

A METHOD OF ESTIMATING A FLOW BREAKDOWN BOUNDARY FOR AEROFOILS AND SWEEP WINGS IN TRANSONIC FLOW

SUMMARY

A method is given of estimating a boundary related to flow separation for two-dimensional aerofoils and swept wings in transonic flow in terms of lift coefficient and freestream Mach number. Here, transonic flow refers to the range of subsonic freestream speeds for which the flow around the aerofoil or wing has one or more supersonic regions which may be terminated by shock waves. The method relates to a separated flow condition called 'flow breakdown'. This condition corresponds to the situation where, with increasing lift coefficient or with increasing freestream Mach number, the turbulent boundary layer in the flow over the upper surface becomes completely separated from the foot of a shock to the trailing edge. Such a condition occurs for a range of geometries and freestream Mach numbers, but there are cases where the types of separation which limit the usable lift are different in form from the one considered here and for these situations the present method does not apply.

The method has been derived using experimental data related to a number of aerofoils and wings covering a variety of streamwise pressure distributions, aspect ratios and sweep angles. The wing flows considered (and to which the method is restricted) are quasi two-dimensional, *i.e.* the spanwise variations of flow parameters are small. The validity of the method has not been checked extensively against experimental data other than those from which it was derived, but the checks that have been applied have shown that the method predicted the flow breakdown boundary with an r.m.s error of 0.04 in lift coefficient where the flow breakdown condition varies slowly with Mach number, and of the order 0.01 in Mach number where the flow breakdown condition varies slowly with lift coefficient.

1. INTRODUCTION

The aerodynamic manoeuvre limits of modern aircraft (*i.e.* stall, buffet and various forms of dynamic instability) are the results of significant boundary layer separation from the upper surface of the wing. For aircraft operating at transonic speeds, shock waves have a large influence on the development of such separations and knowledge of the conditions corresponding to the boundary between acceptable and unacceptable separations for a wing is very important both during the design process and subsequently.

Prediction by methods of computational fluid dynamics (CFD) is possible for both two-dimensional and three-dimensional separated flows. However, prediction of shock-induced separation boundaries by other methods is at present important and likely to remain so. These alternative (and simpler) methods are those which make use of correlations of observations, usually in combination with calculations of either non-separated (*i.e.* attached) flows or flows having very limited regions of separation. One such prediction method is given in Reference 1 and although this is ostensibly a method for two-dimensional aerofoil sections, it has also been adapted and applied to finite wings. For the extension to three-dimensional flows to be valid the type of wing flow where the critical separations occur must be quasi two-dimensional (*i.e.* spanwise variations of the relevant flow parameters must be small). The method of Reference 1 uses as a measure of significant flow separation the effect such separation has in reducing the trailing edge pressure coefficient.

The method presented here is based on the work of Fulker and Ashill². In contrast to the method of Reference 1, the present method yields a boundary related to a condition termed 'flow breakdown'. The separations with which the present method (and the method of Reference 1) are concerned, occur on the upper surface of the wing when the freestream speed is subsonic but is sufficiently high for the flow to develop one or

more local supersonic regions terminated by a shock wave[†]. Lower surface separations are not considered. Flow breakdown (defined in Section 2.2) corresponds to the condition where, with increasing lift coefficient or with increasing freestream Mach number, the turbulent boundary layer in the flow over the upper surface becomes completely separated from the foot of a shock to the trailing edge. Compared with the separated flow condition of Reference 1, the present method almost always relates to a greater degree of separation. The exceptions correspond to those circumstances where the flow separates virtually instantaneously from the shock to the trailing edge. As with the method of Reference 1, the present method is applicable to two-dimensional and three-dimensional flows provided the latter are quasi two-dimensional.

The ways in which various types of relevant shock-induced separation occur and develop have been analysed and classified by Pearcey, Haines and Osborne³. Pearcey *et al* recognised that, as well as the local interaction occurring at the foot of the shock (involving the shock and the turbulent boundary layer – see Sketch 1.1), other features could also be important in determining the development of significant separation. In particular, especially for highly loaded wings of modern design, the presence of a continuous adverse pressure gradient in the subsonic flow over the rear of the wing downstream of the shock could provoke a rear separation prior to the development of a separation bubble at the foot of the shock. In other cases the developing rear separation could be influenced by the shock together with a separation bubble at its foot. Consideration was also given to the effects of varying Reynolds number on the different types of shock wave/boundary layer interaction since it was apparent that this was of great importance in assessing the application of wind tunnel tests to flight conditions. It was observed that for ‘Model A’ separation development³ (*i.e.* one where separation occurs initially as a bubble at the foot of the shock and this bubble spreads rearwards to the trailing edge with increasing freestream Mach number) there is a relative insensitivity to increase of Reynolds number. However for ‘Model B’ separations, where a rear separation region develops (either with or without a separation at the foot of the shock), there is generally a much greater sensitivity to increases in Reynolds number and a corresponding greater uncertainty arising when wind tunnel tests are applied to flight conditions.

