ESDU Copyright material. For current status contact ESDU.

ESDU 91025

Issued December 1991
With Amendment E
March 2007
Re-issued June 2011

Non-Newtonian fluids: Frictional pressure loss prediction for fully-developed flow in straight pipes

Associated software: ESDUpac A9125A and A9125B

Endorsed by
The Institution of Chemical Engineers
The Institution of Mechanical Engineers

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Non-Newtonian Flow Working Party, on behalf of the Internal Flow Panel, and had the following membership:

Prof. P.F.G. Banfill	— Herriot Watt University
Prof. H.A. Barnes	— Unilever Research
Dr D.C-H. Cheng	— Independent Consultant
Dr N.I. Heywood	— AEA Technology plc
Dr. P.J.G. Long	— Cambridge University
Dr A. Tatsis	— ETSU, AEA Technology.

The Internal Flow Panel first met in 1979 and had the following membership:

Chairman	
Dr J.A. Eaton	— University College Galway, Ireland
Members	
Mr D.A. Campbell	— Rolls-Royce plc, Derby
Mr J. Campbell	— Independent
Dr C.J. Clark	— BP International Ltd
Prof. D.H. Freeston*	— Auckland University, New Zealand
Dr W.R. Geddes	— BNFL Engineering
Dr M.E. Gill	— W.S. Atkins Science and Technology
Mr A.J. Green	— BHR Group Ltd
Prof. J.L. Livesey	— University of Salford
Dr. P.J.G. Long	— Cambridge University
Dr M. Moore	— Independent.

* Corresponding Member

The technical work involved in the assessment of the available information and the construction and subsequent development of the Data Item and computer programs was undertaken by

Dr S.A.Logan	— Group Head, Software Development.

(*Continued on inside back cover*)

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

NON-NEWTONIAN FLUIDS: FRICTIONAL PRESSURE LOSS PREDICTION FOR FULLY DEVELOPED FLOW IN STRAIGHT PIPES
PART I: BACKGROUND, ANALYSES AND SOLUTION OUTLINE

CONTENTS

		Page
1.	NOTATION AND UNITS	1
2.	BACKGROUND	3
2.1	Purpose and Scope	3
2.2	Definition of Non-Newtonian Fluids	4
2.3	Examples of Non-Newtonian Fluids	4
3.	RHEOLOGICAL MODELS	5
3.1	General	5
3.2	Metzner-Reed Model	5
3.3	Shear Thinning/Shear Thickening Fluids	6
3.3.1	Power-law model	7
3.4	Bingham Fluid Model	7
3.5	Herschel-Bulkley Fluid Model	7
4.	DETERMINATION OF FLOW REGIME	8
4.1	General Case	8
4.2	Reynolds Number for Power-law Model	9
4.3	Reynolds Number for Bingham Model	9
4.4	Reynolds Number for Herschel-Bulkley Model	10
5.	FRICTIONAL LOSSES IN LAMINAR FLOW IN PIPES	11
5.1	Method I	11
5.1.1	Power-law model	12
5.1.2	Bingham model	12
5.1.3	Herschel-Bulkley model	12
5.2	Method II	13
6.	FRICTIONAL LOSSES IN TURBULENT FLOW IN SMOOTH PIPES	14
6.1	Method of Dodge and Metzner (Method I)	14
6.1.1	Applicability to power-law fluids	14
6.1.2	Applicability to models with a yield stress	15
6.2	Method of Bowen (Method II)	15
6.2.1	Description	15
6.2.2	Pressure loss for other pipe diameters	17

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

7. FRICTIONAL LOSSES IN TURBULENT FLOW IN ROUGH-WALLED PIPES 18
7.1 Intermediate Rough-walled Pipes ($\varepsilon/D \leq 0.1$) 18
7.2 Fully Rough-walled Pipes 18

8. FRICTIONAL LOSSES IN TRANSITIONAL REGION FLOW IN SMOOTH PIPES 19

9. WORKED EXAMPLES 20
9.1 Example 1: Laminar Flow 20
9.1.1 Method I from Section 5.1 22
9.1.2 Method II from Section 5.2 22
9.2 Example 2: Turbulent Flow of a Power-Law Fluid 23
9.2.1 Method I from Section 6.1 24
9.2.2 Method II from Section 6.2 25
9.3 Example 3: Turbulent Flow of a Bingham Fluid 30

10. DERIVATION AND REFERENCES 34
10.1 Derivation 34
10.2 References 35

PART II: COMPUTER PROGRAMS

11. INTRODUCTION 39
11.1 Running the Programs 39

12. METHOD OF DODGE AND METZNER: PROGRAM A9125A 40
12.1 Notes 40
12.2 Program Input 40
12.3 Program Output 45
12.4 Examples 45
12.5 Fitting the Herschel-Bulkley Model to Experimental Data 52
12.5.1 Applying the Herschel-Bulkley model to ($\dot{\gamma}$, τ_w) data pairs 52
12.5.2 Applying the Herschel-Bulkley model to ($8V/D$, τ_w) data pairs 52
12.5.3 Finding n', K', Re' and f 53

13. METHOD OF BOWEN: PROGRAM A9125B 55
13.1 Notes 55
13.2 Program Input 55
13.3 Program Output 59
13.4 Examples 59

APPENDIX A – DERIVATION OF LAMINAR FLOW EQUATIONS 67
A.1 POWER-LAW MODEL 67
A.2 BINGHAM MODEL 67
A.3 HERSCHEL-BULKLEY MODEL 68

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

APPENDIX B – DETERMINATION OF THE OPTIMUM TURBULENT FLOW CORRELATION 69
B.1 EQUATIONS USED 69
B.2 DATA ANALYSIS 70
B.3 REASONS FOR USING THE DODGE-METZNER CORRELATION 72

APPENDIX C – ADDITIONAL FRICTIONAL PRESSURE DROP ESTIMATION METHODS 74
C.1 NOTES 74
C.2 FRICTIONAL PRESSURE LOSS FROM LAMINAR FLOW VISCOMETRIC DATA NOT FITTED TO A FLOW CURVE 74
C.3 FRICTIONAL LOSSES IN TURBULENT FLOW IN SMOOTH PIPES FROM LAMINAR FLOW VISCOMETRIC DATA NOT FITTED TO A FLOW CURVE 76
C.4 FRICTIONAL LOSSES IN TURBULENT FLOW IN SMOOTH PIPE FROM LAMINAR FLOW VISCOMETRIC DATA FITTED TO A BINGHAM PLASTIC MODEL 76
C.5 FRICTIONAL LOSSES IN TURBULENT FLOW IN SMOOTH PIPE FROM LAMINAR FLOW VISCOMETRIC DATA FITTED TO A HERSCHEL-BULKLEY MODEL 77
C.6 FRICTIONAL LOSSES IN TURBULENT FLOW IN ROUGH PIPES FROM LAMINAR FLOW VISCOMETRIC DATA NOT FITTED TO A FLOW CURVE 78
C.7 FRICTIONAL LOSSES IN TURBULENT FLOW IN ROUGH PIPES FROM LAMINAR FLOW VISCOMETRIC DATA FITTED TO A HERSCHEL-BULKLEY MODEL 78

