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Minimum size of disc cams with radial translating roller followers

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The Mechanisms Committee, which first met in 1975, is continuing the work that previously was the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

The ESDU Mechanisms Series is recognised as a valuable source of validated data and methods by the Institution of Mechanical Engineers and its Mechatronics, Informatics and Control Group is involved in continually reassessing the scope and focus of the content of this Series.

MINIMUM SIZE OF DISC CAMS WITH RADIAL TRANSLATING ROLLER FOLLOWERS

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MINIMUM SIZE OF DISC CAMS WITH RADIAL TRANSLATING ROLLER FOLLOWERS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI but any coherent system of units may be used. All angles are measured positively in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

		Units
f	arbitrary function	—
H	total follower lift corresponding to one segment (see Sketch 2.1)	m
O	cam centre of rotation and origin of co-ordinate axis system	—
R_a	prime circle radius	m
R_b	reference circle radius	m
R_g	radius of cutter or grinding wheel	m
R_r	roller radius	m
u	normalised cam angle, defined by Equation (2.2)	—
$(u_\alpha)_{max}$	normalised cam angle corresponding to position of maximum pressure angle	—
w	normalised follower displacement, defined by Equation (2.3)	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
X_c, Y_c	axes fixed in, and rotating with, cam with origin at point O	—
X_m, Y_m	axes fixed in machine, with origin at point O	—
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
y_m	height of centre of roller above X_m -axis	m
y_0	height of centre of roller above X_m -axis at beginning of rise segment or end of return segment	m
α	pressure angle	degree
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m

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Φ	total angle of cam rotation corresponding to one segment	degree
ϕ	cam angle of rotation, measured from X_m -axis, in anticlockwise sense	degree
ϕ_0	cam angle of rotation at beginning of a segment	degree
ω_c	angular velocity of cam	rad/s

Subscripts

p	refers to cam profile
0	refers to initial value
max	indicates maximum magnitude
min	indicates minimum magnitude
ret	refers to return segment
$rise$	refers to rise segment

2. INTRODUCTION

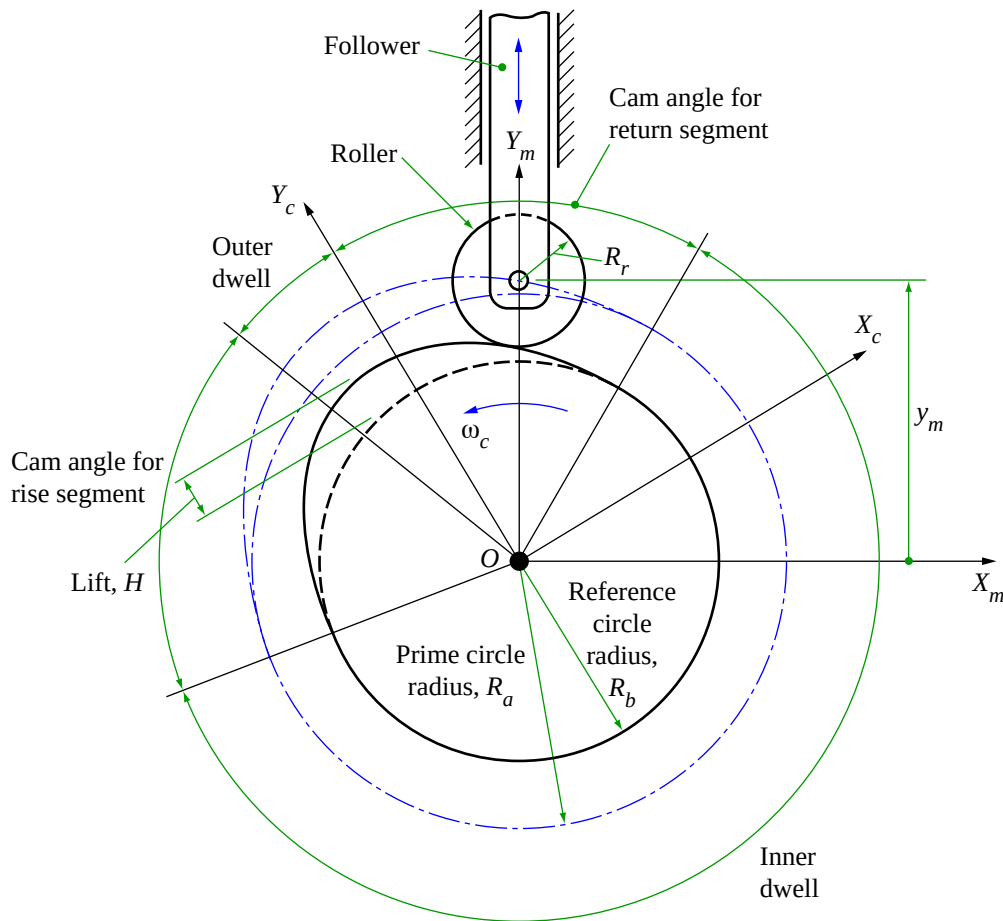
This Data Item describes a procedure for determining the minimum allowable size of a disc cam driving a radial translating roller follower (see Sketch 2.1). Minimisation of disc cam size is an important objective in the design of industrial machinery because it leads to:

- (a) minimum space and weight,
 - (b) minimum camshaft out-of-balance force,
 - (c) minimum angular velocity and minimum fluctuation of angular acceleration of roller follower and hence increased life
- and (d) minimum cam blank material cost and cutting cost.

Estimating the minimum allowable cam size at an early stage of the detail design may lead to a significant reduction in the number of subsequent iterations in the design process (see Derivations 1 and 2). An appropriate space envelope can be determined for the cam/follower subsystem.

Cam size is characterised by the largest distance from the cam centre to the cam profile. This is the sum of the reference circle radius and the maximum follower lift. Usually the lift is a known value, set during the design specification, and shown on a timing diagram. However the reference circle radius is often chosen arbitrarily. A realistic initial estimate of the minimum allowable value of reference circle radius for a given lift is obtained in this Item by consideration of the kinematic constraints (see Section 2.2). The procedure is most easily implemented using a computer program supplied with this Data Item. However, hand calculation is possible, in many cases, using the graphical design data (see Figures 1 to 12).

It is important to realise that the value of reference circle radius obtained using the procedure in this Item is the limiting minimum value for design purposes. To check that contact stress does not exceed an allowable value see ESDU 93001 (Reference 13) and to ensure that roller size is suitable see ESDU 91026 (Reference 11).



Sketch 2.1 Disc cam driving radial translating roller follower

The method described in this Item is applicable only to either a DRD (dwell-rise-dwell or dwell-return-dwell) or a DRRD (dwell-rise-return-dwell) follower motion defined by a single-law. Follower displacement for a single-law motion segment is defined by the equation(s) for one cam law (see Reference 6). ESDU 92014 and ESDU 93002 (References 12 and 14) show the extension of this method to determine the minimum size of cams with more complex motion segments formed by blending parts of single-law profiles.

ESDU 82023 (Reference 7) presents a geometric construction procedure by which the minimum reference circle radius may be obtained. This geometric procedure is applicable to offset as well as radial followers but it is iterative and so does not lead directly to the optimum solution.

2.1 Dimensionless Form of Displacement, Velocity and Acceleration

Sketch 2.1 shows two pairs of co-ordinate axes having origins coincident with the centre of cam rotation. The X_m , Y_m -axes are fixed to the machine frame whilst the X_c , Y_c -axes are fixed to the cam and rotate with it. This Item is concerned only with the common case in which the follower path coincides with the Y_m -axis (i.e. there is no offset).

A cam law is the mathematical function defining the normalised follower displacement, w , during a single motion segment in terms of normalised cam rotation, u . It is conveniently expressed as

$$w = f(u). \quad (2.1)$$

This function is evaluated readily by substituting the equation for the relevant cam law. In this Item the cam laws are identified by their generic names, for example Cycloidal or Modified Sinusoidal Acceleration. The equations of motion for commonly-used cam laws are given as formulae and graphically in ESDU 82006 (Reference 6). For a disc cam driving a radial translating follower, (see Sketch 2.1), the normalised cam rotation and normalised follower displacement for each motion segment are given by

$$u = (\phi - \phi_0)/\Phi \quad (2.2)$$

$$\text{and} \quad w = (y_m - y_0)/H. \quad (2.3)$$

Making the usual assumption that the cam rotates at constant angular velocity, the dimensionless forms of the follower velocity and acceleration are defined as

$$w' = \frac{\dot{y}}{\omega_c H} \left\{ \frac{\pi \Phi}{180} \right\} \quad (2.4)$$

$$\text{and} \quad w'' = \frac{\ddot{y}}{\omega_c^2 H} \left\{ \frac{\pi \Phi}{180} \right\}^2. \quad (2.5)$$