It is apparent that “correlation/calculation”[‡] methods for predicting separation boundaries, such as those of Reference 1 and the present method, should adopt flow models that conform reasonably closely with observation, but at the same time they utilise strategies whereby the detailed prediction of flow fields containing regions of separated flow is avoided^{††}. In Reference 1 this is achieved by defining two criteria, one relating to the direct effects of separation at the shock, and the other to the effects of trailing edge separation. These criteria are developed in terms of parameters calculated using a computer program (the viscous Garabedian and Korn program⁴ is recommended) which does not model separated flow. Nevertheless, such a program is found to be sufficiently predictive of separations in the sense that its results may be ‘calibrated’ against experimental data relating to flows which do involve separation. The accuracy of the resulting method is generally good (for a detailed discussion see Reference 1) but it has been found that a considerable underestimate of the observed trailing edge pressure divergence condition (which was the basis of the separation boundary of Reference 1) might occur when the wing developed a long separation bubble extending downstream from the shock without the trailing edge pressure being significantly disturbed. The present method should deal with this situation more reliably.

The flow model adopted by the present method is described in detail in Sections 4 and 5. The major feature

[†] The pressure distributions of some wing and aerofoil flows may indicate the presence of more than one shock. Sketch 4.3 depicts an example with two shocks. However, in the most common situation where there is significant separation, there is a single dominant supersonic flow region terminated by a shock or steep recompression, and it is to this situation that the present procedure is principally (although not exclusively) directed.

[‡] This name implies that whereas calculations of flow fields are required, these are not the only source of information and correlations based on observations are also utilised.

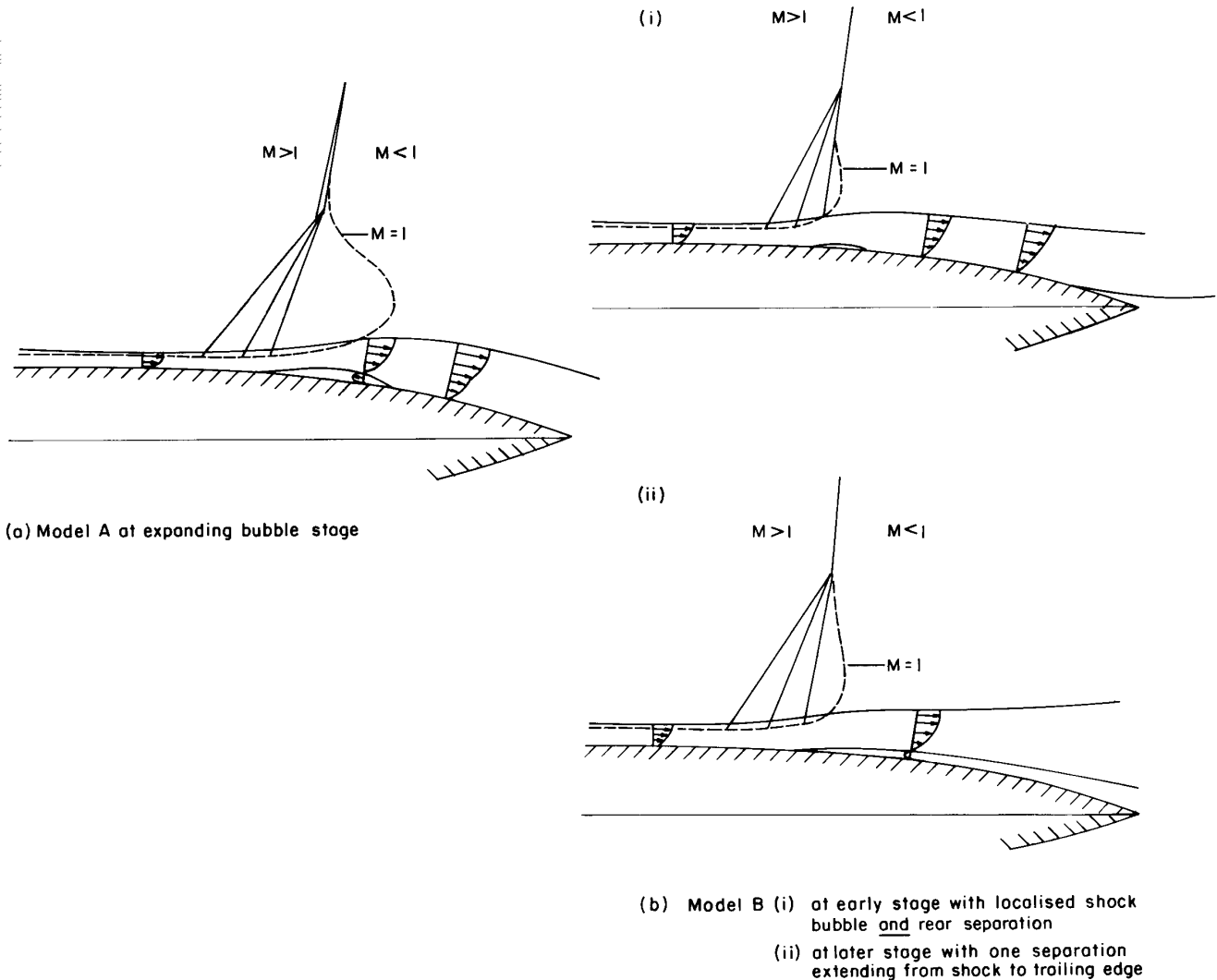
^{††} In some applications of the method it might be appropriate to use measured flow parameters made in the wind tunnel tests on the wing of interest. In these cases the objective would normally be to predict separation boundaries at higher Reynolds numbers than those of the tests.

