



# ESDU 91043

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## Mean wind speeds over hills and other topography

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## THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Wind Engineering Panel. This Panel, which took over the work on wind engineering previously monitored by the Fluid Mechanics Steering Group, first met in 1979 and now has the following membership:

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## MEAN WIND SPEEDS OVER HILLS AND OTHER TOPOGRAPHY

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## MEAN WIND SPEEDS OVER HILLS AND OTHER TOPOGRAPHY

### 1. NOTATION

SI Units

|                                |                                                                                                                             |   |
|--------------------------------|-----------------------------------------------------------------------------------------------------------------------------|---|
| $a$                            | parameter in Equation (4.2) giving variation of $s$ with height                                                             |   |
| $ar_1, ar_2$                   | hill aspect ratios; $W_{01}/L_{01}$ , $W_{02}/L_{01}$ (see Sketch 5.1)                                                      |   |
| $f$                            | Coriolis parameter                                                                                                          |   |
| $f_{ar}$                       | correction factor for effect on $\Delta K_{L2d}$ of three-dimensional topography                                            |   |
| $f_h$                          | correction factor accounting for blockage effect when $H/h > 0.04$                                                          |   |
| $f_r$                          | factor giving $\Delta s$ due to difference in local shape between equivalent embankment and actual topography               |   |
| $h$                            | local boundary layer height; $u_*/(6f)$ or 1500 m (default value)                                                           | m |
| $H$                            | maximum height of hill cross section (AA, BB) through site                                                                  | m |
| $H_0$                          | maximum height of topographic feature (negative if valley)                                                                  | m |
| $H_f$                          | maximum height of surface irregularity (Sketch 6.1)                                                                         |   |
| $H_x$                          | local height of topography at position $x$                                                                                  | m |
| $\Delta H_r$ ,<br>$\Delta H_x$ | local difference between hill height and its equivalent embankment due to rounding at specified key points (see Sketch 4.1) | m |
| $K_L$                          | speed-up factor on mean wind speed; $V_{z,hill}/V_{z,flat}$                                                                 |   |
| $\Delta k_L$                   | fractional speed-up factor; $K_L - 1$                                                                                       |   |
| $\Delta K_{L2d}$               | value of $\Delta K_L$ for two-dimensional topography                                                                        |   |
| $\Delta K_{L3d}$               | increment in $\Delta K_{L2d}$ due to three-dimensional topography                                                           |   |
| $\delta K_L^2$                 | measure of adverse pressure gradient on downslope surface (Equation (7.4))                                                  |   |
| $L$                            | local length scale of topographic feature; $H/\phi$ (see Table 4.1)                                                         | m |
| $L_{01}, L_{02}$               | overall hill length scales from summit to half height along 0x axis (see Sketch 5.1)                                        |   |

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|                  |                                                                                                                                                         |     |
|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| $L_d, L_u$       | local length scales of hill section (AA, BB) defined by maximum slope downwind and upwind of crest respectively; $H/\phi_d$ and $H/\phi_u$ respectively | m   |
| $P$              | length of flat top of embankment or equivalent embankment                                                                                               |     |
| $s$              | incremental speed-up parameter for two-dimensional topography; $(K_L - 1)/(2\phi)$                                                                      |     |
| $\Delta s$       | $s_{hill} - s_{emb}$ at $z = 0$                                                                                                                         |     |
| $u_*$            | surface friction velocity                                                                                                                               | m/s |
| $V_z$            | mean wind speed at location of interest at height $z$ above topographic feature                                                                         | m/s |
| $V_{z, flat}$    | mean wind speed at location of interest at height $z$ above flat ground                                                                                 | m/s |
| $W_{01}, W_{02}$ | overall hill width scales from summit to half height along $Oy$ axis (see Sketch 5.1)                                                                   | m   |
| $x$              | distance measured from summit along $Ox$ axis                                                                                                           | m   |
| $x_o$            | distance measured from crest (O) of equivalent embankment in wind direction (see Sketch 3.5)                                                            | m   |
| $y$              | distance measured from summit along $Oy$ axis (see Sketch 5.1)                                                                                          |     |
| $z$              | height above local ground level                                                                                                                         | m   |
| $z_i$            | inner-layer height above surface within which separated flow effects occur                                                                              | m   |
| $z_0$            | terrain roughness parameter                                                                                                                             | m   |
| $\theta$         | flow deviation from undisturbed wind direction                                                                                                          | rad |
| $\phi$           | $H/L$                                                                                                                                                   | rad |
| $\phi_d, \phi_u$ | maximum slope of hill section (AA, BB) downwind and upwind of crest respectively                                                                        | rad |
| $\phi_{max}$     | value of $\phi_u$ above which $K_L$ remains constant due to separated flow effects                                                                      | rad |
| $\psi$           | angle between hill ( $-Ox$ ) axis and wind direction (see Sketch 5.1)                                                                                   | rad |