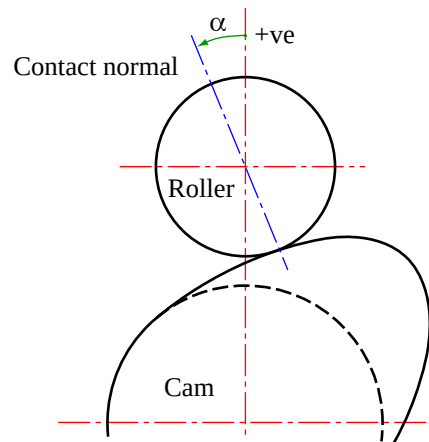
2.2 Kinematic Constraints

This Data Item facilitates the calculation of the minimum allowable reference circle radius compatible with constraints on two kinematic properties of the cam: pressure angle and radius of curvature.

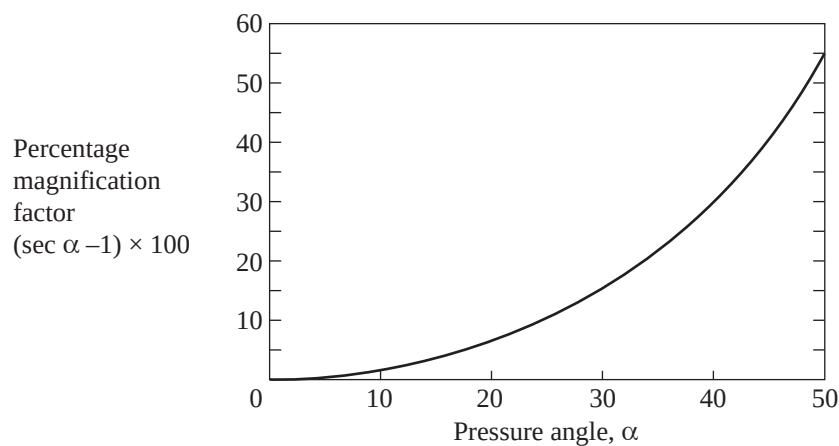
Maximum pressure angle increases as the cam size is reduced. Control of the maximum pressure angle (see Sketch 2.2) is necessary because of the effect this quantity has in magnifying the contact force between the cam and roller. Large values of contact force lead to high contact stress, and increased bearing loads between the follower and its guide and in the roller itself. The increase in contact force due to pressure angle, α , may be expressed by a magnification factor which is given by

$$\text{magnification factor} = \sec(\alpha) - 1 \quad (2.6)$$

and the effect of the pressure angle on this factor is shown in Sketch 2.3. In addition, high values of pressure angle may cause over-running during the return segment. Specification of a maximum allowable value of pressure angle (see Section 4.2) leads to an estimate of minimum cam size.



Sketch 2.2 Pressure angle



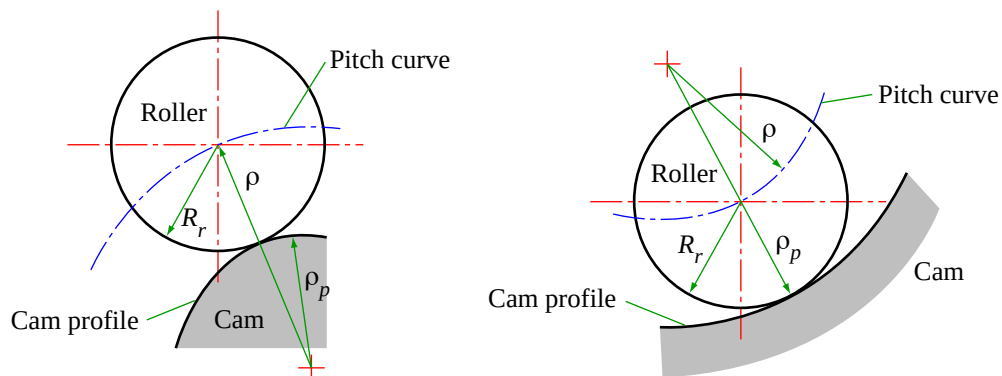
Sketch 2.3 Effect of pressure angle on contact force