is a separation bubble which forms at the foot of the shock and which grows in extent as the shock becomes stronger (corresponding to the freestream Mach number or lift coefficient being raised). The model links the growth of this separation bubble to the form of the attached flow pressure distribution. If this distribution has no severe adverse gradient in the region downstream of the shock, then the bubble grows steadily with increasing lift coefficient or Mach number and the bubble length is then related only to the strength of the shock and to the momentum thickness of the boundary layer immediately ahead of the shock. However, if the pressure distribution does have a region of relatively high adverse pressure gradient then, at some stage, this will induce the separation bubble to 'burst' (*i.e.* there will cease to be a reattachment of the boundary layer, and the separated flow will consequently extend from the shock to the trailing edge) which is, of course, the flow breakdown condition defined in Section 2.2. This bursting may either be associated with a 'linking-up' of a rear separation with the bubble formed at the foot of the shock or it may simply be the result of a very rapid (*i.e.* catastrophic) growth of the bubble. The prediction method based on the flow model thus comprises three elements dealing with, respectively, the length of the (unburst) bubble, the shock strength, and the position of the reattachment point of the bubble when bursting is imminent.

In terms of the flow models introduced by Pearcey, Haines and Osborne³ (reproduced in Figure 15 of Reference 10) the successive stages of flow development which 'bracket' the flow breakdown condition are as follows. Also given is an indication of the flow models to which the flow breakdown model may not be applied.

Model A	Stages V and VI
Model B1	Stages III and IV
Model B2(a)	Stages III and IV
Model B2(b)	Current model not applicable
Model B3(a)	Stages III and IV
Model B3(b)	Current model not applicable.

Further remarks concerning the applicability of the present method, together with its expected accuracy and limitations, are presented in Section 4. To some extent the present method can be seen as complementary to the method of Reference 1. There is a situation where the present method would be preferred (see above), but where trailing edge separation is dominant the reverse is likely to be the case, but it is important to note that the two methods correspond to different degrees of separation.



Sketch 1.1 Some details (schematic) of the region of interaction between turbulent boundary layer and shock wave in flow over aerofoil (from Reference 3)

2. NOTATION AND DEFINITIONS

2.1 Notation

All quantities of length except η_g made non-dimensional are based on local chord, c , in streamwise direction.

$\bar{c}$	geometric mean chord
C_L	lift coefficient (of aerofoil or wing)
C_p	pressure coefficient
f_1, f_2, f_3	denote functional relationships
g	maximum adverse pressure gradient in region downstream of shock, see Sketch 4.4
l_B	non-dimensional length of separated flow bubble prior to bursting
l'_B	non-dimensional length of separated flow bubble, see Equation (4.5)
$\bar{H}$	transformed boundary layer shape parameter
M_∞	freestream Mach number
M_L	local Mach number on aerofoil or wing surface
R	point on pressure distribution, upstream of the point (or points) of tangency of T , where C_p differs from T by 0.05 or, if gradient of T is not severe, x_R is set to 1.0, see Sketch 4.4
R_c	Reynolds number based on chord of aerofoil or on local streamwise chord (for a wing)
$R_{\bar{c}}$	Reynolds number based on geometric mean chord
R_θ	Reynolds number based on local boundary layer momentum thickness
T	tangent drawn to streamwise pressure distribution, downstream of shock and any separation bubble, at point (or points) of maximum adverse pressure gradient, see Sketch 4.4
u	flow speed in direction of flow at outer edge of boundary layer, non-dimensionalised by freestream speed
x	non-dimensional streamwise distance measured from leading edge (aerofoil) or from local leading edge (wing)
y	non-dimensional distance measured normal to chord
α	angle of incidence
η_g	spanwise distance measured from wing-fuselage or wing centre-line, non-dimensionalised by gross wing semi-span

θ non-dimensional momentum thickness of boundary layer in streamline coordinates (or non-dimensional 'streamwise' momentum thickness)

Λ sweep angle (positive for sweepback)