## Subscripts

|             |                                                       |
|-------------|-------------------------------------------------------|
| A – G, O, P | relate to specific locations on hill (see Sketch 4.1) |
| <i>emb</i>  | relates to value for equivalent embankment            |
| <i>hill</i> | relates to value for hill (or valley)                 |
| <i>i</i>    | relates to value at $z = z_i$                         |
| <i>z</i>    | relates to value at height $z$                        |

|        |                                                       |
|--------|-------------------------------------------------------|
| $xyz$  | relates to value at location $(x, y, z)$              |
| $2d$   | relates to value for two-dimensional topography       |
| $3d$   | relates to value for three-dimensional topography     |
| $\psi$ | relates to value for wind inclined to hill ridge line |

## Superscript

|   |                                                                                                                       |
|---|-----------------------------------------------------------------------------------------------------------------------|
| ' | refers to values for local surface irregularities (bumps and depressions) considered separately from basic hill shape |
|---|-----------------------------------------------------------------------------------------------------------------------|

## 2. PURPOSE AND SCOPE

This Data Item presents simple methods for estimating speed-up factors on mean wind speed due to flow over topography such as hills, embankments, escarpments and valleys. The speed-up factor when the wind direction is along a section of a hill (or valley) normal to the local ridge line is given by

$$K_L = 1 + \Delta K_{L2d} f_{ar} + \Delta K_{L3d}.$$

Two methods are given in Section 4 for estimating  $\Delta K_{L2d}$  assuming the hill can be considered as two-dimensional (that is, very wide compared to its length). The first method (Section 4.1) is a very simple graphical approach which, for rounded hills, only takes into account the overall characteristics of the topography. The second method (Section 4.2) takes more account of the detailed shape of the hill when the topographic profile shape is reasonably regular without significant bulges or depressions near the location of interest. An extension to this method is given (Section 6) to deal with the effect of such irregularities when they are likely to influence the speed-up factors at the point of interest and also to take into account the influence of other nearby hills.

Three-dimensional effects caused by the diversion of wind around the sides of a hill, decreasing  $K_L$  over the central region and increasing  $K_L$  near the sides, are represented by the parameters  $f_{ar}$  and  $\Delta K_{L3d}$  (Section 5.1). The effect of wind direction (Section 5.2) is dealt with by vectorily combining the speed-up effects on the normal and tangential components of the approaching wind speed by the hill sections normal and parallel to the local ridge line.

The derivation of the basic methods is explained in Appendix A. They are derived from the more complex theoretical methods which involve solution of the Navier-Stokes equations governing flow in the atmospheric boundary layer. In particular, the method of Deaves<sup>10,11</sup> has been used, not only because it gives results which compare favourably with measurements but also because it is compatible with the Harris-Deaves mean wind speed model which is the basis for other ESDU Data Items.

The methods described in this Item can all be calculated by hand. However, these methods have been incorporated into the software package VIEWpac E0108<sup>7</sup> and it is recommended that this program be used to avoid repeated hand calculations. This program gives values of wind speed and turbulence properties for specified conditions and can be used to estimate topographic effects at a site including the effect of terrain roughness changes upwind of the site. Operating instructions are contained in ESDU 01008<sup>7</sup>.

In summary the following features apply especially to the more detailed method.

- (i) The method accounts, realistically, for speed-up factors on all regions of the hill, including the shelter regions near the foot of the hill, upwind and downwind.
- (ii) The local profile of the hill can be taken into account by the method.
- (iii) Three-dimensional effects are included.
- (iv) The effect of wind inclination to the ridge or valley line can be accounted for.
- (v) Guidance is provided for steep topography when flow separation from the hill surface occurs.
- (vi) The more detailed method includes a means for assessing the effect on speed-up factors of other nearby topography such as individual hills or generally rough, hilly terrain.

The methods do not require the need to digitise the topographic contour shapes although it is necessary to obtain (for each site considered) the cross sectional shape of two orthogonal sections through the site.

In addition to the effects on mean wind speed, turbulence over hills is distorted near the ground and gust speeds can be significantly increased over those for flat terrain; guidance on this aspect will be provided in ESDU 01008.

General background information to flow over hills, *etc.* is given in Section 3. An example on the use of the methods is given in Section 8 and the Appendix contains general notes on the derivation of the methods.

A summary calculation sheet for the more detailed method is provided in Section 10.

## 2.1 Uncertainties in the Methods

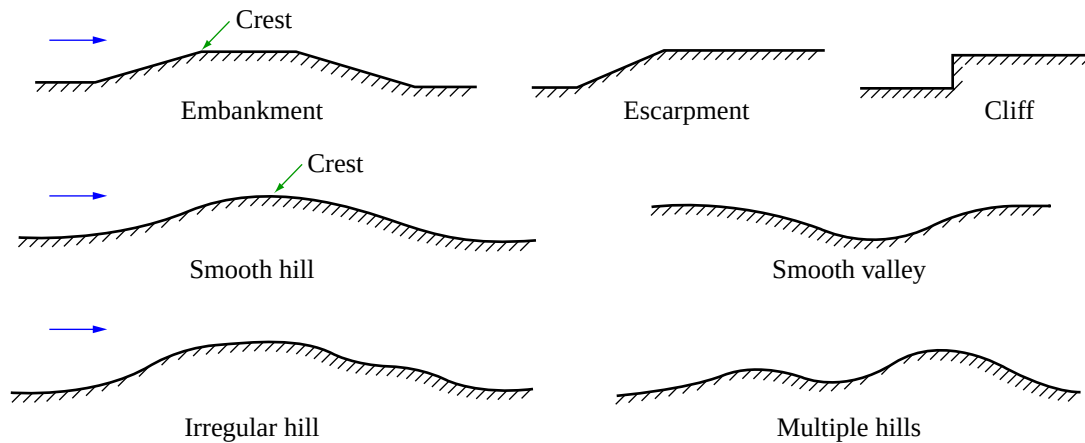
The methods presented are based on calculations made using much more complex methods which solve the Navier-Stokes equations governing the flow over topography in the neutral atmospheric boundary layer. The question of uncertainty in using the methods presented can therefore be considered in two parts: (i) deviations from the accepted complex theoretical methods and (ii) the accuracy of the complex theoretical methods when compared with measurements.