A second phenomenon that may preclude the use of cams below a certain size is undercutting. This occurs if the radius of curvature of the cam profile is too small to permit correct follower movement or if the cam profile cannot be properly manufactured by a cutter of given radius. Both concave and convex sections of cam profile (see Sketch 2.4) must be checked to ensure that radii obey the following conditions. The minimum radius of curvature of the cam profile during concave sections should exceed the radii of both the roller and the cutter. To avoid undercutting the following condition must be satisfied.

$$|(\rho_p)_{min}| > R_r \text{ and } R_g \quad (2.7)$$

In addition, during sections of convex cam profile, the minimum radius of curvature of the pitch curve should exceed the radii of both the roller and the cutter. To avoid undercutting the following condition must be satisfied.

$$\rho_{min} > R_r \text{ and } R_g \quad (2.8)$$



Sketch 2.4 Profile curvature

For further information and explanation of profile undercutting see Derivation 4 and Reference 17. Section 3 of this Item contains background information on derivation of the design data and Section 4 describes two procedures for determining the minimum cam size consistent with the kinematic constraints.

3. DERIVATION OF THE DESIGN DATA

This section provides an explanation of the methods used to compile the design graphs (Figures 1 to 6 and 7 to 12).

3.1 Pressure Angle Design Data

For disc cams driving radial translating roller followers, the pressure angle equation derived in ESDU ME2 (see Derivation 3) simplifies to

$$\tan(\alpha) = \frac{180 H w'}{\pi \Phi y_m} \quad (3.1)$$

The height of the roller centre above the X_m -axis during a rise beginning at inner dwell is given by

$$y_m = R_a + w H \quad (3.2)$$

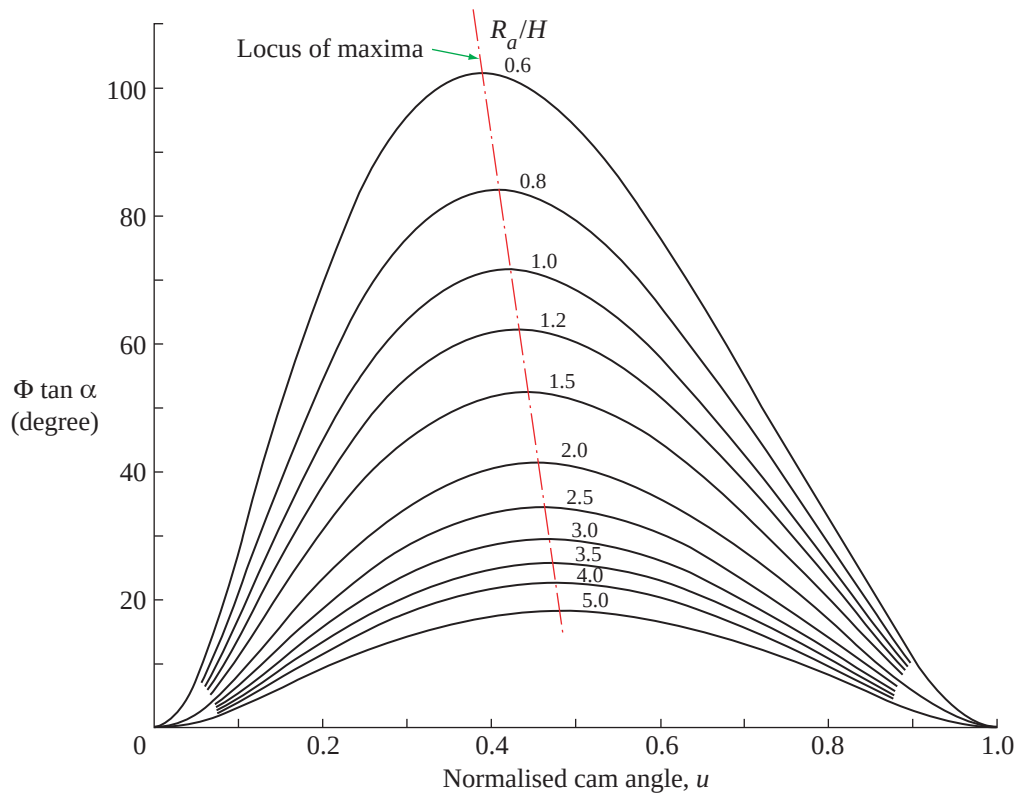
Equation (3.2) is substituted into Equation (3.1) and the resultant expression is rearranged in dimensionless form to give, for a rise motion,

$$\Phi \tan(\alpha) = \frac{\left(\frac{180}{\pi}\right) w'}{\left(\frac{R_a}{H} + w\right)} \quad (3.3)$$

Equation (3.3) is applicable to any cam law. If the expressions for normalised displacement, w , and non-dimensional velocity, w' , for a particular cam law are substituted into Equation (3.3), then a family of curves relating normalised cam angle, u , to the parameter $\Phi \tan(\alpha)$ can be drawn.

