



Simplified method for the prediction of aerofoil excrescence drag magnification factor for turbulent boundary layers at subcritical Mach numbers

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SIMPLIFIED METHOD FOR THE PREDICTION OF AEROFOIL EXCRESCENCE
DRAG MAGNIFICATION FACTOR FOR TURBULENT BOUNDARY LAYERS AT
SUBCRITICAL MACH NUMBERS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	2
3. BACKGROUND TO METHOD	2
4. DRAG MAGNIFICATION FACTOR	3
5. APPLICABILITY AND ACCURACY	4
5.1 Applicability	4
5.2 Accuracy	5
6. DERIVATION AND REFERENCES	5
6.1 Derivation	5
6.2 References	6
7. EXAMPLE	6
APPENDIX A PROGRAM FOR CALCULATION OF AEROFOIL EXCRESCENCE DRAG MAGNIFICATION FACTOR FOR TURBULENT BOUNDARY LAYERS AT SUBCRITICAL MACH NUMBERS	12
A1. INTRODUCTION	12
A2. INPUT AND OUTPUT	12
A3. EXAMPLES OF INPUT AND OUTPUT	15
A3.1 Example 1	15
A3.2 Example 2	17
A3.3 Example 3	19
A3.4 Example 4	21

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SIMPLIFIED METHOD FOR THE PREDICTION OF AEROFOIL EXCRESCENCE DRAG MAGNIFICATION FACTOR FOR TURBULENT BOUNDARY LAYERS AT SUBCRITICAL MACH NUMBERS

1. NOTATION AND UNITS

		SI	British
C_p	pressure coefficient		
c	aerofoil chord	m	ft
I	parameter $(M_1 T_1 / M_\infty T_\infty)^4$		
i	index relating to general chordwise location, x/c		
j	particular value of i ($2 \leq j \leq N$)		
M	Mach number		
m_d	excrement drag magnification factor		
N	value of i at trailing edge		
T	static temperature	K	K
u	velocity	m/s	ft/s
x	streamwise co-ordinate, measured along chord from leading edge	m	ft
θ	boundary-layer momentum thickness	m	ft
ρ	density of air	kg/m ³	slug/ft ³
<i>Subscripts</i>			
E	denotes value at excrement location		
i	denotes value at general chordwise station		
T	denotes value at trailing edge		
1	denotes value at edge of boundary layer		
∞	denotes free-stream value		

2. INTRODUCTION

This Item provides a means of calculating the drag magnification factor, m_d , in two-dimensional flow due to a local excrescence on an aerofoil with turbulent boundary layers at any subcritical Mach number. The need for the Item is outlined in Section 3 while Section 4 presents the Nash and Bradshaw equation (Derivation 4) for m_d , extended (Derivation 5) for the effects of compressibility, and details a simplified means whereby it may be evaluated in the practical situation in which only the aerofoil pressure distribution is known. The applicability and accuracy of the compressible flow form of the Nash and Bradshaw equation, and the simplified method in particular, are discussed in Section 5. Section 6 gives the Derivation and References, while Section 7 presents a detailed Example, evaluated using the simplified method, for which details of a computer program are provided in Appendix A.

Item No. 91029 (Reference 14) provides data, calculated using the program, for the families of aerofoils described in Item No. 71020 (Reference 11) at their design condition. The corresponding profile drag data are given in Item No. 67011 (Reference 9).

3. BACKGROUND TO METHOD

The introductory Item (No. 90029) to aircraft excrescence drag (Derivation 7) notes that Item No. 87004 (Derivation 6) in the Transonic Aerodynamics Sub-series provides three methods of estimating the drag magnification factor, m_d , due to the effects of non-uniform pressure distribution on a local excrescence on transonic aerofoils. The three methods differ in terms of the amount of information required and the complexity of the resulting procedures.

The most accurate method of Item No. 87004 involves the use of a computer program in which the BVGK method (Reference 13) was adapted (as BVGKE) to provide a direct means of establishing m_d via calculations of drag with and without a step increment in momentum thickness representing a local excrescence.