It is not possible to quantify the uncertainties in speed-up factor very precisely because there is a lack of sufficient good quality measurements from which to make an overall assessment. The more detailed method given in Section 4.2 agrees closely with the results obtained using the more complex methods (such as Deaves) and, in general, deviations no greater than about  $\pm 15\%$  in the incremental speed-up factor,  $\Delta K_L$ , occur. The uncertainties of the more complex methods, and the present methods, when compared with full-scale and wind-tunnel measurements are greater. In the absence of other information, it is suggested that an overall uncertainty of  $\pm 30\%$  in  $\Delta K_L$  is assumed where flow separation effects do not influence the location of interest (see Section 7). In making this recommendation it is assumed that other factors influencing the wind speed profile, in particular terrain roughness changes upwind of the site, are taken into account. For an extreme wind speed of 25 ms at  $z = 10$  m, an uncertainty of  $\pm 30\%$  in  $\Delta K_L$  translates, typically, into an uncertainty of about  $\pm 2.5$  m/s in mean wind speed but this will vary with hill slope and height above the surface.

## 3. BACKGROUND TO THE METHODS OF CALCULATION

### 3.1 Types of Topography

The following types of topography are covered by the methods presented.



Sketch 3.1

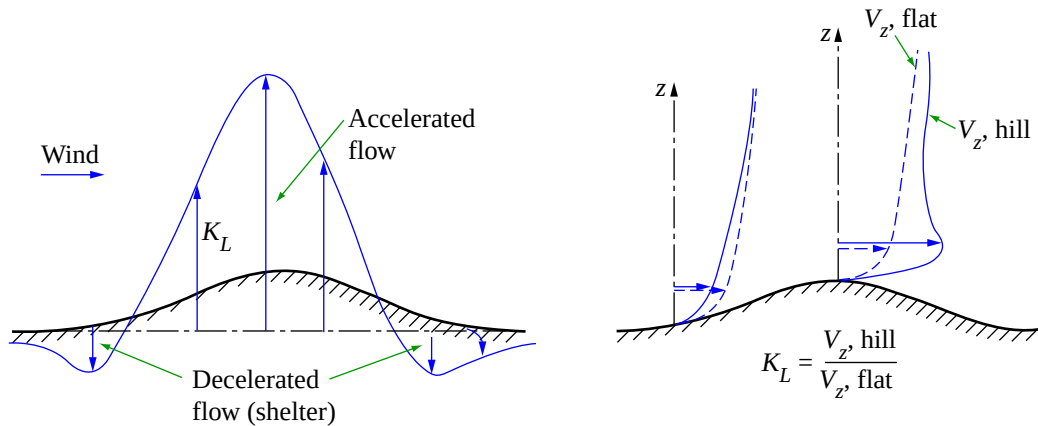
### 3.2 Features of the Flow Over Hills and Similar Topography

The following descriptions apply to hills. Valleys can be treated as inverted hills so that the general features for hills apply except that accelerated flow is replaced by decelerated flow and vice versa.

As the wind approaches a topographic feature, such as illustrated in Sketch 3.1, it is first decelerated near the foot of the hill before being accelerated over the top. On the downwind side the flow is again decelerated eventually returning to the same conditions as for the undisturbed wind upstream of the hill. These flow features are illustrated in Sketch 3.2 which shows typical distributions of speed-up factor near the ground over a hill and examples of the variation of wind speed with height. The following points should be noted.

- (i) There are considerable regions of shelter on both the upwind and downwind approaches to the hill crest.
- (ii) For two-dimensional hills the local incremental speed-up factor,  $\Delta K_L$ , is proportional to the maximum hill (or valley) slope upwind of the crest but on the lee side becomes increasingly dependent on the maximum slope downwind of the crest.
- (iii) The maximum speed-up factor near the ground at the crest can exceed a value of 2 for certain types of topography (*i.e.* the mean wind speed can be doubled).
- (iv) The maximum speed-up factor for rounded hills generally occurs just upwind of the highest point (crest) and not at the crest.