A typical set of curves (for Modified Sinusoidal Acceleration cam law) is shown in Sketch 3.1 and the locus of the maxima of these curves is indicated.

The corresponding design graph (Figure 4) for the Modified Sinusoidal Acceleration cam law was formed by plotting the values of $\Phi \tan(\alpha_{max})$ and the normalised cam angles at which they occur, $(u_{\alpha})_{max}$, against (R_a/H) . The pressure angle design graphs for the other common cam laws (Figures 1, 2, 3, 5 and 6) were generated in the same way. The curves may be used for return motions (see Examples in Section 6).



Sketch 3.1 Pressure angle parameter versus normalised cam angle for Modified Sinusoidal Acceleration motion during a rise segment

3.2 Profile Curvature Design Data

The dimensionless form of the radius of curvature of the pitch curve at any intermediate value of normalised cam angle, u , is given (see Derivations 3 and 4) by

$$\frac{\rho}{H} = \frac{\left[\left(\frac{R_a}{H} + w \right)^2 + \left(\frac{180w'}{\pi\Phi} \right)^2 \right]^{3/2}}{\left[\left(\frac{R_a}{H} + w \right)^2 + 2 \left(\frac{180w'}{\pi\Phi} \right)^2 - \left(\frac{R_a}{H} + w \right) \left(\frac{180}{\pi\Phi} \right)^2 w'' \right]}, \quad (3.4)$$

where convex curvature is positive and concave curvature is negative.

Values of (ρ/H) were determined by solving Equation (3.4) numerically for a range of values of Φ , u and R_a/H for each cam law. The minimum positive and negative values of the non-dimensional quantity $(\rho/H)_{min}$ were determined numerically. These data, for each common cam law, are presented in carpet form in Figures 7 to 12.

Minimum values of curvature sometimes occur at the boundaries of the motion segment (*i.e.* at $u = 0$ or $u = 1$). Because these conditions are not included on the design graphs the curvature at these positions must be determined separately by substituting the relevant values of normalised follower displacement, w , velocity, w' , and acceleration, w'' , into Equation (3.4).

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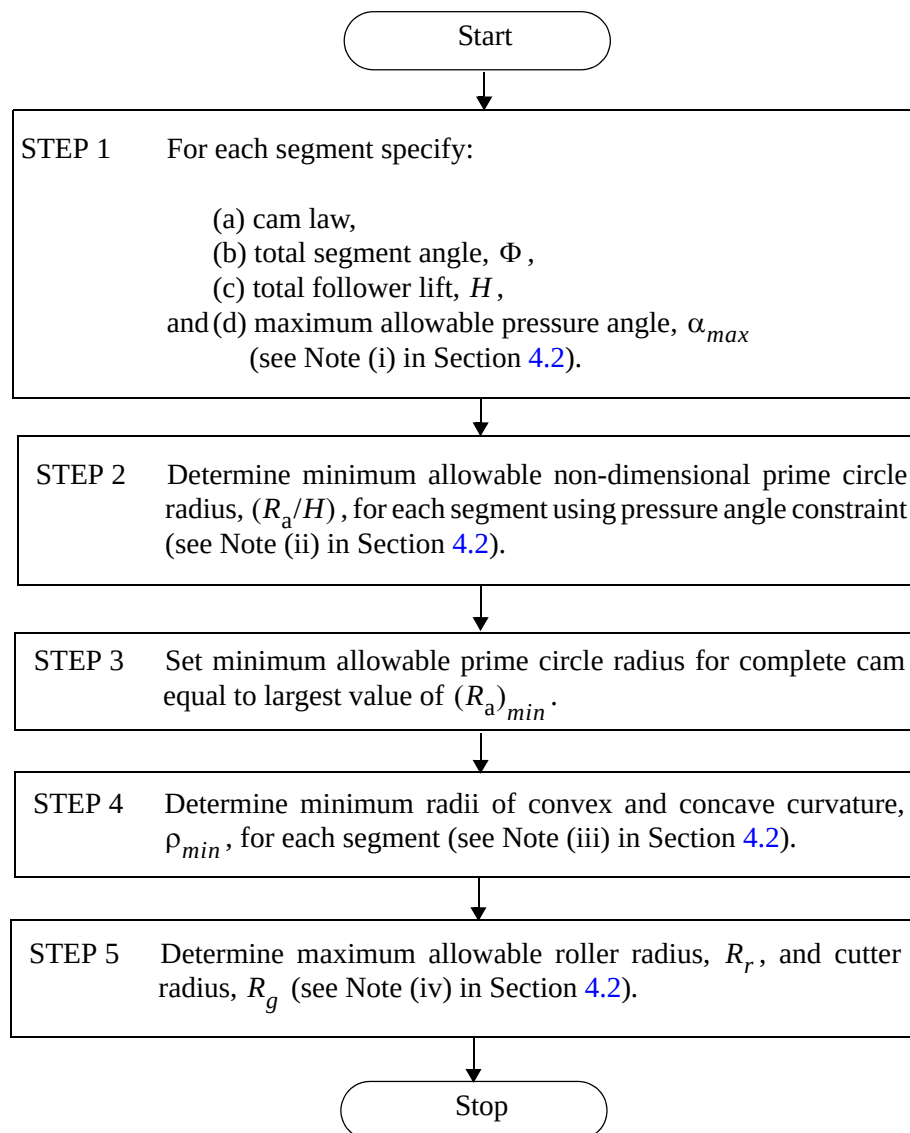
4. DESIGN PROCEDURES

