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Supersedes ESDU 67016 and ESDU 68007

## **Forced convection heat transfer in straight tubes. Part 1: turbulent flow**

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(Continued on inside back cover)

## FORCED CONVECTION HEAT TRANSFER IN STRAIGHT TUBES.

### Part 1: turbulent flow

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## FORCED CONVECTION HEAT TRANSFER IN STRAIGHT TUBES.

### Part 1: turbulent flow

#### 1. NOTATION AND UNITS

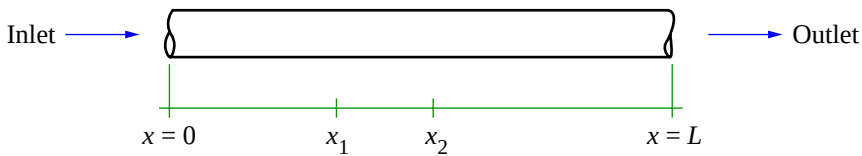
|           |                                                        | Units                    |                                                            |
|-----------|--------------------------------------------------------|--------------------------|------------------------------------------------------------|
|           |                                                        | SI                       | British                                                    |
| $A$       | heat transfer area of tube internal surface, $\pi D x$ | $\text{m}^2$             | $\text{ft}^2$                                              |
| $c_p$     | specific heat capacity at constant pressure            | $\text{J/kg K}$          | $\text{Btu/lb } ^\circ\text{F}$                            |
| $D$       | tube internal diameter                                 | $\text{m}$               | $\text{ft}$                                                |
| $f$       | friction factor, $2\tau/\rho V^2$                      | —                        | —                                                          |
| $g$       | acceleration due to gravity                            | $\text{m/s}^2$<br>(9.81) | $\text{ft/h}^2$<br>( $4.17 \times 10^8$ )                  |
| $\dot{m}$ | mass flux                                              | $\text{kg/m}^2$          | $\text{lb/h ft}^2$                                         |
| $\dot{M}$ | total mass flowrate per tube                           | $\text{kg/s}$            | $\text{lb/h}$                                              |
| $Gr$      | Grashof number, see Equation (4.5)                     | —                        | —                                                          |
| $L$       | tube length                                            | $\text{m}$               | $\text{ft}$                                                |
| $Nu$      | Nusselt number, $\alpha D/\lambda$                     | —                        | —                                                          |
| $n$       | exponent in Equations (4.3) and (4.4)                  | —                        | —                                                          |
| $Pr$      | Prandtl number, $c_p \eta/\lambda$                     | —                        | —                                                          |
| $\dot{q}$ | local heat flux                                        | $\text{W/m}^2$           | $\text{Btu/ft}^2\text{h}$                                  |
| $\dot{Q}$ | total heat duty                                        | $\text{W}$               | $\text{Btu/h}$                                             |
| $Re_s$    | Reynolds number, $\dot{m}D/\eta$                       | —                        | —                                                          |
| $r$       | radial co-ordinate                                     | $\text{m}$               | $\text{ft}$                                                |
| $S$       | tube cross-sectional area, $\pi D^2/4$                 | $\text{m}^2$             | $\text{ft}^2$                                              |
| $T$       | temperature                                            | $\text{K}$               | $^{\circ}\text{R}$<br>( $\approx ^{\circ}\text{F} + 460$ ) |
| $u$       | local fluid velocity in axial direction                | $\text{m/s}$             | $\text{ft/h}$                                              |
| $V$       | mean fluid velocity in axial direction                 | $\text{m/s}$             | $\text{ft/h}$                                              |
| $x$       | axial or streamwise co-ordinate                        | $\text{m}$               | $\text{ft}$                                                |

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|           |                                     |                    |                         |
|-----------|-------------------------------------|--------------------|-------------------------|
| $\alpha$  | heat transfer coefficient           | W/m <sup>2</sup> K | Btu/ft <sup>2</sup> h°F |
| $\beta$   | coefficient of volumetric expansion | 1/K                | 1/°F                    |
| $\eta$    | dynamic viscosity                   | kg/m s             | lb/ft h                 |
| $\lambda$ | thermal conductivity                | W/m K              | Btu/h ft °F             |
| $\rho$    | density                             | kg/m <sup>3</sup>  | lb/ft <sup>3</sup>      |
| $\tau$    | wall shear stress                   | N/m <sup>2</sup>   | lbf/ft <sup>2</sup>     |