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

NON-NEWTONIAN FLUIDS: FRICTIONAL PRESSURE LOSS PREDICTION FOR FULLY DEVELOPED FLOW IN STRAIGHT PIPES

PART I: BACKGROUND, ANALYSES AND SOLUTION OUTLINE

1. NOTATION AND UNITS

		Units	
		SI	British
A	experimental coefficient used in Bowen's method (see Section 6.2)	N s^b/m^{2+b-c}	lbf s^b/ft^{2+b-c}
b	experimental coefficient used in Bowen's method (see Section 6.2), or regression coefficient in Equation (9.16)	–	–
c	experimental coefficient used in Bowen's method (see Section 6), or coefficient in Darby and Melson equation (see Appendix C.4)	–	–
D	pipe diameter	m	ft
f	friction factor (= $\tau_w / ½\rho V^2$)	–	–
He_B	Bingham plastic Hedstrom number given by Equation (4.12)	–	–
He_{HB}	Herschel-Bulkley Hedstrom number given by Equation (4.16)	–	–
K	consistency coefficient or power law coefficient	N s^n/m^2 (Pa s^n)	lbf s^n/ft^2
K_{HB}	Herschel-Bulkley consistency coefficient	N s^n/m^2 (Pa s^n)	lbf s^n/ft^2
K'	local consistency coefficient	N $s^{n'}/m^2$ (Pa $s^{n'}$)	lbf $s^{n'}/ft^2$
L	pipe length	m	ft
m	coefficient in Equation (C.7)	–	–
n	flow behaviour index or power law exponent	–	–
n'	local flow behaviour index	–	–
Δp	frictional pressure drop through pipe	N/m^2 (Pa)	lbf/ft^2
Q	volume flowrate	m^3/s	ft^3/s
r	radial coordinate	m	ft

Issued December 1991 – 78 pages
With Amendment E – March 2007
Re-issued June 2011 – 78 pages

ESDU Copyright material. For current status contact ESDU.
Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

Re	Reynolds number ($= \rho VD/\mu$)	–	–
Re'	generalised Reynolds number (see Equation (4.4))	–	–
Re'_P	power-law Reynolds number (see Equation (4.7))	–	–
Re_B	Bingham Reynolds number (see Equation (4.9))	–	–
Re_{HB}	Herschel-Bulkley Reynolds number (see Equation (4.13))	–	–
v	local instantaneous axial velocity	m/s	ft/s
V	mean axial velocity	m/s	ft/s
x	ratio of yield shear stress to wall shear stress (τ_{yB}/τ_w or τ_{yHB}/τ_w)	–	–
x_c	critical value of x for Bingham laminar limit (see Section 4.3)	–	–
$\dot{\gamma}$	shear rate	1/s	1/s
ε	pipe roughness height	m	ft
μ	dynamic viscosity	N s/m^2 (Pa s)	lbf s/ft^2
μ_B	Bingham plastic viscosity (see Equation (3.6))	N s/m^2 (Pa s)	lbf s/ft^2
ρ	fluid density	kg/m^3	slug/ft^3
τ	shear stress	N/m^2 (Pa)	lbf/ft^2
τ_w	wall shear stress ($= D\Delta p/4L$)	N/m^2 (Pa)	lbf/ft^2
τ_y	yield shear stress	N/m^2 (Pa)	lbf/ft^2
τ_{yB}	Bingham yield shear stress	N/m^2 (Pa)	lbf/ft^2
τ_{yHB}	Herschel-Bulkley yield shear stress	N/m^2 (Pa)	lbf/ft^2

Subscripts

c	refer to critical value
lam	refers to laminar flow conditions
min	refers to turbulent flow conditions
r	refers to rough pipe
s	refers to smooth pipe
$turb$	refers to turbulent flow conditions

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

2. BACKGROUND

2.1 Purpose and Scope

This Data Item is one of a group on the flow properties of non-Newtonian fluids. The first, ESDU 97034[26], provides an introduction to the identification, classification and characterisation of non-Newtonian fluids, including the measurement methods. The second, ESDU 95012[34], provides guidance on obtaining viscometric data necessary for estimating frictional pressure drop in pipeflow using the methods of the present Item, which is the third in the group. Finally, ESDU 04005[27] contains a worked example that illustrates the use of a tube viscometer to measure viscometric data and develop the flow curve.

This item is applicable to fluids that can be considered as pseudohomogeneous (non-setting) under conditions of steady shear. Methods are given in this Item for estimating frictional pressure losses for the case of fully-developed isothermal flow of these non-Newtonian fluids flowing in straight pipes of uniform circular cross section.

To use this Item it is necessary to (a) know the rheological category of the fluid or (b) conduct some rheological tests in order to categorise it. The user should refer to ESDU 95012 for further explanation and practical guidance on flow curve measurement.

In this Item Section 3 deals with the various rheological descriptions in regular use and explains the mathematical models used to depict the observed behaviour.

Once the type of fluid has been established and an appropriate model chosen the flow regime is found from Section 4.

Knowing both the category and the flow regime the user may determine frictional losses using the methods of Section 5 for laminar flow, Section 6 for turbulent flow in smooth pipes or Section 7 for turbulent flow in rough pipes. Laminar flow in pipes is amenable to an exact mathematical analysis that is well supported by experiment. Turbulent flow is not fully described by any theoretical analyses and the calculation procedures are based on the correlation of experimental data.

Although transitional flows are best avoided, Section 8 outlines an approximate method for predicting frictional losses in the transitional regime.

Section 9 gives worked examples of the estimation of frictional losses for laminar and turbulent flows. These worked examples illustrate the use of the graphical methods.

Section 10 lists the sources used in the preparation of this Item and indicates further references.

The derivations of the laminar flow equations in Section 5 are given in Appendix A.

Appendix B gives the results of the analysis of the data sets used to establish the choice of turbulent correlation.

