

EXAMPLE OF STATISTICAL TECHNIQUES APPLIED TO AUTOLAND TOUCHDOWN DISPERSION (Confidence Intervals for Normal and Skew Distributions)

1. NOTATION AND UNITS

cf	cumulative frequency	
e	expected frequency (within an interval)	
f	observed frequency (within an interval)	
n	size of sample	
q	probability	
s_{TD}	touchdown distance	ft
z	margin from mean of probability distribution (standardised Normal variable)	
χ^2	“Chi-squared” test statistic, see Section 5.1 of Reference 1	

Subscript

I	denotes I th interval in subdivision of touchdown distances
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2. INTRODUCTION

This Data Item uses the analysis of two sets of autoland touchdown dispersion data to provide examples of the following.

The use of normal/probability graph paper (see Section 3.3 of Reference 1),
the application of the χ^2 test (see Section 5.1 of Reference 1),
the analysis of a skew distribution using a “log” transformation (see Section 6 of Reference 1),
and
the estimation of the probability of an event (for example, a touchdown outside an acceptable dispersion) by setting up confidence intervals based on the touchdown dispersion data (see Section 3.2.1. of Reference 1).

An introduction to statistical techniques for use in analysis of aircraft performance data is given in Data Item No. 91017 (Reference 1).

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3. THE DATA

When developing autoland systems it is important to ensure that the performance of the system/aircraft combination will be adequate to achieve acceptably low probabilities of

landing too short (undershooting the runway), or
landing too long (potential overrun accident).

Extensive simulation programmes, verified by a smaller number of flight-test measurements are usually run to check this aspect of autoland performance, giving rise to such results as those presented in Tables 3.1 and 3.2.

3.1 Touchdown Dispersion for Standard Approach Angle and Standard Glide-Slope Origin

Table 3.1 presents the results of a sample of 1480 simulated landings using a standard approach angle (nominal 3°) and a standard glide-slope origin (1500 ft from the runway threshold). The data are presented in the form of a tabulation of the frequency of occurrence of touchdowns in each of 13 one hundred feet long “intervals” between points 951 ft and 2250 ft from the runway threshold, plus one interval covering those touchdowns less than 951 ft from the threshold.

Table 3.1 is completed by calculating values of cumulative frequency for each interval and the percentage cumulative frequency.

Note: Following conventional practice the percentage cumulative frequency is calculated for the I th interval,

$$cf_I\% = \frac{cf_I \times 100}{n + 1} \quad (3.1)$$

TABLE 3.1

Touchdown distance, s_{TD} ft	Frequency	Cumulative frequency, cf_I	Cumulative frequency, $cf_I\%$
≤ 950	7	7	0.47
951 – 1050	30	37	2.50
1051 – 1150	55	92	6.21
1151 – 1250	100	192	12.96
1251 – 1350	177	369	24.92
1351 – 1450	240	609	41.12
1451 – 1550	229	838	56.58
1551 – 1650	225	1063	71.78
1651 – 1750	180	1243	83.93
1751 – 1850	107	1350	91.15
1851 – 1950	70	1420	95.88
1951 – 2050	41	1461	98.65
2051 – 2150	14	1475	99.59
2151 – 2250	5	1480	99.93
No. of test points, n	1480		

3.2 Touchdown Dispersion for Steep Approach Angle, Displaced Glide-Slope Origin and Modified Control Laws

A second set of simulations was carried out to check the capability of the system in a steep-approach, short-field (STOL) type of landing. Consequently, Table 3.2 presents the results of this second sample of 1480 simulated landings using a 7.5° nominal approach angle, a glide-slope origin 1000 ft from the runway threshold and control laws modified to reduce any tendency to land “short”. The results are presented in Table 3.2 in a similar form to Table 3.1 but this time there are 8 one hundred feet long intervals between 751 ft and 1550 ft plus one interval covering the touchdowns shorter than 751 ft.

The percentage cumulative frequencies are calculated in the same way as for Table 3.1, using Equation (3.1).

TABLE 3.2

Touchdown distance, s_{TD} ft	Frequency	Cumulative frequency, cf_I	Cumulative frequency, $cf_I\%$
≤ 750	4	4	0.27
751 – 850	47	51	3.44
850 – 950	201	252	17.02
951 – 1050	414	666	44.97
1051 – 1150	400	1066	71.98
1151 – 1250	253	1319	89.06
1251 – 1350	107	1426	96.29
1351 – 1450	43	1469	99.19
1451 – 1550	11	1480	99.93
No. of test points, n	1480		