Subscripts

|     |                                   |
|-----|-----------------------------------|
| $a$ | arithmetic mean value             |
| $b$ | bulk fluid value                  |
| $c$ | constant fluid property value     |
| $f$ | fully-developed flow value        |
| $i$ | inlet conditions                  |
| $l$ | logarithmic mean value            |
| $m$ | mean value over axial distance    |
| $o$ | outlet conditions                 |
| $w$ | conditions at wall                |
| $x$ | local value at axial position $x$ |



Sketch 1.1 Axial co-ordinate notation

## 2. INTRODUCTION

This Data Item is one of two on forced convective heat transfer in straight tubes, and supersedes ESDU 67016 and 68007. The second Item, ESDU 93018, supersedes ESDU 68006. The methods of this Item apply to forced convective heat transfer in straight tubes for turbulent flow of a single-phase Newtonian fluid. The effects on heat transfer of a number of secondary factors are discussed.

Section 3 provides definitions of the bulk fluid temperature, the heat transfer coefficient, Nusselt number, the laminar, transition and turbulent flow regimes and a note concerning the evaluation of fluid properties. A recommended equation for the estimation of Nusselt number and hence heat transfer coefficient is presented in Section 4.1. Whilst this Data Item makes clear recommendations for the use of this equation over a specified range of flow conditions, many other equations are available for the prediction of forced convective heat transfer coefficients in turbulent flow. Five commonly-used alternative equations are presented in Appendix A. The methods of accounting for the effects of fluid property variations and surface roughness are discussed in Sections 4.2 and 4.3 respectively. In Sections 4.4 and 4.5 guidance is given on the effects of mixed convection and entrance region heat transfer on the heat transfer coefficient. The use of the recommended equation and determination of the influence of secondary effects is illustrated by the worked examples of Section 5.

In Appendix A, detailed information is provided on the accuracy of all six equations compared with a comprehensive experimental data set. Figures A1 to A3 are plots of the correlating equations over their stated ranges of use and are useful in comparing the predictions of the equations.

## 3. DEFINITIONS

### 3.1 Bulk Fluid Temperature

The local value of the fluid temperature  $T$  is a function of axial position  $x$ , and radial position  $r$ . To achieve a simple representation of the heat transfer a characteristic temperature of the fluid, namely the local bulk fluid temperature  $T_{b,x}$ , is defined at axial position  $x$ .

$$\dot{m} c_{p,x} T_{b,x} = \frac{1}{S} \int_S \rho c_p u T \, dS. \quad (3.1)$$

This definition is difficult to use unless the specific heat capacity,  $c_p$ , is assumed to be constant over the tube cross-section. The definition of the local bulk fluid temperature,  $T_{b,x}$ , then simplifies to

$$T_{b,x} = \frac{1}{\dot{m} S} \int_S \rho u T \, dS. \quad (3.2)$$

The variation of the local bulk temperature along the tube is related to the local wall heat flux. Between any two axial stations  $x_1$  and  $x_2$  the change in local bulk temperature is given by

$$T_{b,x_2} - T_{b,x_1} = \frac{\pi D}{\dot{M}} \int_{x_1}^{x_2} \frac{\dot{q}}{c_p} dx. \quad (3.3)$$

When the specific heat capacity is assumed to be constant between the two axial stations the change in the local bulk temperature simplifies to

$$T_{b,x_2} - T_{b,x_1} = \frac{\pi D}{\dot{M} c_p} \int_{x_1}^{x_2} \dot{q} \, dx. \quad (3.4)$$

The mean bulk temperature between two axial stations along the tube is given by

$$T_{b,m} = \frac{T_{b,x_1} + T_{b,x_2}}{2}. \quad (3.5)$$

### 3.2 Heat Transfer Coefficients

The heat transfer between the tube wall and a fluid moving relative to the wall can be expressed in terms of a local heat transfer coefficient defined as

$$a_x = \frac{\dot{q}}{|T_w - T_b|_x}. \quad (3.6)$$

The definition of the flow-length mean heat transfer coefficient consistent with Equation (3.6) is

$$\alpha_m = \frac{1}{L} \int_0^L \alpha_x dx. \quad (3.7)$$

In practice, it is usually more convenient to use a mean heat transfer coefficient defined in terms of the total heat load over the entire tube and a characteristic mean temperature difference. There are two tube wall thermal boundary conditions for which the definition is relatively simple.