The design data may be used in several ways to determine features of a disc cam and roller follower subassembly. The choice of approach depends on the known quantities and the objectives of the design. Two strategies are presented on flowcharts in Section 4.1 and detailed instructions are given in the notes in Section 4.2. A computer program that implements both design strategies is supplied with this Data Item. Guidance on the use of this program is given in Section 4.3 and the program is described in Section 7.

4.1 Design Flowcharts

4.1.1 Method for determination of minimum allowable prime circle radius and corresponding maximum allowable roller/cutter radius

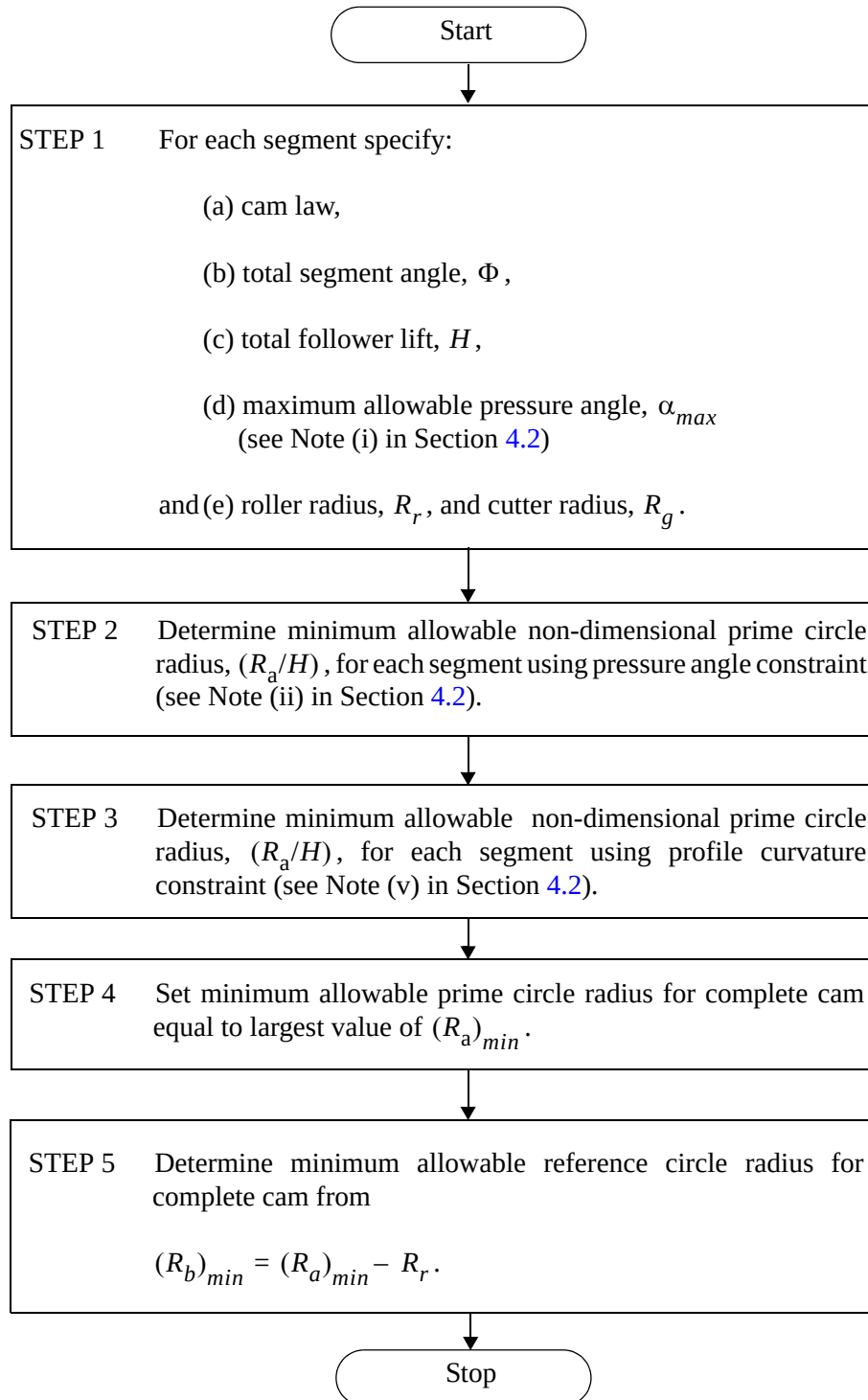
Use this method to find the minimum allowable cam size when the roller and cutter sizes are currently unknown. When the actual roller and cutter radii are chosen the minimum allowable reference circle radius may be finalised using the method described in Section 4.1.2.