Appendix C provides a summary of some additional frictional pressure drop estimation methods.

Two computer programs are presented in Part II, together with examples of their use.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

2.2 Definition of Non-Newtonian Fluids

When the fluid viscosity is independent of the velocity gradient, the fluid is termed Newtonian and the relationship between applied shear rate and the resulting shear stress, for both laminar and turbulent flow, is written

$$\tau = \mu\dot{\gamma}, \tag{2.1}$$

where, for laminar pipe flow,

$$\dot{\gamma} = \frac{dv}{dr}. \tag{2.2}$$

Fluids for which this relationship does not apply are, by definition, non-Newtonian. Various models have been proposed to describe the shear stress/shear rate relationship for non-Newtonian[26] flow. Guidance on the selection of an appropriate model for a particular application is given in Section 3.

2.3 Examples of Non-Newtonian Fluids

Listed in Table 2.1 are fluids that may exhibit non-Newtonian behaviour[14]. It is difficult to predict whether a particular fluid will exhibit non-Newtonian behaviour but suspensions of solids in liquids, high molecular weight polymer solutions and many multiphase fluids often show non-Newtonian characteristics. In any event a rheological test is the only sure method of ascertaining the rheological behaviour of a fluid. The user should refer to ESDU 95012 for guidance on rheological tests.

TABLE 2.1 Some Examples of Fluids Exhibiting Non-Newtonian Behaviour

Adhesives	Peat slurries
Biological fluids	Plastic melts
Cement slurries	Polymer solutions
Chalk slurries	Printing inks
Chocolate melts	Quicksand
Coal slurries	Rock slurries
Detergent slurries	Rubber solutions
Food sauces	Sand slurries
Greases	Sewage sludges
Handcreams	Shampoo
Margarine	Soap slurries
Mayonnaise	Starch solutions
Metal oxide slurries	Tomato paste
Oilwell drilling muds	Toothpaste
Paints	Wet beach sand
Paper pulp	

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

3. RHEOLOGICAL MODELS

3.1 General

Many models have been proposed to describe the observed relationship between the imposed shear rate and resultant shear stress for non-Newtonian fluids. The majority of these are of little value for engineering design purposes and serve more as theoretical analyses. However, there are several well established models used widely in the prediction of non-Newtonian fluid rheology. Section 3.2 gives a general model applicable to any fluid type. Three models, each specific to one fluid type, are given in Sections 3.3, 3.4 and 3.5.

The plot of shear stress, τ, against shear rate, $\dot{\gamma}$, is known as the flow curve. Determining the flow curve from viscometric data can be a complex matter and is described in ESDU 95012. Choosing a suitable model to fit a particular flow curve is a matter of some engineering judgement. In addition, the designer should note that the rheology of many non-Newtonian fluids will be described by more than one model *depending on the shear rate and shear stress ranges*. For example, the power-law model outlined in Section 3.3.1 can characterise adequately the behaviour of many fluids over a certain range of shear rate but may fail as $\dot{\gamma}$ approaches zero in one extreme and high values at the other. This does not, however, negate the suitability of the power-law model if it is an adequate representation over *the operational shear stress range of the pipeline*, but extrapolation *outside* this range is invalid. This important point must be borne in mind when designing the laminar characterisation tests used to determine the rheological constants. It is crucial that these tests are conducted over a similar wall shear stress range and at the same temperature as would be expected in the actual pipeline.

Guidance on appropriate shear stress/shear rate windows is given in Section 5.1 of ESDU 95012.

3.2 Metzner-Reed Model

Metzner and Reed[3] developed a completely general technique for expressing the rheology of *any* fluid in circular section pipe flow by considering wall shear stress, $\tau_w (= D\Delta p/4L)$, as a function of the notional* wall shear rate $8V/D$. This relationship is

$$\frac{D\Delta p}{4L} = K'\left(\frac{8V}{D}\right)^{n'}. \tag{3.1}$$

In physical terms n' can be interpreted as that physical property of a fluid which characterises its degree of non-Newtonian behaviour (analogous to n from the power law model of Section 3.3.1). Similarly K' is analogous to K and ultimately to viscosity; the higher the value of K' the more viscous the fluid. On a log-log plot of wall shear stress against $8V/D$, n' is a best fit tangent drawn through the data at $8V/D$, and $\log_{10} K'$ is the y-axis intercept of the tangent. Equation (3.1) can be rewritten to define n' as

$$n' = \frac{\mathrm{d}\left[\log_{10}\dfrac{D\Delta p}{4L}\right]}{\mathrm{d}\left[\log_{10}\left(\dfrac{8V}{D}\right)\right]}. \tag{3.2}$$

For many fluids K' and n' are constants over wide ranges of $8V/D$ and wall shear stress (when the fluid falls into the power-law model category of Section 3.3.1) but some care must be taken to determine whether this actually is the situation. In general different values of K' and n' would have to be used for each value

* That is, $8V/D$ would be the wall shear rate if the fluid were Newtonian. For a non-Newtonian fluid flowing in a circular-section pipe, actual wall shear rate is related to $8V/D$ by Equation (5.3) of ESDU 95012.

ESDU Copyright material. For current status contact ESDU.
Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

of $8V/D$, and Equation (3.2) would be the equation of the tangent to the logarithmic plot of τ_w against $8V/D$ at that point. In any event the easiest and most rigorous way of applying these relationships to a real fluid is to measure the pressure drop and flowrate in a tube viscometer under laminar flow conditions and over the wall shear stress range of interest. This method is outlined in ESDU 95012 and will permit rapid evaluation of $8V/D$ and wall shear stress, τ_w. Values of K' and n' are then found from a logarithmic plot of wall shear stress against $8V/D$. If the log-log plot is not a straight line (that is, the fluid is not a power-law type), care must be taken to ensure that the estimates of K' and n' are taken at a value, or over a range of values, of wall shear stress that would be appropriate to the scaled-up case*.

