

$$\overline{\Delta SAR}_2 = -3.45\% ,$$

with a standard deviation

$$s_{\Delta SAR_2} = 0.88\% .$$

* Before combining the results of these two samples to obtain pooled estimates of the population mean and standard deviation, it is first necessary to check that the characteristics of the two samples are sufficiently similar for them to be treated as coming from the same population. This can be done by applying the F Variance Ratio test (see Section 5.2 of Reference 1) to check for similarity in variance and by using Student's "t" to test the difference between the means of the two samples (see Section 4.2 of Reference 1).

The F Variance Ratio test is applied as follows.

Since the variance of the first sample was the greater, the F Variance Ratio is calculated from Equation (5.2) of Reference 1, i.e.

$$F = \frac{1.31^2}{0.88^2} = 2.22 . \quad (3.1)$$

* With such small samples the technique of pooling samples does not result in a large saving in calculation effort but with bigger samples significant savings can be made, hence its use here as an illustration.