The first is the uniform heat flux (UHF) thermal boundary condition for which the mean heat transfer coefficient,  $\alpha_{am}$ , is based on the arithmetic mean temperature difference (AMTD).

$$\alpha_{am} = \frac{\dot{Q}/A}{\Delta T_{am}}, \quad (3.8)$$

where 
$$\Delta T_{am} = \frac{(T_w - T_{b,i}) + (T_w - T_{b,o})}{2}. \quad (3.9)$$

The second thermal boundary condition is the uniform wall temperature (UWT) thermal boundary condition for which the mean heat transfer coefficient,  $\alpha_{lm}$ , is based on the logarithmic mean temperature difference (LMTD).

$$\alpha_{lm} = \frac{\dot{Q}A}{\Delta T_{lm}}, \quad (3.10)$$

where 
$$\Delta T_{lm} = \frac{(T_w - T_{b,i}) - (T_w - T_{b,o})}{\log_e \left[ \frac{(T_w - T_{b,i})}{(T_w - T_{b,o})} \right]}. \quad (3.11)$$

These conditions are closely approached in practice by a number of situations.

The UWT thermal boundary condition is closely approached when the outside tube surface is heated by an isothermally condensing fluid, or cooled by an isothermally boiling fluid. It is also approached in a number of other situations, such as in co-current (parallel) heat exchangers where the heat transfer coefficient and flowing heat capacity ( $\dot{M}c_p$ ) for the external flow are approximately equal to the corresponding values for the flow inside the tubes, or in heat exchangers where the heat transfer coefficient and flowing heat capacity ( $\dot{M}c_p$ ) for the shell-side are much larger than the corresponding values for the flow inside the tubes.

The UHF thermal boundary condition solutions are generally applied to the practical situations of electric resistance heating and counter-current heat exchangers where the values of heat transfer coefficient and flowing heat capacity ( $\dot{M}c_p$ ) for the shell-side are approximately equal to the corresponding values for the flow in the tubes.

When the actual thermal boundary conditions cannot be approximated by either the UHF or UWT thermal boundary conditions it is recommended that the LMTD method be used to evaluate the heat transfer area, with

$$\Delta T_{lm} = \frac{(T_w - T_{b,i}) - (T_w - T_{b,o})}{\log_e \left[ \frac{(T_w - T_{b,i})}{(T_w - T_{b,o})} \right]}. \quad (3.12)$$

The influence of the thermal boundary condition on the heat transfer coefficient is generally more significant in laminar flow (see ESDU 93018) than in turbulent flow. The experimental turbulent flow heat transfer data used in the preparation of this Item show no influence of the thermal boundary condition on the fully-developed heat transfer coefficient. Therefore it is recommended that the methods presented here be used to predict  $\alpha_x$ ,  $\alpha_{lm}$  and  $\alpha_{am}$ .

## 3.3 Nusselt Number

The local Nusselt number is defined as

$$Nu_x = \frac{\alpha_x D}{\lambda} \quad (3.13)$$

and the mean Nusselt number is defined as

$$Nu_m = \frac{\alpha_m D}{\lambda} \quad (3.14)$$

## 3.4 Flow Regime

The dependence of the heat transfer coefficient on the Reynolds number indicated in Equation (4.1) is important because the mechanism of forced convective heat transfer is influenced by the type of flow regime that exists.

In the absence of free convection effects the type of flow regime is categorised by the flow Reynolds number. For the purposes of the equations presented in the two Data Items on forced convective heat transfer the following regime boundaries are defined.

LAMINAR :  $Re < 2300$

TRANSITION :  $2300 \leq Re \leq 4000$

TURBULENT :  $Re > 4000$

Data in this Item are for turbulent flow. Laminar and transition flows are considered in ESDU 93018.

## 3.5 Evaluation of Fluid Properties

Except where stated otherwise the data for the calculation of local heat transfer coefficients apply for fluid properties evaluated at the bulk temperature,  $T_b$ , at any axial position along the tube. The data for the calculation of the mean heat transfer coefficient apply for fluid properties evaluated at the mean bulk temperature,  $T_{b,m}$ , between the two axial stations along the tube.

## 4. HEAT TRANSFER IN FULLY-DEVELOPED TURBULENT FLOW

### 4.1 Representation of Heat Transfer Coefficient Data and Estimation of Nusselt Number for Use under Idealised Conditions

Convective heat transfer coefficients are generally expressed in dimensionless form by the Nusselt number ( $Nu = \alpha D / \lambda$ ). Under conditions where the factors listed in Section 2 and discussed in Sections 4.2 to 4.5 are ignored the Nusselt number can be represented as a function of Reynolds number and Prandtl number. In some of the following equations Nusselt number is given as a function of Reynolds number, Prandtl number and the dependent variable friction factor. The equation recommended here has a theoretical basis and this is considered an advantage over the empirical correlations commonly used.