Further, the relationship between K' and K and n and n' for a power-law fluid (see Section 3.3.1) can be seen to be

$$n' = n$$

and

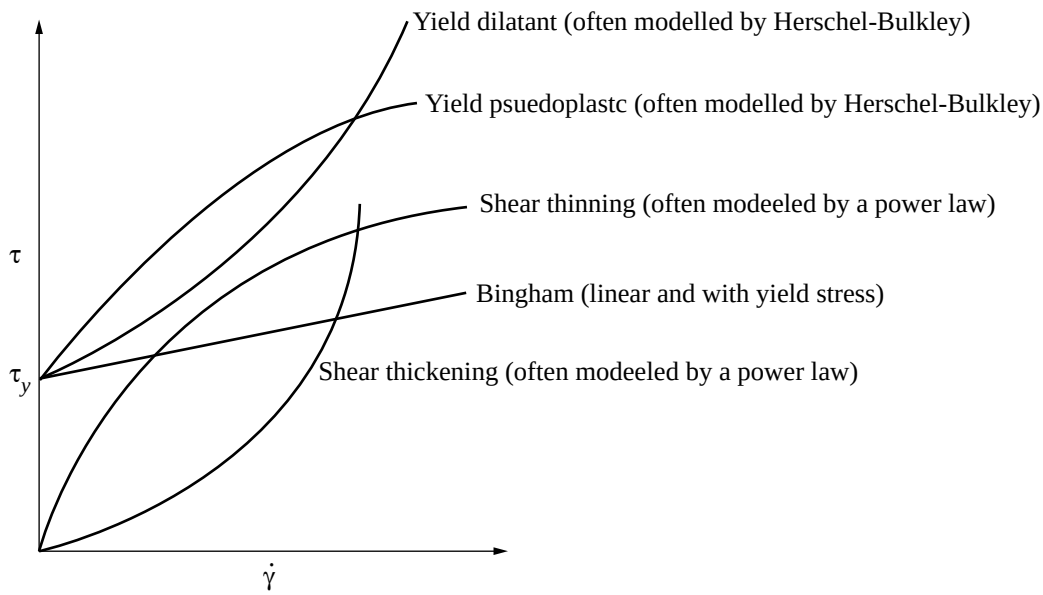
$$K' = K\left(\frac{1+3n}{4n}\right)^n . \tag{3.3}$$

For a Newtonian fluid, $n' = 1$ and K' = dynamic viscosity, μ.

3.3 Shear Thinning/Shear Thickening Fluids

These fluids are those for which an infinitesimal shear stress will initiate motion (the flow curve passes through the origin). Forms of the relationship between shear rate and shear stress (that is, the flow curve) are shown in Sketch 3.1.

Shear thinning or pseudoplastic fluids exhibit a non-linear shear stress/shear rate relationship in which the rate of increase of shear stress with shear rate decreases with increasing shear rate.



Sketch 3.1 The flow curve: Flow type and flow models

Shear thickening or dilatant fluids exhibit a similar non-linear shear stress/shear rate relationship except that the rate of increase of shear stress with shear rate increases with increasing shear rate.

* In most cases the pipeline that the engineer is designing is of much larger diameter than the tube used in the characterisation experiments. Using small bore tube viscometers under laminar-flow conditions allows a higher $8V/D$ value than would be obtainable in a full size pipe. Hence, in most cases, engineers would be scaling-up their characterisation tests and lower values of $8V/D$ would be involved.

ESDU Copyright material. For current status contact ESDU.
Documents Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

3.3.1 Power-law model

The shear stress/shear rate relationship for both shear thinning and shear thickening fluid types can often (but not always) be described by a power-law model,

$$\tau = K\dot{\gamma}^n, \tag{3.4}$$

where K is the consistency coefficient or power-law coefficient and n is the flow behaviour index or power-law exponent. For shear thinning (pseudoplastic) behaviour n is less than unity whereas for shear thickening (dilatant) behaviour n is greater than unity. This relationship may hold over a very wide range of shear rate*, only failing at very low or very high $\dot{\gamma}$. Note that the power-law equation reduces to the Newtonian relationship

$$\tau = \mu\dot{\gamma} \tag{3.5}$$

when $n = 1$, giving K = dynamic viscosity, μ.

3.4 Bingham Fluid Model

Bingham fluids are those in which a finite shear stress is required to initiate motion (the flow curve does not pass through the origin) and for which there is a linear relationship between shear stress and shear rate beyond that point (see Sketch 3.1).

The constitutive relationship can be written

$$\tau = \tau_{yB} + \mu_B\dot{\gamma}, \tag{3.6}$$

where τ_{yB} is the Bingham yield shear stress, that is the shear stress required to initiate movement, and μ_B is termed the Bingham 'plastic' viscosity. Note that this expression also collapses to that for the Newtonian case when τ_y equals zero.

3.5 Herschel-Bulkley Fluid Model

Other time-independent non-Newtonian fluids can sometimes be considered as combinations of the above types. Yield pseudoplastic and yield dilatant fluids require a finite shear stress to initiate motion but behave as their non-yield counterparts above that shear stress level. A general constitutive equation, known as the Herschel-Bulkley model (or sometimes the generalised Bingham model), can be written:

$$\tau = \tau_{yHB} + K_{HB}\dot{\gamma}^n. \tag{3.7}$$

This collapses to Equation (3.4) or (3.6) when suitable values of K_{HB}, n, and τ_{yHB} are chosen.

* Other relationships such as the Cross or Sisko models (see ESDU 97034[26]) may better approximate the behaviour of real fluids over a wider range but for engineering purposes the power-law model is often sufficient.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

4. DETERMINATION OF FLOW REGIME

4.1 General Case

As in the Newtonian case the flow of non-Newtonian fluids can be laminar, transitional or turbulent. Before the frictional losses in a pipeline can be established the flow regime must be known. Metzner and Reed[3] proposed a generalised Reynolds number definition to cover any rheological model. They took the concept of friction factor as used in *laminar* Newtonian flow when

$$f = \frac{16}{Re}, \tag{4.1}$$

where

$$f = \frac{\tau_w}{\frac{1}{2}\rho V^2}, \tag{4.2}$$

and inserted the model of Equation (3.1) to give a non-Newtonian relationship for *laminar flow,*

$$f = \frac{16}{Re'}, \tag{4.3}$$

where Re' is a generalised Reynolds number and is defined as

$$Re' = \frac{D^{n'} V^{2-n'} \rho}{8^{n'-1} K'}, \tag{4.4}$$

where n' is given by Equation (3.2) as

$$n' = \frac{\mathrm{d}[\log_{10}\tau_w]}{\mathrm{d}\left[\log_{10}\left(\frac{8V}{D}\right)\right]}, \tag{4.5}$$

in which $\tau_w = D\Delta p/4L$, and K' is given by

$$K' = \frac{\tau_w}{\left(\frac{8V}{D}\right)^{n'}}. \tag{4.6}$$

These values of n' and K' must be estimated from the flow curve at the appropriate value of ($8V/D$).