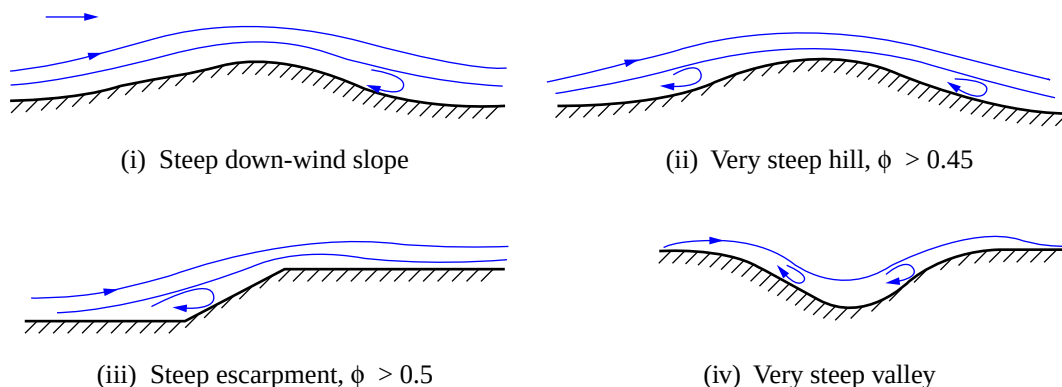
- (v) When the level plateau downwind of the hill is lower, or higher than, the ground level upwind of the hill the wind speed does not return, immediately, to its undisturbed value. When the downwind plateau is higher than the upwind level there is an effective blocking effect on the wind flow which increases the mean wind speed from its undisturbed value, and vice versa when the plateau is lower.



**Sketch 3.2 Illustration of wind speed variation near the ground over a hill**

With increasing height above the surface of the topographic feature, the speed-up factor,  $K_L$ , gradually decreases to a value of unity. The rate at which this takes place varies with location on the hill but, in very general terms, at a height of about  $2L_u$  (where  $L_u = H/\phi_u$ )  $K_L$  is close to unity. In contrast, the actual local wind speed ( $K_L V_{z, flat}$ ) near the crest can reach a maximum value at a height near the ground as illustrated in Sketch 3.2.

For steeper topography the flow will separate from the hill surface. For rounded hills (and embankments) this first occurs at the downwind end of the hill (see Sketch 3.3) where, if the flow has lost sufficient



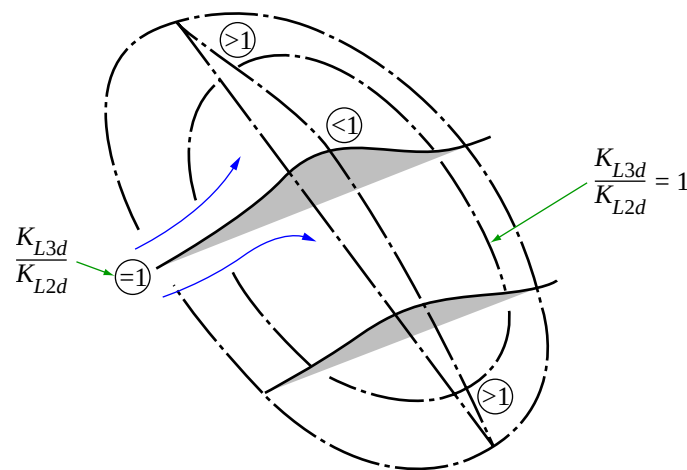
**Sketch 3.3 Examples of flow separation**

momentum in fighting against the adverse pressure gradient induced by the rapid acceleration and deceleration over the hill, it separates from the surface. For hills of increasing steepness this flow separation point moves forward from the lee side towards the crest region until for a very steep hill much of the surface is in a region of separated flow. The effect of this is generally to reduce the speed-up factors where flow

separation has occurred. In the particular case of an escarpment, the adverse pressure gradient is less severe due to the flat plateau region following the crest and separation effects are less severe compared with short embankments and hills.

Additionally, when the upwind slope,  $\phi_u$ , is greater than a certain value the flow is unable to sustain sufficient momentum to remain attached to the surface and it will separate at the upwind base of the hill. The separated flow region forms an effective boundary (as illustrated in Sketch 3.3) of slope  $\phi_{max} < \phi_u$  which is approximately constant for  $\phi_u > \phi_{max}$ .

In general, hills are not two-dimensional; they are of finite width or span. Three-dimensional effects cause part of the flow over the central part of the hill to be deflected around the sides thus reducing the speed-up factor over the central part of the hill span near the crest but increasing it towards the sides as illustrated in Sketch 3.4. This effect is more pronounced at the top of the hill than near the base so that speed-up factors in the upwind (and downwind) regions of the hill are less affected by three-dimensional effects. However, three-dimensional effects are not generally large and they can be represented by the relatively simple methods described in Section 5.1.



**Sketch 3.4 Three-dimensional hill**

### 3.3 Overview of the Methods

The methods presented are based on three main observations in relation to a two-dimensional section of a hill (or valley).

- (i) The fractional speed-up parameter  $s = \Delta K_L / (2\phi)$  is essentially independent of hill slope (assuming flow separation does not occur).
- (ii) The variation of the parameter  $s$  around a hill section is similar to that for an idealised equivalent embankment having the same maximum height and the same maximum slope as illustrated in Sketch 3.5. The hill-section length scale is defined as  $L = H/\phi$  and it is found that, for a given hill slope,  $s$  is dependent only on  $x_o/L$  and  $z/L$ . For an unsymmetrical hill separate length scales based on the maximum upwind, and the maximum downwind, slopes are defined.
- (iii) The variation of  $s$  with  $x_o/L$  for a hill and its equivalent embankment differ in detail due to local variations in surface slope (and curvature) near the foot of the hill and in the crest region. These differences in  $s$  can be accounted for using linear perturbation theory whereby differences in local section shape are related directly to increments in  $s$  (or  $\Delta K_L$ ).