#### Recommended equation

In this section an equation is presented for the prediction of Nusselt number for the fully-developed turbulent flow of gases and liquids under idealised conditions where the factors of Sections 4.2 to 4.5 do not influence the heat transfer. This equation can also be applied in practical cases where the degree of accuracy required does not justify these effects being considered.

Theoretical analysis by Petukov (Derivation 40) yields expressions for temperature and velocity profiles, heat transfer coefficients and skin friction for forced convective turbulent flow heat transfer. The predictions of the analytical solution for heat transfer are expressed in terms of Nusselt number as a function of Reynolds number and Prandtl number. The following two equations (presented by Petukov) are recommended for  $4000 \leq Re \leq 5 \times 10^5$  and  $0.6 \leq Pr \leq 60$ :

$$Nu = \frac{(f/2)RePr}{1.07 + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)}, \quad (4.1)$$

$$\text{and} \quad f = \frac{1}{4(1.82 \log_{10} Re - 1.64)^2}. \quad (4.2)$$

The variation of Nusselt number in the stated ranges of Reynolds number and Prandtl number is shown in Figure 1. The curves correspond to Equation (4.1). Information on the accuracy of this equation is given in Table A2 of Appendix A. The experimental data against which these comparisons have been made are drawn from both uniform heat flux (UHF) and uniform wall temperature (UWT) experimental boundary conditions.

Petukov presented a second equation expressing Nusselt number as a function of Reynolds number and Prandtl number. This more complex equation, presented in Appendix B, fits the analytical solution more accurately than Equation (4.1) over the range  $5 \times 10^5 Re \leq 5 \times 10^6$  and  $60 \leq Pr \leq 2000$ . Therefore it is recommended for calculation of Nusselt number outside the ranges of Reynolds number and Prandtl number for which experimental data are available. Figure 2 shows the variation of Nusselt number predicted by Equation (B1.1).

#### Other commonly-used equations

Many other equations are available for the prediction of Nusselt number for the fully-developed turbulent flow of gases and liquids. Appendix A lists five commonly used equations and shows how their performance against the experimental data set compares with that of Equation (4.1).

## 4.2 Variable Fluid Properties

Radial variations of the fluid properties will occur under conditions where the difference between the wall temperature and fluid bulk temperature is large, or when operating near the critical point. Variations of viscosity, thermal conductivity, specific heat capacity and density will affect the prediction of heat transfer. The correction factors presented for liquids and gases are considered to be the most accurate available for use with Equation (4.1).

The variation of density can give rise to free convection (buoyancy) effects. Guidance on the significance of free convection effects is given in Section 4.4.

### Liquids

For liquids far from their critical points, the thermal conductivity, specific heat capacity and density are weak functions of temperature, but the dynamic viscosity varies significantly with temperature. The relative change of Nusselt number due to viscosity variations with temperature can be expressed as

$$\frac{Nu}{Nu_c} = \left( \frac{\eta_b}{\eta_w} \right)^n \quad (4.3)$$

where Derivation 40 states the value of  $n = 0.11$  for heating and  $n = 0.25$  for cooling. Equation (4.3) is valid over the range  $0.08 \leq \eta_w/\eta_b \leq 40$ ,  $10^4 \leq Re \leq 1.25 \times 10^5$  and  $2 \leq Pr \leq 140$ .

### Gases

For gases other than those considered in this section the use of a correction factor is not recommended.

For certain specific gases in cases where there is a large temperature difference between the tube wall and gas flow the available data have been used to develop temperature correction factors. If the variations of the fluid properties with absolute temperature are assumed to follow simple power laws, which is a reasonable approximation for gases, the relative change of Nusselt number due to property variations with temperature may be expressed by

$$\frac{Nu}{Nu_c} = \left( \frac{T_w}{T_b} \right)^n \quad (4.4)$$

where the value of  $n$  depends upon the gas being considered. Values of  $n$  for the heating of several gases and steam are given in Table 4.1 together with the ranges of  $T_b, T_w/T_b$  and static pressure for which the exponent has been established empirically. The values of  $n$  are based on experimental data for substantially fully-developed flow (that is  $x/D > 10$ ) reported in the Derivations shown in Table 4.1. In all cases the experimental data were correlated with an *rms* error of between 7.5 and 11 per cent.