For Newtonian fluids in fully-developed isothermal pipe flow, the critical Reynolds number denoting the onset of turbulence occurs at about 2100 for most engineering situations. This corresponds to a critical friction factor value of approximately 0.008.

The following Newtonian definitions of laminar, transition and turbulent regions are suggested. These serve only as a guide and are not rigorous:

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

laminar	$Re' < 2100$	$(f > 0.008)$
transition	$2100 \le Re' < 3500$	$(0.005 < f \le 0.008)$
turbulent	$Re' \ge 3500$	$(f \le 0.005)$.

For non-Newtonian fluids both the critical Reynolds number and the range of the transition region vary from fluid to fluid.

4.2 Reynolds Number for Power-law Model

For a power-law fluid the Reynolds Number can be obtained from Equations (4.4) and (3.3) as

$$Re'_P = \frac{D^n V^{2-n} \rho}{8^{n-1} K} \left(\frac{4n}{1+3n} \right)^n . \tag{4.7}$$

The power law Reynolds number collapses to the Reynolds number for a Newtonian fluid when $n = 1$ and $K = \mu$.

The power-law Reynolds number for the laminar limit is given as[30]

$$Re'_{P,c} = \frac{6464 n (n+2)^{\left(\frac{n+2}{n+1}\right)}}{(1+3n)^2} . \tag{4.8}$$

For a wide range of n (approximately 0.2 to 1.0), $Re'_{P,c}$ varies between 1800 and 2400.

4.3 Reynolds Number for Bingham Model

For a Bingham fluid the calculation of n' and K' may be of considerable difficulty (depending on the type of viscometer used in the characterisation test) and for purposes of establishing the flow regime a Bingham Reynolds number can be defined as

$$Re_B = \frac{\rho V D}{\mu_B} , \tag{4.9}$$

where μ_B is the Bingham plastic viscosity, defined by Equation (3.6). The Bingham Reynolds number for the laminar limit is illustrated in Figure 2 and can be estimated from[13]

$$Re_{B,c} = \frac{He_B}{8 x_c} \left(1 - \frac{4}{3} x_c + \frac{1}{3} x_c^4 \right) , \tag{4.10}$$

where x_c is the critical value of the ratio τ_{yB} / τ_w, given over the range $10^4 < He < 10^7$ by

$$\frac{x_c}{(1-x_c)^3} = \frac{He_B}{16\ 800} , \tag{4.11}$$

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

and the Bingham Hedstrom number, He_B, is defined as

$$He_B = \frac{\rho D^2 \tau_{yB}}{\mu_B^2}. \tag{4.12}$$

An iterative calculation is required for x_c.

4.4 Reynolds Number for Herschel-Bulkley Model

A Herschel-Bulkley Reynolds number can be written in a similar form to the power-law Reynolds number of Equation (4.7) as

$$Re_{HB} = \frac{D^n V^{2-n} \rho}{8^{n-1} K} \left(\frac{4n}{1+3n} \right)^n . \tag{4.13}$$

The Herschel-Bulkley Reynolds number for the laminar limit is given by[13]

$$Re_{HB,c} = \frac{6464n}{(1+3n)^n} (2+n)^{\frac{2+n}{1+n}} \frac{\left[\frac{(1-x_c)^2}{1+3n} + \frac{2x(1-x_c)}{1+2n} + \frac{x_c^2}{1+n} \right]^{2-n}}{(1-x_c)^n} \tag{4.14}$$

where x_c is the critical value of the ratio τ_{yHB} / τ_w, which is estimated by iteration from

$$He_{HB} = \frac{3232}{n} (2+n)^{\frac{2+n}{1+n}} \left[\frac{x_c}{1-x_c} \right]^{\frac{2+n}{n}} \left[\frac{1}{(1-x_c)^n} \right]^n \tag{4.15}$$

in which the Herschel-Bulkley Hedstrom number, He_{HB}, is defined as

$$He_{HB} = \frac{\rho D^2}{\tau_{yHB}} \left(\frac{\tau_{yHB}}{K_{HB}} \right)^{2/n} . \tag{4.16}$$

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

5. FRICTIONAL LOSSES IN LAMINAR FLOW IN PIPES

Two methods of predicting laminar flow frictional losses are described in this Section.

Method 1 (Section 5.1) uses one of the rheological models from Sections 3.3 to 3.5 and integrates over the shear stress range from $\tau = 0$ at the pipe centre to $\tau = \tau_w$ at the pipe wall. This method is easily applied once the form of the flow curve is known (see ESDU 95012).

Method 2 (Section 5.2) is appropriate when the flow curve (data) is not model-fitted and where the laminar characterisation experiment is conducted in a tube viscometer.

An additional laminar flow frictional pressure loss prediction method is described in Appendix C.2, for the case where the flow curve obtained through viscometry is complicated and cannot readily be fitted to the rheological models of Section 3.

It should be noted that pipe wall roughness does not affect the frictional pressure loss in laminar flow.

5.1 Method I

For the special case of laminar flow, the relationship between volume flowrate, pressure drop, pipe length and pipe diameter is amenable to mathematical analysis. Regardless of the model chosen to represent the rheological behaviour of the fluid there are two important general equations on which the laminar analysis is based. Firstly a general equation linking volume flowrate and shear stress is

$$Q = \frac{\pi D^3}{8\tau_w^3}\int_0^{\tau_w} \tau^2 \mathrm{fn}(\tau)\,\mathrm{d}\tau, \tag{5.1}$$

where $\dot{\gamma} = \mathrm{fn}(\tau)$.

The derivation requires the following assumptions.

(i) The flow is steady and constant with time.

(ii) The fluid is incompressible.

(iii) The shear rate is a function of shear stress only.

(iv) The fluid properties are independent of time.

(v) There are no pipeline entrance and exit effects.

(vi) There is no slip between the fluid and the pipe wall.

A second general relationship, obtained by a simple force balance, that is valid for *any* fully-developed circular-section pipe flow (laminar, transitional or turbulent) in equilibrium is

$$\tau_w = \frac{D\Delta p}{4L}. \tag{5.2}$$

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

By substituting the relevant constitutive equation (Equation (3.4), (3.6) or (3.7)) and the wall shear/pressure loss relationship, Equation (5.2), into the general volume flowrate relationship, Equation (5.1), and integrating, the following expressions are obtained. (Appendix A outlines the mathematical procedure used to derive Equations (5.3), (5.4) and (5.7).)