The simple method (METHOD 1) in Section 4.1 gives the variation in graphical form of the speed-up parameter,  $s$ , for two-dimensional embankments at  $z \rightarrow 0$  (Figure 1) together with a correction factor giving the variation of  $s$  with height measured from the hill surface. For simple two-dimensional hills with a rounded top, similar data are presented giving the variation of  $s$  with  $x_o/L$  and  $z/L$  (Figure 3) based on the assumption of a typical simple hill shape. This is satisfactory if the hill profile, particularly in the region of interest, is regular and the determination of  $L_u$  and  $L_d$  is straightforward as shown in Sketch 3.5.

The second method (METHOD 2) in Section 4.2 allows more accurately for the difference in shape between a two-dimensional rounded hill and its equivalent embankment. This is based on the theoretical result that perturbations in hill shape produce perturbations in  $s$  independent of the original hill shape. Computed differences ( $\Delta s$ ) in  $s$  between the actual hill and its equivalent embankment for a number of hill shapes are found to follow a similar trend. The local perturbations,  $\Delta s$ , can be related to the maximum local deviation ( $\Delta H_r/H$ ) in hill height within each of three regions encompassing the foot of the hill (upwind and downwind) and the crest region. An approximately universal relationship is found to apply (for regular-shaped hills) between the ratio  $\Delta s/[\Delta H_r/H]$  and  $x_o/L$  for each of the three regions.

An extension of the second method (Section 6.1) is to be used when the hill shape is not regular and it is necessary to take into account the existence of significant deviations from a regular contour shape as illustrated in Sketch 3.6. This is achieved by applying the linear perturbation method but in this case to larger deviations in contour shape. Bulges (or depressions) superimposed on an otherwise smooth contour shape can be treated as separate hills (or valleys) and their perturbations in terms of the incremental speed-up factor,  $\Delta K'_L$ , can be superimposed (added to) those evaluated for the smooth contour shape.

A further extension, based on identical principles, is to multiple hills (Section 6.2) whereby the overall distribution of  $\Delta K_L$  is obtained by superimposing the perturbations in  $\Delta K_L$  due to each component hill considered independently.

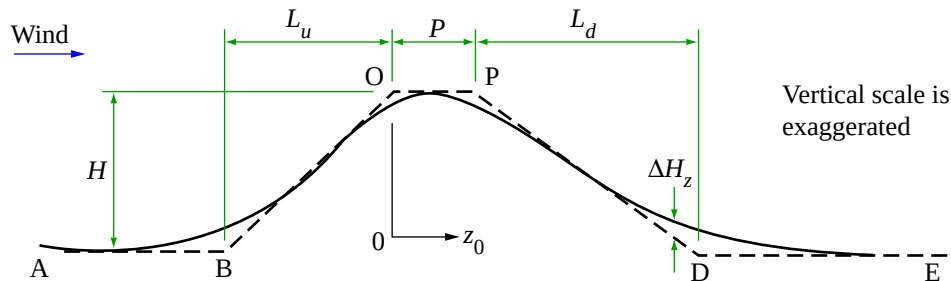
Corrections to the basic methods are given (in Section 5) to account for three-dimensional topography, inclined winds and blockage effects due to high hills.

Steep hills are a particular problem because regions of separated flow occur on either, or both, faces of the hill. The methods given here (Section 7) are less accurate than for the attached flow case but they can be used to give corrections which will have the correct trends.

The method has been calibrated against hills for which measured data (wind tunnel and full scale) are available; the most recent and exhaustive international study on Askervein in the Outer Hebrides (Scotland) is used as the major illustrative example (Section 8).

## 3.4 Input Data for the Methods

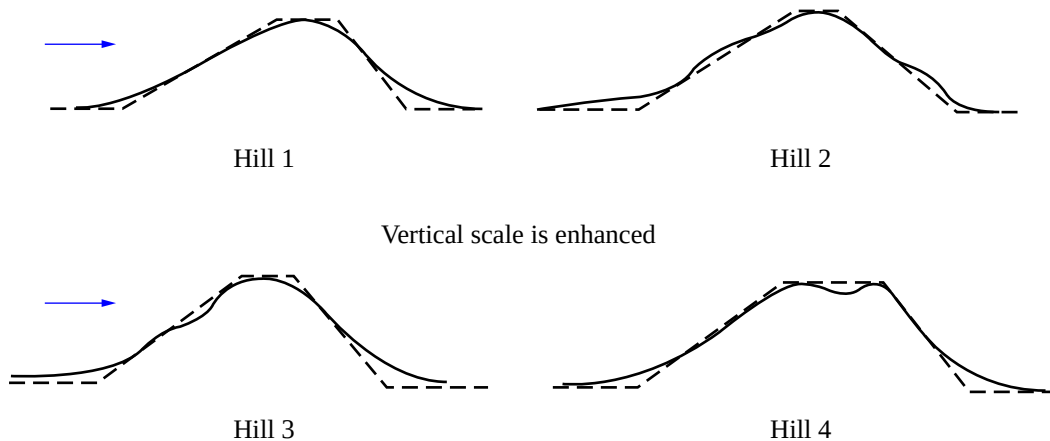
### 3.4.1 Equivalent embankment concept for hills and valleys