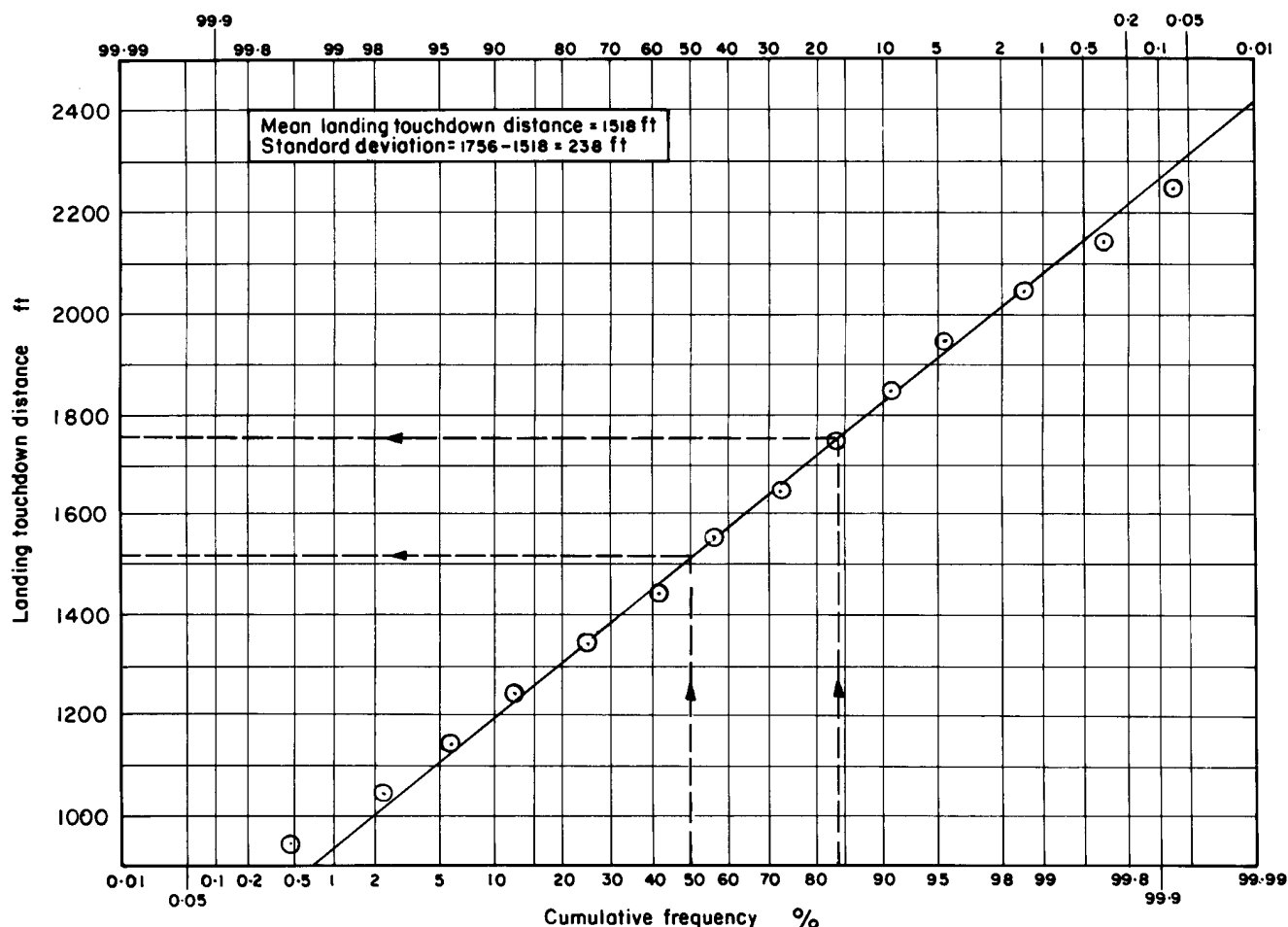
4. THE PERFORMANCE ANALYSES

The purpose of the performance analyses for both sets of data was to establish the following parameters, to check the performance of the system against the requirements of References 2 and 3.

- the mean touchdown distance,
- the probability of touching down on the runway short of a point 200 ft from the threshold,
- the probability of touching down on the runway beyond the “touchdown” zone lighting (*i.e.* more than 3000 ft from the threshold), and
- the touchdown distance that is three standard deviations beyond the mean touchdown point of (a), above.

4.1 The Standard Approach Angle and Standard Glide-Slope Origin Case

The percentage cumulative frequency data of Table 3.1 are plotted against touchdown distance in Sketch 4.1, using Normal/probability graph paper (see Section 3.3 of Reference 1). The “best straight line” is drawn through the data yielding a mean touchdown distance of 1518 ft and a standard deviation of 238 ft (see Section 3.3 of Reference 1). However, before using these values of mean and standard deviation, since the data are not a “perfect fit”, it would be as well to check that the “Normal Distribution” represented by the straight line is, in fact, an acceptable approximation to the sample distribution. This is done by applying the χ^2 test (see Section 5.1 of Reference 1).



Sketch 4.1 Touchdown dispersion cumulative frequency plot (standard approach)

The χ^2 calculation is summarised in Table 4.1. The interval limits and the observed frequencies for each interval are taken from Table 3.1 and the expected frequencies are derived from the expected percentage frequencies for each interval, which were in turn derived from the expected cumulative frequencies read from the straight line of Sketch 4.1 at each interval boundary.

The final column in Table 4.1 collects together the values of $(f - e)^2/e$, needed to calculate χ^2 in accordance with Equation (5.1) of Reference 1.

The value of χ^2 from Table 4.1 is 9.01 and this is appropriate to 13 degrees of freedom, one less than the number of intervals.