5.1.1 Power-law model

For the power-law model, the pressure loss can be expressed explicitly as shown in the following equation:

$$\Delta p = \frac{4LK}{D}\left[\frac{8Q}{\pi D^3}\frac{(3n+1)}{n}\right]^n . \tag{5.3}$$

5.1.2 Bingham model

For the Bingham model, the flowrate is given by

$$Q = \frac{\pi D^3 \tau_w}{8\mu_B}\left[\frac{1}{4} - \frac{1}{3}\left(\frac{\tau_y}{\tau_w}\right) + \frac{1}{12}\left(\frac{\tau_y}{\tau_w}\right)^4\right] . \tag{5.4}$$

Equation (5.4) is explicit only for flowrate and must be solved iteratively in conjunction with Equation (5.2) for pressure drop, diameter or pipe length. However, if the yield stress/wall shear stress ratio is less than 0.5 (that is $\tau_y/\tau_w < 0.5$), the last term can be ignored and Equation (5.4) can be written as

$$Q = \frac{\pi D^3 \tau_w}{8\mu_B}\left[\frac{1}{4} - \frac{1}{3}\left(\frac{\tau_y}{\tau_w}\right)\right] , \tag{5.5}$$

which can be rearranged in conjunction with Equation (5.2) to give pressure loss explicitly, with an error of less than 7 per cent, as

$$\Delta p = \frac{128\mu_B L Q}{\pi D^4} + \frac{16}{3}\frac{L\tau_y}{D} . \tag{5.6}$$

5.1.3 Herschel-Bulkley model

For the Herschel-Bulkley model, the flowrate is given by

$$Q = \frac{\pi D^3}{8}\frac{n}{1+3n}\left(\frac{\tau_w}{K_{HB}}\right)^{\frac{1}{n}}(1-x)^{\frac{1}{n}}\left\{1 - \frac{x}{1+2n}\left[1 + \frac{2nx}{(1+n)}(1+nx)\right]\right\} . \tag{5.7}$$

An iterative approach must be used to find pressure drop, diameter or pipe length.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

5.2 Method II

This is the preferred method when the fluid cannot adequately be described as power-law, Bingham or Herschel-Bulkley and when the laminar characterisation experiment is conducted in a tube viscometer.

In most cases of scale-up the required flowrate, diameter and length of the full size pipeline are known and pressure loss is the unknown. Under such conditions the following procedure should be followed.

(i) Conduct a laminar characterisation experiment in a tube viscometer (see Section 6.1 of ESDU 95012) to give a plot of τ_w against $8V/D$.

(ii) Evaluate $8V/D$ for the full size pipeline.

(iii) From the plot of step (i) read off the value of τ_w corresponding to the $8V/D$ value of step (ii).

(iv) Find n' and K' at this point (from Section 3.2).

(v) Calculate Re' (from Equation (4.4)) for the full size pipeline and check that it is less than 2100. If not then the flow may be in the transitional or turbulent regimes, where this method is invalid.

(vi) Find pressure loss from Equation (5.2) using the τ_w value from step (iii).

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

6. FRICTIONAL LOSSES IN TURBULENT FLOW IN SMOOTH PIPES

There are several turbulent flow correlations available for predicting f. Most are based on the power-law model and require some modifications to be used for Bingham or Herschel-Bulkley fluids. All assume flow in smooth walled pipes. Equations for rough walled pipes are presented in Section 7.

Extensive analysis was conducted on the available data sets[4,5,9,17,19] to determine the optimal predictive correlation for friction factor against Reynolds number. All available correlations were found to have serious limitations with no single expression adequately predicting all the data. Appendix B gives a full account of the correlating methods used and presents a table listing the errors. As a consequence of this uncertainty two methods of predicting f are recommended and presented in this Item: the first uses the Dodge and Metzner[6] semi-theoretical equation and the second is based on the empirical work of Bowen[10].

The choice of method is determined by the experimental data available. If only laminar flow data are available, use Method I of Section 6.1. If laminar and turbulent flow pressure drop and flowrate data are available for at least two pipe diameters, use Method II of Section 6.2.

Three additional methods for turbulent flow in smooth pipes worthy of note are presented in Appendices C.3 to C.5. These methods cover the cases where laminar flow viscometric data are, respectively, not fitted to a flow curve, are fitted to a Bingham plastic model or are fitted to a Herschel-Bulkley model.

6.1 Method of Dodge and Metzner (Method I)

6.1.1 Applicability to power-law fluids

Dodge and Metzner[6] developed a Nikuradse-type implicit equation linking f, n and Re' and fitted it to experimental data. The resulting correlation is recommended as the most appropriate of those available in the literature and cannot be significantly improved upon given the spread of the available results (see Appendix B). The equation is

$$\frac{1}{\sqrt{f}} = \frac{4.0}{n^{0.75}}\log_{10}\left[Re' f^{\left(1-\frac{n}{2}\right)}\right] - \frac{4.0}{n^{1.2}} \tag{6.1}$$

and can be considered to give a fair estimate of f over the range $3500 < Re' < 100\ 000$ and a guide to f in the transition range $2100 < Re' < 3500$ (see Appendix B for details of errors). Figure 1 shows this relationship for various values of n.

The method can be summarised as follows.

It is necessary to determine Re' to use this method and that requires evaluation of n and K. These values may be determined directly from experiment or from the flow curve (see ESDU 95012).

(i) Preliminary experiment

(a) Decide the approximate $8V/D$ range over which the pipeline is to operate.

(b) There are two possibilities depending on the type of viscometer available. If the preferred tube viscometer is to be used then conduct a laminar tube viscometer experiment and plot τ_w against $8V/D$ on log-log paper over the range determined in part (a) above. This allows determination of n and K for the fluid, if necessary as a function of $8V/D$ (see Section 6.1 of ESDU 95012).

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

Otherwise obtain the flow curve for the fluid with whatever viscometer is available (see ESDU 95012). It is possible to convert the τ versus γ curve into a τ_w versus $8V/D$ curve by application of the Rabinoswitsch-Mooney equation (Equation (5.3) of ESDU 95012. If the fluid is not a power-law type this is a complex procedure and best undertaken by computer. The accompanying program performs all the necessary calculations. An alternative method that does not require a computer is to approximate the τ versus γ curve with a power-law model over the region of interest. Thus n and K can be found by local application of Equations (3.3).

(ii) ***To Determine f.***

Either use Figure 1 or carry out the following procedure.

(a) Assume a pressure drop, Δp, then calculate τ_w from Equation (5.2).

(b) Find the values of n and K that correspond to this value of τ_w from the log-log plot of τ_w against $8V/D$.