**Sketch 3.5 Equivalent embankment for a hill**

The equivalent embankment concept is illustrated in Sketch 3.5 for a section through a hill normal to the local ridge line. Superimposed on the hill shape is the equivalent embankment having the same overall height and the same maximum upwind slope,  $\phi_u = H/L_u$ , and the same downwind maximum slope,  $\phi_d = H/L_d$ . As explained elsewhere, the distribution of the fractional speed-up parameter,  $s$ , over the hill follows a similar distribution to that over the equivalent embankment. The methods presented are based on the premise that all topographical effects can be related (but not equated) to those occurring on the equivalent embankment.

In determining the dimensions of the equivalent embankment the following guidelines should be noted.



**Sketch 3.6 Examples of hills and their equivalent embankments**

- (i) Hill 1 is relatively smooth and regular and the determination of the equivalent embankment slope is straightforward. Methods 1 and 2 apply.

- (ii) Hill 2 exhibits several “bulges”. The influence of the “bulge” is very localised in practice so that if the site of interest on the hill is not located on the bulge then the bulge can be generally ignored and a smoothed hill profile used to determine the equivalent embankment<sup>\*</sup>. Both methods then apply. In this case the values of  $L$  and  $\phi$  will depend on the chosen smoothed profile. The final results are not too sensitive to this choice because if a larger effective value of  $\phi$  is chosen, tending to give larger values of  $K_L$ , the corresponding value of  $P/L$  will be smaller, tending to reduce  $K_L$ .
- (iii) Hill 3 exhibits a depression and the same remarks as for Hill 2 apply.
- (iv) In the case of Hill 4 it is likely that the depression in the central crest region will influence the speed-up factors in a region of interest (*e.g.* near the crest). The local effect of a depression, or a bulge, can be considered separately as described in Section 6.1.

### 3.4.2 Topographic input data

The geometric data required are as follows. Usually, they can be obtained from local Ordnance Survey maps. In the most general case they are required for the two orthogonal sections AA and BB (see Sketch 5.1) which pass through the site with AA normal to the local ridge line.

- (i) The maximum height of the hill section (or embankment).
- (ii) The effective maximum slope ( $\phi_u$ ) upwind of the crest.
- (iii) The effective maximum slope ( $\phi_d$ ) downwind of the crest.
- (iv) The length  $P$  of the flat top of the equivalent embankment.
- (v) For the more detailed method, the difference ( $\Delta H_r$ ) in local height between the hill and its equivalent embankment at various key points is also required (see Section 4.2).

Because the equivalent embankment concept is used for all topography it is convenient to choose the crest of the equivalent embankment at the top of the upwind slope as the reference point for the two-dimensional calculation. All values of along-wind distance  $x_o$  are thus measured from this point although in practice it may be more convenient to work in terms of  $x$  measured from the actual hill crest.

In the general case of a three-dimensional hill with the wind from any direction, the method described in Section 5.2 requires the data in (i) to (v) for the two orthogonal sections AA and BB (see Sketch 5.1) passing through the site normal to, and parallel to, the local ridge line respectively. Also required are the half-height length scales  $L_{01}$ ,  $L_{02}$ ,  $W_{01}$ ,  $W_{02}$  which are used to determine local three-dimensional effects.

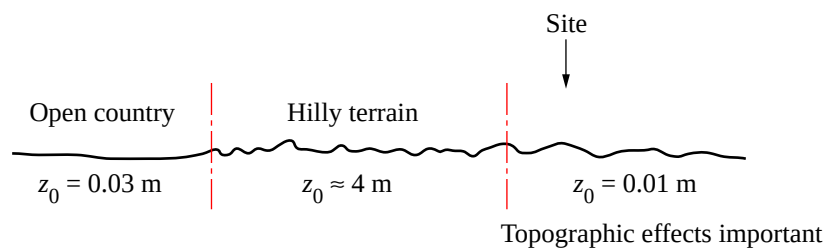
### 3.4.3 Terrain roughness and wind speed profile upwind of hilly site

The speed-up factors,  $K_L$ , are applied to the mean wind speeds ( $V_{z,flat}$ ) that would exist over the site if the surface terrain was flat. Where changes in terrain roughness occur upwind of the site, the variation of  $V_{z,flat}$  with height is calculated at the site location, assuming flat terrain, using, for example, ESDU 82026, ESDU 84011 or the computer program VIEWpac E0108 described in ESDU 01008<sup>7</sup>.

\* The implication of this is that in wind-tunnel testing for terrain effects it is only necessary to model in detail the local profile of the hill in the immediate vicinity of the site under consideration.

The speed-up factors are dependent to some extent on wind speed through the blockage parameter  $H/h$  where  $h$  is the local boundary layer height defined as  $u_*^*/(6f)$ . The effect on values of  $s$  is described in Section 5.1 and is represented by a correction factor,  $f_h$ . For the purpose of estimating  $f_h$  by hand calculation, it is adequate to assume that  $h \approx 1500$  m. There is also a small effect on  $K_L$  of  $H/z_0$ ; this is less than about  $\pm 2\%$  and is ignored.