Suffices

act denotes actual value of shock strength

all denotes allowable value of shock strength

e denotes value at outer edge of boundary layer or external flow

fl.b denotes flow breakdown (when applied to M , freestream Mach number is also denoted)

m denotes furthest aft bubble re-attachment point (corresponding to imminent 'bursting' of bubble)

n denotes component normal to shock wave

R denotes value at point R , see Sketch 4.4

r denotes wing root, i.e. body side

sh denotes conditions on aerofoil or wing surface immediately upstream of shock, see Sketch 2.1, called 'shock-upstream' conditions

te denotes conditions at aerofoil or wing trailing edge

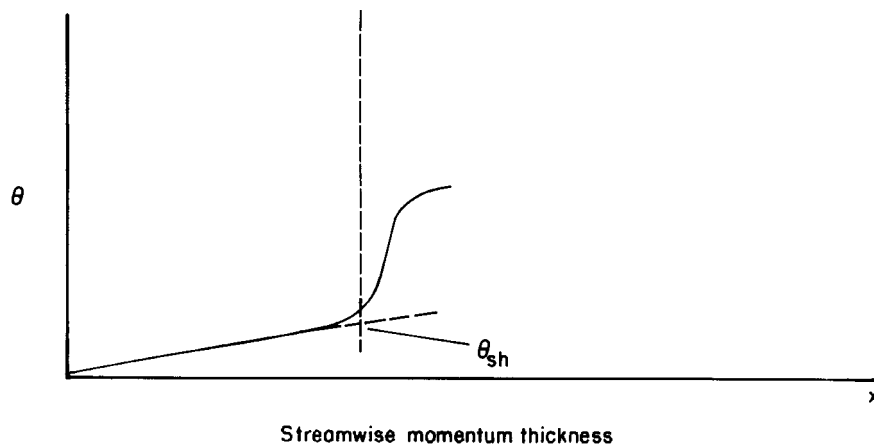
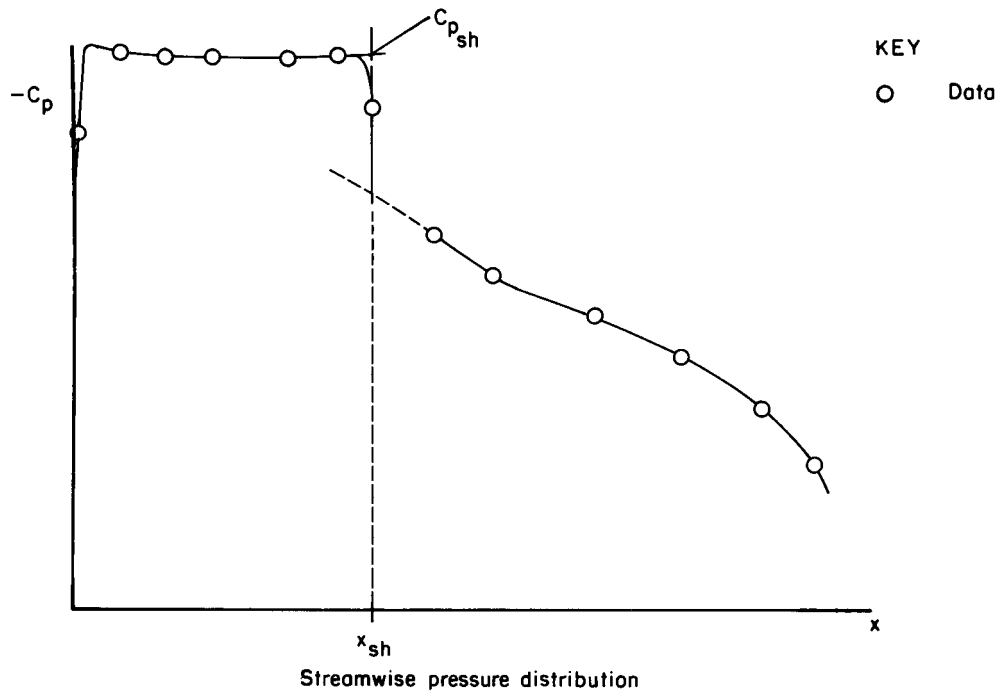
x denotes conditions at streamwise location x

2.2 Definitions

- (i) **Observed flow breakdown condition:** A condition $C_{L_{fl.b}}, M_{fl.b}$ obtained from experimental data corrected to free air conditions[†] at which, on an aerofoil or for at least one section of a wing, as appropriate, for all stations downstream of the shock, the decrease in pressure coefficient below the attached flow trend has, with increasing lift coefficient[‡], reached or exceeded 0.04.
- (ii) **Observed flow breakdown boundary:** The locus in the C_L, M_∞ plane of points $C_{L_{fl.b}}, M_{fl.b}$.
- (iii) **VGK:** An extension to viscous flow of the method of Garabedian and Korn for solving the transonic potential flow about two-dimensional aerofoils (see Reference 4).
- (iv) **Wings and aerofoils:** Unless specifically stated to the contrary, where reference is made to a wing it also implies the aerofoil case. Reference to an aerofoil, on the other hand, does not, unless stated otherwise, include a wing.
- (v) **Shock-upstream location and values (of C_p and θ , for example) relating to the shock-upstream location:** Sketch 2.1 is to be consulted for these definitions.