(c) Calculate Re' from Equation (4.4).

(d) Find f from Equation (6.1).

(e) Find Δp corresponding to that f value from Equations (5.2) and (4.2).

(f) If Δp from step (e) is different from that assumed in step (a), repeat steps (a) to (e) until agreement is obtained.

This is a time-consuming process to undertake by hand so the accompanying computer program performs all the necessary calculations (see Part II).

6.1.2 Applicability to models with a yield stress

Using locally-determined values of n and K (that is, n' and K') appropriate to the relevant $8V/D$ values for turbulent flow, the method of Section 6.1.1 can be applied to models which have a yield stress.

6.2 Method of Bowen (Method II)

6.2.1 Description

Bowen[10] proposed that for a *particular* fluid in the turbulent flow regime a correlation of the form

$$\tau_w = A\frac{V^b}{D^c} \tag{6.2}$$

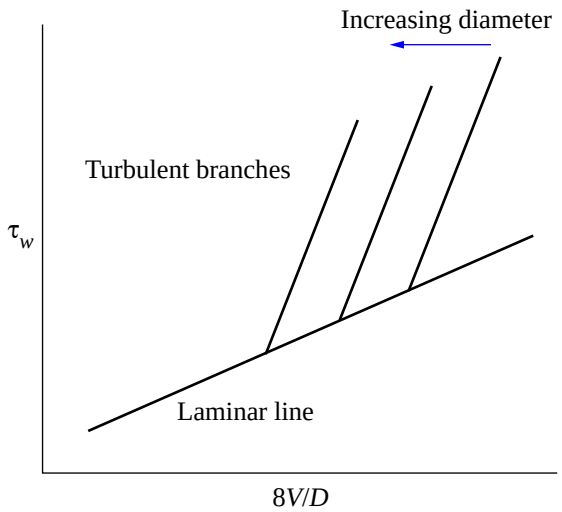
appears possible, with A, b and c being experimentally determined coefficients for a particular fluid and conditions and system of units. He noted that a plot of wall shear stress against $8V/D$ correlates data well for all pipe diameters in the laminar region but that for turbulent flow the data lie on different lines, each line corresponding to a particular pipe size. His empirical method capitalises on this fact to provide a procedure to predict laminar and turbulent conditions and gives guidance on the critical Reynolds number. Because of the explicit inclusion of a diameter effect, this method is to be preferred for scaling-up small pipe experiments to predict pressure losses in larger pipe diameters.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

Sketch 6.1 shows the log-log plot of wall shear stress against $8V/D$ for both laminar and turbulent flow. The three turbulent branches, which all have the same slope, correspond to three different pipe diameters whilst a single laminar line suffices for all diameters assuming that there is no slip between the fluid and the wall. The Reynolds number at transition can be calculated from the value of $8V/D$ at which the turbulent branch departs from the laminar line.



Sketch 6.1 Wall shear stress against $8V/D$ for laminar and turbulent pipe flow

In practice the angle of intersection is less well defined than shown in the sketch and a transition region can be estimated between the laminar straight line and any turbulent straight line (see Section 9.2.2 for an analysis using experimental data). Strictly speaking, only turbulent flow data from the characterisation pipes are valid to allow scale-up to turbulent flow in larger pipes. However, measurements in the laminar and transition regimes are essential in determining fully the rheology of the fluid and help to clarify the transition region, a region that the designer should preferably avoid*. The method can be summarised as follows.

(i) Conduct preliminary tube characterisation tests to measure wall shear stress and $8V/D$ using at least two pipe diameters and under both laminar and turbulent flow conditions.

(ii) Plot all data as wall shear stress against $8V/D$ on log-log paper. The laminar data should collapse onto one master curve and there should be one branching turbulent line for each pipe diameter.

(iii) The power of the velocity term, b, is the slope of the turbulent branch. Ideally all branches would have identical slope but in practice calculate b for each pipe diameter and take a simple arithmetic average. Take care to note the transition region (if any) and only include points that lie on the straight turbulent branches.

(iv) Calculate $LV^b/\Delta p$ for each turbulent data point.

(v) Plot $LV^b/\Delta p$ against D on log-log paper. Draw a best fit straight line through the log-log data. The slope of this line is $1 + c$. Calculate c.

* Whilst for economic reasons it may be advantageous to operate at as low Re' as possible, there are often good engineering reasons for avoiding the transition region. Pressure losses in pipelines in transitional flow can fluctuate unpredictably thus placing the pump and ancillary equipment under undue stress.

ESDU Copyright material. For current status contact ESDU.
Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

(vi) Calculate A for each turbulent data point from a rearrangement of Equations (6.2) and (5.2),

$$A = \left(\frac{D\Delta p}{4L}\right)\frac{D^c}{V^b}. \tag{6.3}$$

Ideally A should be identical for all points but in practice calculate the simple arithmetic average and use this as the A value. (Alternatively plot $D^c\tau_w$ against V on a log-log graph. Log A is the intercept on the $D^c\tau_w$ axis of the best fit straight line through the data.)

The turbulent flow of the fluid in question can now be represented by Equation (6.2). A visual guide to the effectiveness of this method can be seen by plotting $D^c\tau_w$ against V on log-log paper. All the turbulent data should collapse onto a single line with slope equal to b.

6.2.2 Pressure loss for other pipe diameters

Equation (6.2) can now be used directly to give pressure drop for any pipe diameter if the flow is turbulent. Alternatively it can be used to construct a wall shear stress versus $8V/D$ turbulent line for any pipe diameter. The procedure is as follows.

(i) With A, b and c previously obtained and knowing the new pipe size, D, and length, L, plot wall shear stress, τ_w (found from Equation (6.2)), against $8V/D$ (using a range of velocities) on the same log-log graph as the characterisation runs.

(ii) Extrapolate the new pipe diameter line until it intersects the laminar flow line. This intersection marks the absolute lowest limit of applicability of the turbulent line.