The simplest method of the three gives a rapid guide to the likely values of m_d for pressure distributions typical of modern transonic aerofoils. Data for m_d , calculated using the BVGKE method are given in Item No. 87004 for two RAE sections (2822 and 5230) for a range of excrescence locations on their upper and lower surfaces, covering a range of lift coefficients at each of three high subsonic Mach numbers. The corresponding pressure distributions for the clean aerofoil, *i.e.* in the absence of the excrescence, are also given. The user is required to match the pressure distribution for his particular aerofoil as closely as possible to one of those given and then to use the corresponding data for m_d .

The final method given in Item No. 87004 is intermediate to the other two in that it overcomes the limitations to the general use of the simple graphical method but is quicker and easier to use than the BVGKE method and loses little in accuracy provided that there are no shock waves. However, the method, involving the Nash and Bradshaw equation (see Item No. 90029 for a full development), adapted to compressible flow (Derivation 5), does require that pressure distribution and boundary-layer momentum thickness data for the clean aerofoil are available, from test or calculation, for the required flow conditions.

At the stage in the design process when excrescence drag is being assessed it is very likely that the designer would have pressure distributions available, but it is perhaps not so certain that boundary-layer data would be available. The present Item provides a means of evaluating the modified Nash and Bradshaw equation for wholly turbulent flow assuming a knowledge of clean aerofoil pressure distribution only.

4. DRAG MAGNIFICATION FACTOR

Equation (3.13) in Item No. 90029, taken from Item No. 87004, gives the following formula for aerofoil excrescence drag magnification factor due to Nash and Bradshaw (Derivation 4), with first-order effects of compressibility (Derivation 5):

$$m_d = \frac{\rho_{1T}}{\rho_\infty} \left(\frac{u_{1E}}{u_\infty} \right)^{(4.2 - 0.6M_\infty^2)} \left(\frac{u_\infty}{u_{1T}} \right)^{(1 - M_\infty^2)} \left(\frac{\theta_E}{\theta_T} \right)^{0.2} \quad (4.1)$$

For low-speed flows this reduces to the standard Nash and Bradshaw equation

$$m_d = \left(\frac{u_{1E}}{u_\infty} \right)^{4.2} \frac{u_\infty}{u_{1T}} \left(\frac{\theta_E}{\theta_T} \right)^{0.2} \quad (4.2)$$

Equation (4.1) can be evaluated directly for any aerofoil provided that appropriate data are available for the pressure (and hence velocity and density) and momentum thickness at the excrescence location and the aerofoil trailing edge, in the absence of the excrescence. However, in order to cope with the situation in which only pressure distribution is available, some means of estimating the momentum thickness ratio term in Equation (4.1) is required.

The momentum thickness development downstream of the excrescence location in the Nash and Bradshaw method assumed a turbulent boundary layer which was predicted using the Spence method (Derivation 2). For reasons of consistency and compatibility, therefore, the Spence method should be used to derive the momentum thickness ratio. A treatment of compressibility using the Stewartson transformation (Derivation 1) is given in Derivation 3, and for a wholly turbulent boundary layer the momentum thickness ratio is given by

$$\frac{\theta_E}{\theta_T} = \left[\left(\frac{M_{1T}}{M_{1E}} \right)^{4.2} \left(\frac{T_{1T}}{T_{1E}} \right)^{3.6} \frac{\int_0^{x_E/c} (M_1 T_1 / M_\infty T_\infty)^4 d(x/c)}{\int_0^1 (M_1 T_1 / M_\infty T_\infty)^4 d(x/c)} \right]^{5/6} \quad (4.3)$$

It will be noted that the assumption of a wholly turbulent boundary layer has resulted in Equation (4.3) being independent of Reynolds number (see Section 5.2).