[†] The flow conditions include the Reynolds number and upper and lower surface transition locations for which the flow breakdown boundary is required.

[‡] When the flow breakdown boundary in the C_L, M_∞ plane is close to vertical, the determination of the boundary from experimental data may be facilitated by plotting C_{p_x} vs M_∞ for fixed lift coefficient (or incidence).



Sketch 2.1 Definition of shock-upstream location, x_{sh} , and related values of C_p and θ

3. THE CRITERION FOR PREDICTING FLOW BREAKDOWN

Flow breakdown is predicted to occur when $M_{L_{sh}}$ or $M_{L_{nsh}}$, as appropriate, has the value in terms of $R_{\theta_{sh}}$, θ_{sh} , x_R and x_{sh} given by Figure 2a or 2b. The application of this criterion is described in Section 4.

4. THE METHOD

For a specified aerofoil or wing geometry in a flow having specified Reynolds number, R_c or $R_{\bar{c}}$, the method yields a prediction of the associated flow breakdown boundary.

The method utilizes a flow model proposed by Fulker and Ashill (Reference 2) which is described in Section 4.1. It is shown that the flow model leads to two individual data requirements and these are described in detail in Section 4.2. Once these requirements are fulfilled for a particular aerofoil or wing, there is sufficient information available to enable points in the (C_L, M_∞) plane corresponding to flow breakdown to be determined. The detailed process for obtaining the required data and determining those points is presented in Sections 4.3 to 4.6.

4.1 The Flow Model of Fulker and Ashill

For simplicity the flow model is described here for the case of flow over an aerofoil, but the wing case is very similar and presents no additional conceptual difficulties. The extension to the wing case is referred to specifically in Section 4.4.

The major feature of the adopted flow model is a separation bubble which forms at the foot of the shock in the flow over the upper surface. The upstream end of the bubble (the separation point) is taken to coincide with the shock location, denoted x_{sh} . With increasing shock strength (due to increase in either the Mach number, M_∞ , or lift coefficient, C_L) the bubble lengthens. Increasing shock strength generally corresponds, at least initially, to a gradual rearward movement of the shock. Subsequently, the shock may move forward as M_∞ or C_L increases further and separation becomes more extensive. In either situation an increase in shock strength results in the re-attachment point of the separated flow bubble (*i.e.* the downstream extremity of the bubble) moving rearward, as shown in Sketches 4.1 and 4.2.

Flow breakdown is taken to coincide with the separation bubble lengthening to such an extent that it reaches a point (denoted by m) where it 'bursts' or fails to re-attach. The key elements of the flow model are:

- (a) length of the separation bubble prior to bursting, l_B ,
- (b) the furthest aft location at which bubble re-attachment can occur, denoted by x_m .

Fulker and Ashill² found that the length of the 'unburst' bubble, l_B , could be correlated in terms of flow parameters (namely θ , R_θ and M_L) corresponding to the shock-upstream location. Furthermore, by examining various aerofoil and wing pressure distributions it was found that, at the mean streamwise position of the bubble, the local pressure gradient could be correlated by