(iii) If the characterisation runs show a clear transition region, determine the locus of the onset of turbulence and extrapolate this line until it intersects with the new pipe diameter line. This marks the upper end of the transition region. The new diameter turbulent line is valid for values of $8V/D$ and wall shear stress above this critical point.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

7. FRICTIONAL LOSSES IN TURBULENT FLOW IN ROUGH-WALLED PIPES

No experimental data were available on the turbulent flow of non-Newtonian fluids in rough pipes. The prediction methods of Section 7.1 and 7.2 are given only as a guide and should be used with caution. Two additional methods based on the use of laminar flow viscometric data are described in Appendices C.6 and C.7

7.1 Intermediate Rough-walled Pipes ($\varepsilon/D \leq 0.1$)

The friction factor/Reynolds number relationship for the turbulent flow $(Re > 4000)$ of Newtonian fluids can be represented to within 2% for $\varepsilon/D \leq 0.1$ by

$$f = \left\{ 3.6 \log_{10} \left[\frac{6.9}{Re} + \left(\frac{\varepsilon}{3.70D} \right)^{1.1} \right] \right\}. \tag{7.1}$$

A modification of Equation (7.1) can be used to estimate the non-Newtonian frictional losses in non-smooth walled pipes. This is an approximate method. The procedure is as follows

(i) Determine the smooth pipe friction factor, f_s, from Section 6.

(ii) Evaluate the rough-walled friction factor, f_r, from

$$f_r = f_s \frac{\log_{10} \left[\frac{6.9}{Re'} + \left(\frac{\varepsilon}{3.70D} \right)^{1.1} \right]}{\log_{10} \left[\frac{6.9}{Re'} \right]}. \tag{7.2}$$

7.2 Fully Rough-walled Pipes

Torrance[12] proposes the following relationships for fully rough-walled pipes. Neither correlation was supported by any experimental results.

For power-law fluids

$$\frac{1}{\sqrt{f}} = \frac{4.07}{n} \log_{10} \left(\frac{D}{2\varepsilon} \right) + 6.0 - \frac{2.65}{n}. \tag{7.3}$$

For Bingham fluids

$$\frac{1}{\sqrt{f}} = 4.07 \log_{10} \left(\frac{D}{2\varepsilon} \right) + 3.36. \tag{7.4}$$

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011

8. FRICTIONAL LOSSES IN TRANSITIONAL REGION FLOW IN SMOOTH PIPES

For most design purposes it is recommended that the $2200 < Re' < 3500$ range should be avoided. Guidance on frictional losses in transitional flows is given in References 30 and 31. The turbulent relation of Dodge and Metzner[6] (Equation (6.1)) can be used to provide an overestimate of the losses.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011

9. WORKED EXAMPLES

9.1 Example 1: Laminar Flow

Problem: It is required to pump a suspected non-Newtonian fluid over a distance of 52 m through a 0.2 m diameter pipe under isothermal conditions. It is important to keep the flow in the laminar regime. What is the maximum flowrate achievable and what is the pressure drop over the pipe length at this flowrate?

Solution: Firstly the rheological category of the fluid needs to be established (Section 3). To this end rheological characterisation tests* were conducted in a tube viscometer at the same temperature as that at which the proposed pipeline would operate. Values of $8V/D$ and wall shear stress $(D\Delta p/4L)$ were measured. The results are given in Table 9.1 and plotted on log-log axes in Sketch 9.1. The density was found to be 999.5 kg/m^3.

TABLE 9.1 Results of Laminar Characterisation Experiment for Example 1

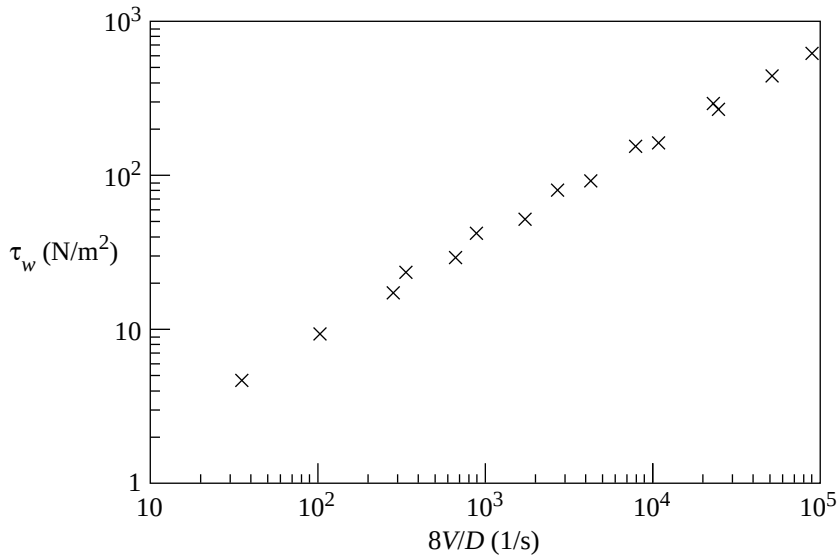
$8V/D$	$\tau_w = D\Delta p/4L$
336	23.3
896	41.9
2690	79.2
8000	153
23100	291
35.2	4.69
104	9.33
284	17.2
677	29.0
1750	51.4
4300	90.3
10900	160
24800	270
51500	429
88800	594

* The data used here are from run numbers 7A, 7B and 7C from Derivation 5.

ESDU Copyright material. For current status contact ESDU.

Documents
Document Sharing Hub

Full document details: ESDU 91025 E-R1 2011



Sketch 9.1 Results of laminar characterisation experiment for Example 1 (flow curve)

The best fit straight line to the log-log data is found by using a least squares method. The equation of this line is

$$\log_{10}\left(\frac{D\Delta p}{4L}\right) = 0.61\log_{10}\left(\frac{8V}{D}\right) - 0.24 \tag{9.1}$$

with a correlation coefficient of 0.997. Thus $n' = 0.61$ and $\log_{10} K' = -0.24$, giving $K' = 0.57$ N s$^{n'}$/m^2. Because n' is constant with $8V/D$ (that is the log-log plot is fitted well by a straight line) the fluid can be considered a power-law type. From Equation (3.3),

$$n = n' = 0.61 \tag{9.2}$$

and

$$K = \frac{K'}{\left(\frac{1+3n}{4n}\right)^n}$$

$$= \frac{0.57}{\left(\frac{1+3(0.61)}{4(0.61)}\right)^{0.61}}$$

$$= 0.52 \text{ N s}^n/\text{m}^2 . \tag{9.3}$$

(The wall shear rate can then be found from the Rabinoswitsch-Mooney equation (Equation (5.3) of ESDU 95012). The flow curve can now be plotted, if required.)

An estimate of the laminar/transitional critical Reynolds number is made from Equation (4.8) as

$$Re'_{P,c} = \frac{6464n(n+2)^{\left(\frac{n+2}{n+1}\right)}}{(1+3n)^2} = 2331 . \tag{9.4}$$

ESDU Copyright material. For current status contact ESDU.
Documents
Document Sharing Hub
Full document details: ESDU 91025 E-R1 2011