On the assumption that pressure distribution on the clean aerofoil is available, the other variables required to evaluate Equation (4.1) using Equation (4.3) follow from the corresponding Mach number distribution for isentropic flow (with the ratio of specific heat capacities taken as 1.4), *i.e.* in general

$$\frac{M_1}{M_\infty} = \left\{ \frac{5}{M_\infty^2} \left[\frac{1 + 0.2M_\infty^2}{(1 + 0.7M_\infty^2 C_p)^{2/7}} - 1 \right] \right\}^{1/2} \quad (4.4)$$

Therefore, from the isentropic flow relationships, the required variables are

$$\frac{u_1}{u_\infty} = \frac{M_1}{M_\infty} \left(\frac{T_1}{T_\infty} \right)^{1/2}, \quad (4.5)$$

and
$$\frac{\rho_1}{\rho_\infty} = \left(\frac{T_1}{T_\infty} \right)^{5/2}, \quad (4.6)$$

where
$$\frac{T_1}{T_\infty} = \frac{1 + 0.2M_\infty^2}{1 + 0.2M_\infty^2 (M_1/M_\infty)^2}. \quad (4.7)$$

The particular values at the excrescence location and the trailing edge are simply given by adding respective subscripts E and T to C_p , M_1 , u_1 , T_1 and ρ_1 in the above general equations.

The evaluation of Equation (4.1) using Equations (4.3) to (4.7) is straightforward, except for Equation (4.3) which in general requires numerical integration, although in many cases a simple approximation is adequate, see the example in Section 7. However, for routine use a computer program for evaluating the equations is given, see Appendix A.

5. APPLICABILITY AND ACCURACY

5.1 Applicability

This Item provides a relatively straightforward means of calculating the drag magnification factor due to a local excrescence on any aerofoil with turbulent boundary layers at subcritical Mach numbers.

If good quality pressure distribution and boundary layer data for the clean aerofoil are available, then direct use of Equation (4.1) gives good accuracy when compared with BVGKE calculations (see Section 5.2). The simplified method, for use when only pressure distribution is known, involves Equation (4.3) for the boundary-layer momentum thickness ratio and is more than adequate for the purpose of assessing excrescence drag.

The success of the simplified method hinges on the fact that accuracy in calculating momentum thickness is not too crucial, for two main reasons. Firstly, the fact that momentum thickness only appears as the ratio θ_E/θ_T in Equation (4.1) helps to alleviate any errors in θ_E itself. Secondly, the index 0.2 to θ_E/θ_T ensures that the effect on m_d is of much less significance than that due to local velocity ratio.

Derivation 6 notes that certain assumptions and approximations used in deriving Equation (4.1) are invalid for an excrescence placed near the trailing edge and recommends the restriction to excrescences at or forward of $x/c = 0.8$. However, since m_d is generally predicted to be less than unity (implying a drag **reduction** due to non-uniform flow) near the trailing edge it is unlikely to be accounted for in any practical excrescence drag calculations.

5.2 Accuracy

In the work leading to Item No. 87004 the full BVGKE method was compared with calculations using Equation (4.1), in which the required data were taken from the BVGK method (Reference 13) for the clean aerofoil. It was found that for subcritical Mach numbers agreement was generally within $\pm 5\%$. Even for cases with small regions of supercritical flow ($1 \leq M_1 \leq 1.05$, say) the errors were not excessive downstream of such regions. Calculations (Derivation 5) using the BVGKE method also showed that the effect of variation of Reynolds number over a wide range of likely flight values had a negligible effect on m_d , corroborating the independence of Reynolds number in Equation (4.3). The various comparisons therefore indicate the potential of Equation (4.1) when used with good quality pressure distribution and boundary-layer data.