$$\frac{\theta_{sh}}{u_{sh}} \frac{du}{dx} = -3.3 \times 10^{-4}. \quad (4.1)$$

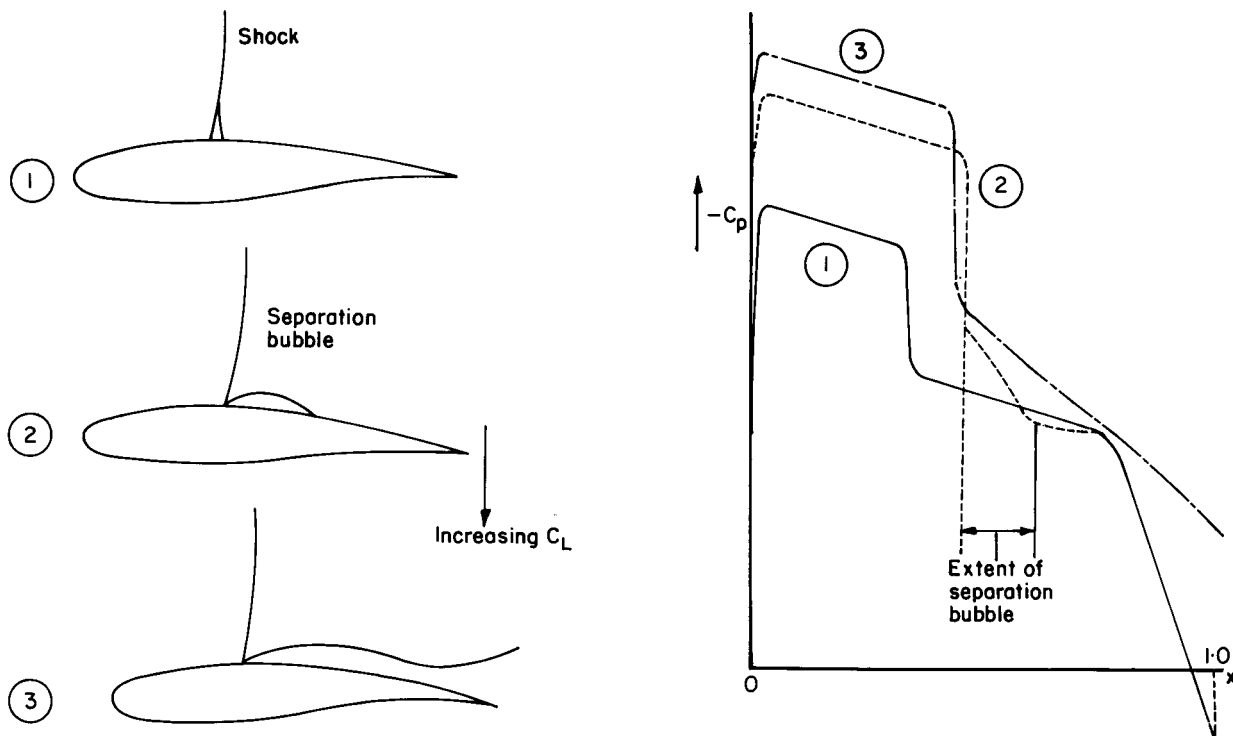
For those pressure distributions which exhibit adverse pressure gradients less severe than indicated by Equation (4.1), it was found that the re-attachment point of the bubble when flow breakdown occurs is close to the trailing edge. However, when the attached flow pressure distributions do have a region of relatively high adverse gradient (*i.e.* in excess of that indicated by Equation (4.1) bursting occurs at an

earlier stage. By introducing R as the effective commencement point[†] of any such high adverse gradient (or else, if no high adverse gradient region exists, by setting $x_R = 1.0$) it is found that x_m (the aft limit for bubble reattachment) is always close to x_R , and, moreover, the small (non-dimensional) chordwise distance, $x_R - x_m$, may be correlated in terms of θ_{sh} and $M_{L_{sh}}$.

An example of the way in which x_R , x_m , x_{sh} and l_B vary with C_L , at fixed M_∞ , is shown for a particular aerofoil in Sketch 4.2. Flow breakdown corresponds to the following equation being satisfied

$$x_m = x_{sh} + l_B, \quad (4.2)$$

and it is seen that this occurs at point P in Sketch 4.2. P thus provides a point $(C_{L_{fl.b}}, M_{fl.b})$ on the flow breakdown boundary of the particular aerofoil illustrated.



Sketch 4.1 Development of typical streamwise pressure distribution with increasing lift coefficient

4.2 Data Required in Order to Make Use of the Flow Model to Predict Flow Breakdown

To use the flow model to predict a flow breakdown boundary, it is first necessary to determine the parameters x_{sh} , l_B and x_m corresponding to a range of (C_L, M_∞) conditions for the aerofoil of interest, and then to determine those conditions for which Equation (4.2) is satisfied.

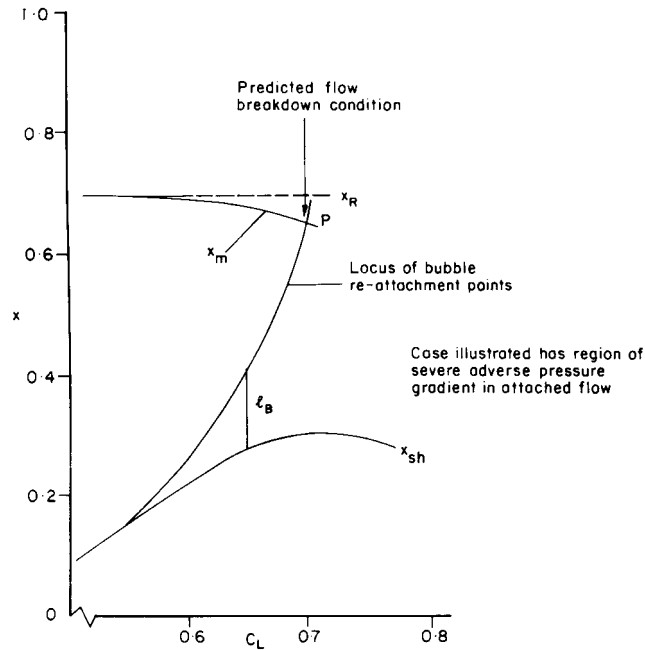
Fulker and Ashill² present a correlation of bubble length, l_B , in terms of the parameters θ_{sh} , $R_{\theta_{sh}}$ and $M_{L_{sh}}$. In functional form this correlation may be expressed as