In cooling, the available experimental data show that the ratio  $T_w/T_b$  has no significant effect on the Nusselt number (that is  $n = 0$ ) for several of the gases listed in the table. These data are for gases preheated above ambient temperature, and data for each gas extends to  $T_b$  in excess of 1800 K.

**TABLE 4.1 Values of the Exponent  $n$  for Use in Equation (4.4)**

| Fluid          | Heating, $T_w/T_b > 1$ |                       |                       |                               |                | Cooling<br>$0.1 < T_w/T_b < 1$ |                             |
|----------------|------------------------|-----------------------|-----------------------|-------------------------------|----------------|--------------------------------|-----------------------------|
|                | $n$                    | Range of $T_b$<br>(K) | Range of<br>$T_w/T_b$ | Range of<br>Pressure<br>(bar) | Derivations    | $n$                            | Derivations                 |
| Air            | -0.40                  | 300 – 1000            | 1.0 – 2.4             | 1.0 – 8.0                     | 19, 25, 27     | 0                              | 6, 16, 17, 24<br>38, 39, 40 |
| Carbon Dioxide | -0.12                  | 300 – 400             | 1.0 – 2.1             | 1.0 – 8.0                     | 19             | –                              | –                           |
| Helium         | -0.38                  | 300 – 1200            | 1.0 – 4.2             | 1.0 – 8.0                     | 19, 27, 33, 36 | 0                              | 38                          |
| Hydrogen       | -0.37                  | 300 – 1000            | 1.0 – 5.0             | 15 – 500                      | 33, 36, 37     | –                              | –                           |
| Nitrogen       | -0.49                  | 120 – 730             | 1.0 – 6.7             | 2.9 – 5.9                     | 34             | 0                              | 43                          |
| Steam          | +0.18                  | 560 – 750             | 1.0 – 1.5             | 21 – 100                      | 20             | –                              | –                           |

It is therefore recommended that the data given in Table 4.1 are used only within the ranges stated.

## 4.3 Surface Roughness

Surface roughness may be classified into two types as follows.

- Natural roughness where the individual roughness elements have an irregular pattern such as in the case of galvanised commercial tubes.
- Artificial roughness where the roughness has a regular pattern such as in tubes containing fins, ribs or screw threads. For such situations ESDU<sup>50</sup> presents methods of predicting the heat transfer coefficient.

Both types of roughness will increase pressure drop, accompanied by some increase in the heat transfer coefficient.

The experimental data (Derivations 13 and 14) for the effect of roughness on heat transfer in commercial tubes are insufficient to yield generalised correlations for every type of commercially rough surface over all tube diameters. The effect of natural roughness enhancement will generally be enough for it to be ignored.

## 4.4 Mixed Convection

The motion which occurs in a fluid as a result of buoyancy forces is called free or natural convection and is characterised by the Grashof number which is defined as

$$Gr = \frac{\beta g D^3 \rho^2 |T_w - T_{b, m}|}{\eta^2} \quad (4.5)$$

In general this type of motion will be present flows to a degree in all forced flows.

Figures 3 and 4, which are based on the correlation of both theoretical and experimental work reported in Derivation 32, show the forced, free and mixed convection regions for the flow of gases and liquids in horizontal and vertical tubes respectively. These Figures cover the range  $0.01 < PrD/L < 1$ .

The limits plotted in Figure 4 have been established for heated upflow and cooled downflow (buoyancy assisted flow). Too few experimental data exist to establish the limits for heated downflow and cooled upflow (buoyancy opposed flow) and no similar figure is available.

By calculating the non-dimensional parameters  $Re, Pr, Gr$  and  $D/L$  and forming the groups shown in Figures 3 and 4, the operating point for any practical application may be determined. **If this point lies in the shaded forced turbulent regime, the effect of free convection may generally be neglected.** However, if the fluid is heated and flows upwards in a vertical tube, degradations of the heat transfer coefficient can occur, even for operating points in the shaded region of Figure 4. Degradations occur when

$$Re < 30Gr^{0.38} \quad (4.6)$$

This criterion has been established over the ranges  $4.2 \times 10^3 \leq Re \leq 1.2 \times 10^5$ ,  $0.7 \leq Pr \leq 7$  and for ratios of  $L/D$  up to 70. The temperature difference  $|T_w - T_{b,m}|$  used in the evaluation of  $Gr$  in this criterion is the value occurring in the absence of significant (i.e. 10% or less) degradation of the heat transfer coefficient\*. The criterion should be used in order to avoid heat transfer degradations with heated upflow in tubes and the consequent local wall temperature rises that occur under uniform heat flux conditions.