Section 6.2 explains how the effect of other nearby topography may be taken into account. However, it is only necessary to undertake detailed calculations when the various features are very close (see Section 6.2). In general it is sufficient to allow for the effects of a fetch of hilly terrain upwind of the site simply by modelling the upwind terrain as flat but with a high value of the terrain roughness parameter,  $z_0$ , to account for energy taken out of the flow. A value of  $z_0$  between 3 m and 5 m is suggested and the related fetch of hilly terrain can be treated simply as an upwind roughness change using the computer program E0108<sup>7</sup> or the methods in other related Data Items.



**Sketch 3.7 An example of complex terrain**

## 4. BASIC METHODS FOR TWO-DIMENSIONAL TOPOGRAPHY; WIND NORMAL TO RIDGE LINE

The following methods apply when the wind direction is along a section through a hill normal to the local ridge line (*e.g.*, section AA in Sketch 5.1) and assumes the section can be treated as two-dimensional (*i.e.*, the hill width is greater than about five times its length). Methods for estimating three-dimensional effects, including the effect of wind direction are given in Section 5.

### 4.1 Simple Method (METHOD 1)

This method only differentiates between specific types of topography, namely embankment (Figure 1a & 1b), escarpment (Figure 2) or rounded hill (or valley) (Figure 3). It takes into account the maximum slope of the hill (or valley) section but in the case of an arbitrarily-defined hill no account is taken of the detailed variation of the local profile shape from that assumed in producing the simple method. A means for correcting for the detailed local hill profile is given in Section 4.2.

In order to use the simple method the only input data required are  $H$ ,  $L_u$ ,  $L_d$  and  $P$  relating to the equivalent embankment.

The speed-up factor for two-dimensional topography when the wind direction is normal to the ridge line is given by

$$K_{L2d} = 1 + 2\phi s, \quad (4.1)$$

where  $\phi = H/L$  (4.1a)

and  $L$  is the effective slope as defined in Table 4.1.

Figures 1 to 3 give values of  $s$  at  $z = 0$ . For convenience the variation of  $s$  with height is given in equation form and is represented by

$$\frac{s_z}{s_{z=0}} = \frac{1}{1 + a |s_{z=0}| (z/L)^{0.8}}, \quad (4.2)$$

noting the use of the modulus of  $s_{z=0}$  on the right-hand side of Equation (4.2). Values of  $a$  depend on  $x_o/L$  and are given by Equations (4.3a) to (4.3e).

$$\text{For } x_o/L \leq -1 \text{ (A to B)} \quad a = 12. \quad (4.3a)$$

$$\text{For } -1 \leq x_o/L < 0.25 \quad a = 5 - 7x_o/L. \quad (4.3b)$$

$$\text{For } 0.25 \leq x_o/L \leq P/L + 1 \quad a = 5 + 7(x_o - P)/L \text{ or } 3.25 \text{ whichever is larger.} \quad (4.3c)$$

$$\text{For } x_o/L \geq P/L + 1 \quad a = 12. \quad (4.3d)$$

For convenience, and with only little loss in accuracy, it is satisfactory to assume that Equation (4.3c) can be simplified to

$$a = 5 = a_{x_o=0} = a_{x_o=P}. \quad (4.3e)$$

For the situation when the upwind and downwind slopes are not the same,  $L$  is equated to  $L_u$  on the upwind slope and, on the downwind slope,  $L$  varies between  $L_u$  and  $L_d$  such that

$$L = L_u - (L_u - L_d) \frac{x_o - P}{L_d}. \quad (4.4)$$

Table 4.1 summarises the values of the parameter  $L$  to be used in conjunction with Equations (4.1a), (4.2) and (4.3).

**TABLE 4.1**

| Region<br>(see Sketch 3.5)       | Effective length scale:<br>* $L$ | Variation of $s$ with<br>$z$ : $a$ in Equation<br>(4.2) |
|----------------------------------|----------------------------------|---------------------------------------------------------|
| Upwind of hill base:<br>A to B   | $L_u$                            | 15                                                      |
| Upwind slope:<br>B to O          | $L_u$                            | Equation (4.3b)                                         |
| Crest region:<br>O to P          | $L_u$                            | Equation (4.3c)                                         |
| Downwind slope:<br>P to D        | $L_u$                            | Equation (4.3d)                                         |
| Downwind of hill base:<br>D to E | Equation (4.4)                   | 15                                                      |
|                                  | $L_d$                            |                                                         |

\* If  $\phi_u (= H/L_u) > 0.4$ , see Section 7.1

The general (simple) procedure for estimating  $K_{L2d}$  is as follows. If only one particular location on the hill is of interest, such as the summit, the procedure is used only once for that location.

- (i) From the given topographic dimensions construct the equivalent embankment (see Section 3.4.1). Obviously, if the topographic feature is an embankment or an escarpment this step is not necessary.
- (ii) Determine the topographic dimensions  $H$  and  $L$  (noting that for a valley,  $H$  is negative). When the upwind and downwind slopes of the equivalent embankment are not equal Table 4.1 summarises the value of  $L$  to be used for the region of interest.
- (iii) From Figure 1 or 3 determine  $s$  (for  $z = 0$ ) for the required location  $x_o$ .
- (iv) For the required height,  $z$ , above the topographic surface evaluate  $s_z/s_{z=0}$  using Equations (4.2) and (4.3) as directed by Table 4.1.
- (v) The speed-up factor at the required location is

$$K_{L2d} = 1 + 2\phi s_z$$

where  $\phi = H/L$  and  $L$  is defined by Table 4.1. For a valley  $H$  (and hence  $\phi$ ) is negative so that if  $s$  is positive then  $K_L$  is less than unity.