$$\frac{l_B}{\theta_{sh}} = f_1(R_{\theta_{sh}}, M_{L_{sh}}). \quad (4.3)$$

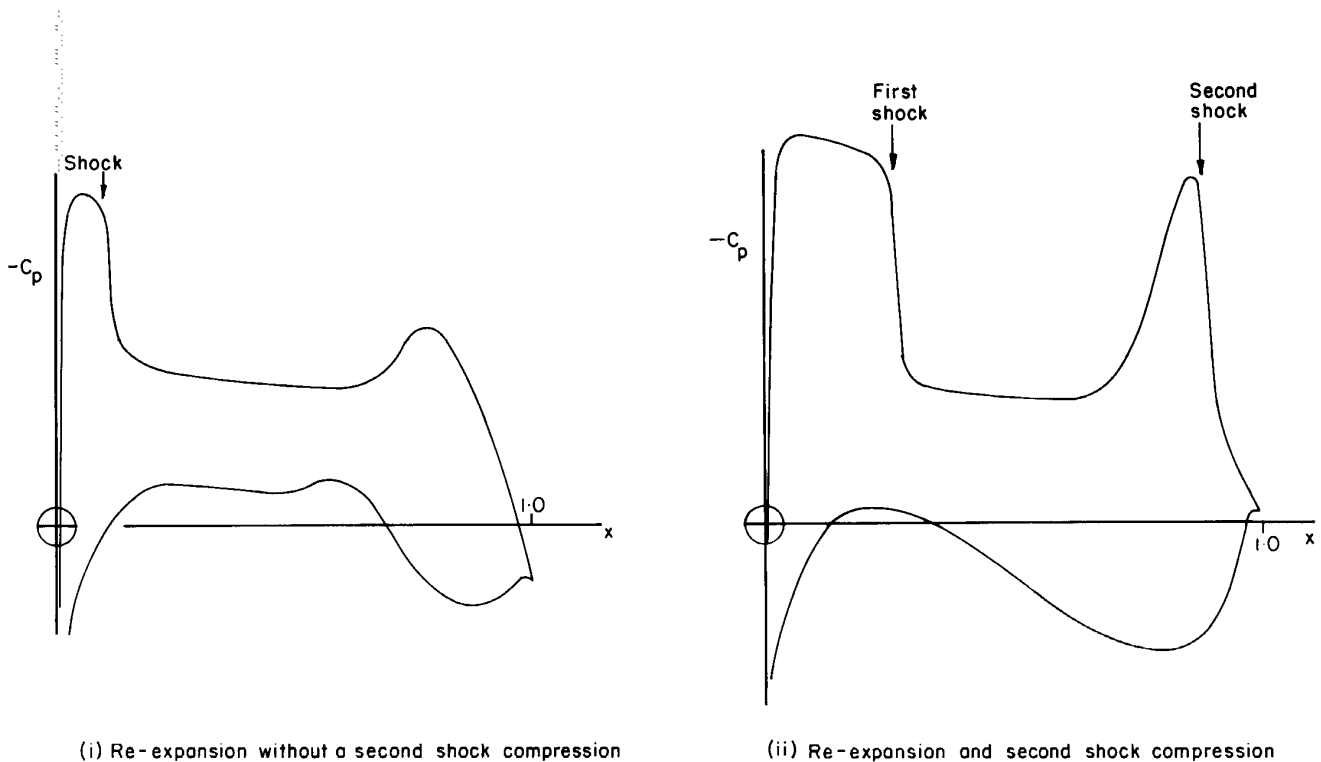
[†] Defined precisely in Section 4.6.1.

Having defined x_R in relation to a suitable pressure distribution, the authors found that $(x_R - x_m)/\theta_{sh}$ could be correlated with $M_{L_{sh}}$, i.e.

$$\frac{x_R - x_m}{\theta_{sh}} = f_2(M_{L_{sh}}). \quad (4.4)$$



Sketch 4.2 Typical variations of shock and separation bubble parameters with C_L at fixed M_∞



Sketch 4.3 Typical pressure distributions with local re-expansion

The nature of the function f_2 is particularly simple. Hence, in order to predict flow breakdown, for the aerofoil of interest, given the correlations represented by Equations (4.3) and (4.4), the requirements which remain may be identified as:

- (1) For each of a range of (C_L, M_∞) conditions it is necessary to find:

$$x_{sh}, M_{L_{sh}}, \theta_{sh} \text{ and } R_{\theta_{sh}}.$$

- (2) It is necessary to determine one or more suitable aerofoil pressure distributions so that x_R may be found for each condition.