The adequacy of Equation (4.3) for θ_E/θ_T was tested against three more modern methods when used in evaluating Equation (4.1). For each comparison the remainder of the data required were the same throughout and were available as output from the complete coupled viscous/inviscid solutions used. The three turbulent-boundary-layer methods used were those of Nash and Macdonald (Reference 8), Horton (Reference 10) and Green, Weeks and Brooman (Reference 12). Four aerofoils, ranging in thickness/chord ratio from 0.06 to 0.14, were used in the comparisons which covered a range of lift coefficients and Mach numbers up to about the critical. Analysis of the results showed that errors in m_d calculated using Equation (4.1) with Equation (4.3) for momentum thickness ratio were generally within about $\pm 3\%$ of those calculated using Equation (4.1) with any of the other methods for momentum thickness. However, this accuracy level was only achieved for boundary-layer transition located within $0.1c$ of the leading edge and for excrescence locations at least $0.1c$ aft of the transition position itself. Errors increased to as much as $\pm 15\%$ for excrescences too close to the transition point.

6. DERIVATION AND REFERENCES

6.1 Derivation

The Derivation lists selected sources of information that have assisted in the preparation of this Item.

1. STEWARTSON, K. Correlated compressible and incompressible boundary layers.
Proc. R. Soc., Series A, Vol. 200, p. 84, 1949.
2. SPENCE, D.A. The development of turbulent boundary layers.
J. aeronaut. Sci., Vol. 23, No. 1, pp. 3 to 15, January 1956.
3. NASH, J.F.
MOULDEN, T.H.
OSBORNE, J. On the variation of profile drag coefficient below the critical Mach number.
ARC CP 758, 1963.
4. NASH, J.F.
BRADSHAW, P. The magnification of roughness drag by pressure gradients.
J.R. aeronaut. Soc., Vol. 71, pp. 44 to 46, January 1967.
5. PATEL, B.
LOCK, R.C. Calculation of excrescence drag magnification using the Nash and Bradshaw formula adapted to compressible flow and a modified BVGK computer program (U).
ESDU Memor. No. 63, 1987 (with Amendment A, 1989).
6. ESDU Calculation of excrescence drag magnification due to pressure gradients at high subsonic speeds.
ESDU International, Item No. 87004, 1987.

7. ESDU An introduction to aircraft excrescence drag.
ESDU International, Item No. 90029, 1990.

6.2 References

The References list sources of information supplementary to that given in this Item.

8. NASH, J.F.
MACDONALD, A.G.J. The calculation of momentum thickness in a turbulent boundary layer at Mach numbers up to unity.
ARC CP 963, 1966.
9. ESDU Profile drag at the drag-rise condition of aerofoils having a specified form of upper-surface pressure distribution at this condition.
ESDU International, Item No. 67011, 1967.
10. HORTON, H.P. Entrainment in equilibrium and non-equilibrium turbulent boundary layers.
HSA Hatfield/Research/1094/HPH, 1969.
11. ESDU Aerofoils having a specified form of upper-surface pressure distribution; details and comments on design.
ESDU International, Item No. 71020, 1971.
12. GREEN, J.E.
WEEKS, D.J.
BROOMAN, J.W.F. Prediction of turbulent boundary layers and wakes in compressible flow by a lag entrainment method.
ARC R & M 3791, 1973.
13. ASHILL, P.R.
WOOD, R.F.
WEEKS, D.J. An improved, semi-inverse version of the viscous Garabedian and Korn method (VGK).
RAE tech. Rep. 87002, 1987.
14. ESDU Excrescence drag magnification factors at the drag-rise condition for aerofoils with a specified form of upper-surface pressure distribution.
ESDU International, Item No. 91029, 1991.

7. EXAMPLE

An assessment of the drag penalty at cruise conditions is required for local excrescences on the upper surface of an aerofoil to be used in the wing of a transport aircraft designed to cruise at a moderately high subsonic speed. As an aid to the assessment, the variation of the excrescence drag magnification factor across the upper surface is required. The aerofoil is designated 102-10-40-75, one of a series of aerofoils designed to have a near sonic roof-top upper-surface pressure distribution at the drag-rise Mach number, details of which are given in Item No. 71020 (Reference 11). The aerofoil designation reveals that the design Mach number (M_∞) is 0.75. Table 7.1 gives details of the upper-surface pressure distribution, taken from Table 6.4 of Item No. 71020 and shown in Sketch 7.1.