For the effects of three-dimensional topography, and wind inclination, see Section 5.

## 4.2 More Detailed Method (METHOD 2)

This method takes into account more properly the actual profile shape of the section under consideration when the hill (or valley) has a rounded crest region. The simple method described in Section 4.1 assumes a typical rounded hill and this is adequate when the hill profile is relatively smooth and regular. However, better estimations of the  $s$  factors can be obtained by taking into account the deviations in shape between the hill (or valley) profile and its equivalent embankment. As explained in Section 3.3, it is possible to relate deviations in  $s$  due to deviations in profile shape between two types of hill by a linear perturbation theory by which hill shapes, and their component variation in  $\Delta K_L$ , are superimposed on each other to obtain an overall solution for the combined shape.

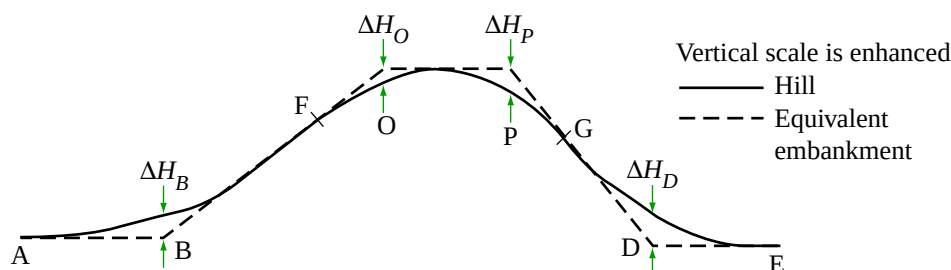
The method described here uses a simplification of this result, as described in Section 3.3 (METHOD 2). It relates values of  $\Delta s = s_{hill} - s_{emb}$  with deviations in  $\Delta H_r/H$  between the hill and its equivalent embankment in three regions at key points where the local deviations, due to rounding of the embankment corners, are a maximum. It is found from theoretical computations that, for hills of regular shape, the ratio

$$f_r = \frac{\Delta s}{\Delta H_r/H} \quad (4.5)$$

is, to a close approximation, independent of hill shape and dependent only on  $x/L$ . This variation is given in Figure 4 for the three regions (defined in Sketch 4.1) encompassing the foot of the hill (A to E and G to E) and the crest region (F to G). The corresponding values of  $\Delta H_r$ , are  $\Delta H_B$ ,  $\Delta H_D$  and, for the crest region, a combination of  $\Delta H_O$  and  $\Delta H_P$ . Note that it is these key values of  $\Delta H_r$  that are relevant in the method and not the values of  $\Delta H_x$  at the location of interest (unless it happens to coincide with one of the key points).

The following steps summarise the procedure for estimating the speed-up factor at a particular location on

a hill. For a valley the procedure is the same but the hill and equivalent embankment shape in Sketch 4.1 are inverted. A calculation sheet is provided in Section 10.



**Sketch 4.1 Definition of various regions on topographical shape**

- (i) For the hill section under consideration determine the equivalent embankment shape using the guidelines in Section 3.4.1.
- (ii) Determine the parameters  $H$ ,  $L_u$ ,  $L_d$  and  $P$  defining the equivalent embankment by graphical means or calculation.
- (iii) For the location ( $x_o$ ) under consideration determine  $s_{emb}$  at  $z = 0$  from Figure 1a (and Figure 1b if necessary) for the equivalent embankment.
- (iv) For the region of the hill under consideration which includes the location of interest, that is region AF, FG or GE, evaluate the parameter  $\Delta H_r/H$  at the point B (region AF), at the points O and P (region FG) or at the point D (region GE).

For the crest region FG, in order to provide continuity in  $\Delta s$  for an unsymmetrical hill, it is necessary to define  $\Delta H_r$  as

$$\Delta H_r = \Delta H_O(1 - x_o/P) + \Delta H_P(x_o/P) \text{ for } 0 \leq x_o \leq P \quad (4.6a)$$

with  $\Delta H_r = \Delta H_O$  for  $x_o < 0$  (4.6b)

and  $\Delta H_r = \Delta H_P$  for  $x_o > P$ . (4.6c)

- (v) Obtain the factor  $f_r$  from Figure 4 for the appropriate location,  $x_o$ .

- (vi) Evaluate  $\Delta s$  for the location under consideration from

$$\Delta s = f_r \times (\Delta H_r/H). \quad (4.7)$$

- (vii) Evaluate  $s_{2d} = s$  for the hill under consideration corresponding to the chosen point from

$$s_z = [s_{emb}(\text{step iii}) + \Delta s(\text{step vi})] \{s_z/s_{z=0}\} \quad (4.8)$$

where  $s_z/s_{z=0}$  is given by Equation (4.2) and Table 4.1.