The first of these requirements might appear particularly demanding, since as the parameter values are required for conditions at least close to those for flow breakdown (so that no extensive extrapolations are involved), then these conditions must correspond to a considerable degree of separation. It might also be supposed that any flow calculation method that yields sufficiently accurate values of x_{sh} , $M_{L_{sh}}$, θ_{sh} and $R_{\theta_{sh}}$ for given values of C_L and M_∞ where there is considerable separation would be capable of yielding flow breakdown directly, *i.e.* without the need for any correlation procedure. However, such suppositions are not correct, and it has been found that good predictions of flow breakdown can be achieved by utilizing certain calculation methods that are not directly applicable to separated flows. The reason for this is that the calculation methods that have been employed (VGK⁴ for aerofoil flows and a viscous coupled Euler method⁷ for three-dimensional flows) can provide a reliable representation of the flow upstream of the shock for a given lift coefficient, with the errors in the calculated load distribution downstream of the shock largely cancelling[†].

When using the present procedure for predicting flow breakdown, it is preferable not to employ a calculation scheme that reproduces too closely all the features of pressure distributions that may actually occur when there is extensive separation. Two such features that are best avoided are:

- (a) the forward movement of the shock with increasing lift coefficient,
- and (b) the reduction in shock strength (associated with reduction in $M_{L_{sh}}$) with increasing lift coefficient.

There are two reasons why these features are best not reproduced by the calculation methods employed. These are:

- (a) When taken into account, forward movement of the shock, and reduction in shock strength each result in the value of C_L corresponding to flow breakdown prediction being increased. (In Sketch 4.7 the way this occurs is illustrated.) In fact flow breakdown is not significantly delayed to higher C_L values by these features; indeed the features are reactions to flow separation which is at least sufficiently severe for flow breakdown to be imminent[‡].
- (b) Quite a different reason is that the prediction method has been developed and checked by making use of calculation methods which do not predict these particular features of separated flows. It is clear that predictions using the present scheme could actually be less accurate if calculation methods were employed which were capable of yielding these features. Even though it would doubtless be possible to modify the prediction process to accommodate more accurate flow calculation methods,

[†] Fulker and Ashill² indicated how improvements to the calculated pressure distribution for the region downstream of the shock might be achieved by making use of correlations for (a) shock strength, (b) the mean pressure gradient in the separated flow bubble, and (c) the bubble length (see Equation (4.3)). These improvements could be valuable if detailed loading or trim data were needed. They are not needed in order to predict flow breakdown.

[‡] By imminent it is implied that the lift coefficient is only very little below the flow breakdown value.

it would be questionable whether it would be worthwhile: a valuable characteristic of the prediction procedure is its ability to utilize calculation methods that are not capable of directly predicting separated flows.

4.3 Organisation of the Procedure for Predicting Flow Breakdown

Conditions corresponding to predicted flow breakdown (see Sketch 4.2) are those where Equation (4.2) is satisfied. In this Section the way these conditions are determined is outlined.

For each of a series of values of C_L (at fixed M_∞), two values of $M_{L_{sh}}$ are derived. These are the “actual” and “allowable” values, denoted by $M_{L_{sh, act}}$ and $M_{L_{sh, all}}$. The actual value of $M_{L_{sh}}$ is simply the value of local Mach number at the shock-upstream location, as determined by the flow calculation method applied at the specified (C_L, M_∞) condition[†]. The allowable value, $M_{L_{sh, all}}$ is the value of the shock-upstream Mach number which would produce a separated flow bubble of length l'_B , where

$$l'_B = x_m - x_{sh}. \quad (4.5)$$

Thus, in terms of the correlation represented by Equation (4.2), $M_{L_{sh, all}}$ is found such that

$$\frac{l'_B}{\theta_{sh}} = f_1(R_{\theta_{sh}}, M_{L_{sh, all}}). \quad (4.6)$$

In Equation (4.5), x_{sh} is found from the calculation method. The value of x_m is found from x_R together with the correlation represented by Equation (4.4).

In Equations (4.5) and (4.6) the values of θ_{sh} and $R_{\theta_{sh}}$ used are those obtained from the flow calculation method. Finally, the predicted flow breakdown condition at the particular fixed value of M_∞ is found from the intersection of the curves represented by the variations

$$M_{L_{sh, act}} \sim C_L,$$

and
$$M_{L_{sh, all}} \sim C_L.$$

In practice, the method of determining flow conditions where Equation (4.2) is satisfied is slightly different from the foregoing. The difference is introduced to make the procedure slightly simpler, and this is done by effectively merging the correlations represented by Equations (4.3) and (4.4). By adding to the bubble length when flow breakdown is imminent, $x_m - x_{sh}$, the length $x_R - x_m$, the separate evaluation and use of x_m can be avoided. This is possible because of the nature of the correlations determined by Fulker and Ashill². Thus, from Equation (4.3),

$$\frac{x_m - x_{sh}}{\theta_{sh}} = f_1(R_{\theta_{sh}}, M_{L_{sh, all}}), \quad (4.7)$$

and from Equation (4.4)

$$\frac{x_R - x_m}{\theta_{sh}} = f_2(M_{L_{sh, all}}). \quad (4.8)$$

[†] For this reason the suffix *act* is omitted in subsequent Sections.