Local heat transfer degradations might also occur for cooled downflow in vertical tubes, but there are no known data to support this possibility. Local degradations are not critical in this case, since they can only cause local reductions in the wall temperature under uniform heat flux conditions.

Data in Derivations 42 and 48 show that for buoyancy-opposed flow (heated downflow and cooled upflow) buoyancy enhances heat transfer. Consequently, in mixed turbulent buoyancy-opposed flow there are no heat transfer crises comparable to those in assisted flow.

It is not generally possible to provide specific data on the variation of  $Nu$  with  $Re, Gr$  and  $Pr$  in the turbulent mixed convection region. For further information on this topic, Derivations 1, 7, 35, 41 and 48 should be consulted.

Figures 3 and 4 suggest that the effects of free convection on turbulent forced convection are suppressed for large values of the ratio  $L/D$ , but this is unlikely to be the case in practice. The upper bounds of the mixed convection-turbulent flow regions in these Figures are strictly valid only for  $L/D$  below about 30, and caution should be exercised in using the Figures for higher values of this ratio.

## 4.5 Tube Entrance Effects

A number of distant thermal entrance conditions can be identified.

The first condition is that in which the velocity and temperature profiles develop simultaneously along the tube (that is the fluid is heated over the complete length of the tube). This condition is appropriate to most practical applications.

\* This criterion has been deduced from a correlation of data for pressurised water at subcritical temperatures given in Derivation 44. The criterion has been extended to encompass data for supercritical pressure helium at temperatures well above the critical (i.e. temperatures greater than 1.5 times the temperature at which  $c_p$  is a maximum in  $c_p$  at any given pressure) given in Derivation 45, and air and water data at atmospheric pressure given in Derivation 41 and 46 respectively.

A second thermal entrance condition is that in which the temperature profile develops after the velocity profile has been fully established. This condition arises when a long unheated length of tube precedes the start of heating (or cooling). Most experimental work on single phase heat transfer in tubes has employed this entrance arrangement.

Clearly a range of possibilities intermediate between the above two conditions can arise. However, in most cases the local heat transfer coefficient decreases from a maximum value at the point of commencement of heating to a constant value at the point where the temperature and velocity profiles become fully-developed. Downstream of this point, under conditions of constant fluid properties\*, the value of the local heat transfer coefficient is constant.

In addition any maldistribution of the flow in the entrance region, the nature of which is dependent on the geometry of the entrance configuration (that is bend, sharp edge, sudden contraction, orifice, etc.), will give rise to further changes in the heat transfer coefficient in the entrance region.

Generally, the length necessary for thermal development to be completed (that is the thermal entrance length) is dependent on the Reynolds number and Prandtl number. The thermal entrance length is usually defined as the tube length necessary for the local Nusselt number to diminish to a value 5 per cent larger than its value in fully-developed flow. Data are available (Derivations 4, 9, 15, 17, 21, 22, 23, 30 and 31) on thermal entrance lengths for the second type of thermal entrance condition described previously, and there is evidence that, as an approximation, these data can be applied to other thermal entrance conditions. Errors arising from this approximation may be quite large for the first few diameters of heated length but will diminish rapidly with distance downstream.

In Table 4.2 approximate values of the thermal entrance length for local conditions are given for different values of Reynolds number and Prandtl number. The effect of the thermal entrance length on the mean Nusselt number will diminish as heated length increases. Some typical values for a given definition of mean Nusselt number are shown in Table 4.3.

Equation (4.1) may be applied without serious error provided the actual tube length exceeds the thermal entrance length given in Tables 4.2 and 4.3. The thermal entrance length is given for local and mean Nusselt numbers respectively. Differences between the values of entrance lengths calculated using local and mean Nusselt numbers arise from the definition of mean Nusselt number.

**TABLE 4.2 Thermal Entrance Lengths Defined by Reference to the Local Nusselt Number**

| $Pr$   | Thermal Entrance Lengths, Diameters |                      |             |
|--------|-------------------------------------|----------------------|-------------|
|        | $Re = 10^4$                         | $Re = 3 \times 10^4$ | $Re = 10^5$ |
| 0.7    | 10                                  | 10                   | 10          |
| 5 – 10 | 8                                   | 5                    | 2           |
| 50     | —                                   | 2                    | —           |
| 75     | —                                   | 1                    | —           |

\* Perfect constant property conditions correspond to isothermal flow. Constant property conditions are approximated when the radial and axial temperature variations in the pipe are small enough for physical properties variations to be negligible.