Sketch 4.1 Design Flowchart 1

4.1.2 Method for determination of minimum allowable reference circle radius compatible with roller/cutter of known radius

Use this approach when the radii of the roller and the cutter are known and the minimum cam size is to be found.



Sketch 4.2 Design Flowchart 2

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4.2 Design Instructions

The following notes contain guidance on the use of the design data and should be read in conjunction with the flowcharts (see Sketches 4.1 and 4.2).

Notes

- (i) Specification of maximum allowable pressure angle

An empirical default value of maximum pressure angle of 30° is recommended. However, if the contact force due to working load and follower system inertia is known to be low during a particular motion segment, then a larger value of pressure angle may be tolerable.

- (ii) Determination of minimum allowable prime circle radius using pressure angle constraint

Calculate $\Phi \tan(\alpha_{max})$ and then select the appropriate graph from Figures 1 to 6 (according to cam law type). Read the value of (R_a/H) from the graph and, if required, multiply by the known value of follower lift, H , to give the value of $(R_a)_{min}$ for the motion segment. If cam law, total follower lift and maximum pressure angle are the same for all motion segments then $(R_a)_{min}$ need only be found for the motion segment with the smallest total segment angle.

- (iii) Determination of minimum radii of convex and concave curvature of pitch curve

Read the intermediate minimum non-dimensional radii of convex and concave curvature of the pitch curve, (ρ_{min}/H) , from the relevant graph (Figures 7 to 12). Determine separately the values of pitch curve curvature at the ends of the motion segments that blend with the inner dwell (when $u = 0$ for rise motion and when $u = 1$ for return segment) using Equation (3.4).

- (iv) Determination of maximum allowable roller radius and cutter radius (see Sketch 2.4 and Equations (2.7) and (2.8))

- (a) For concave sections of cam profile the theoretical maximum allowable value of roller and cutter radii are given by

$$(R_r)_{max} = (R_g)_{max} = |(\rho_p)_{min}| = |\rho_{min}| + (R_r)_{max}. \quad (4.1a)$$

In practice the following relationship is recommended.

$$\begin{aligned} (R_r)_{max} &= (R_g)_{max} = |\rho_{min}| + 0.9(R_r)_{max}, \\ \text{therefore} \quad (R_r)_{max} &= (R_g)_{max} = 10|\rho_{min}|. \end{aligned} \quad (4.1b)$$

- (b) For convex sections of cam profile the maximum allowable value is

$$(R_r)_{max} = (R_g)_{max} = \rho_{min}. \quad (4.2)$$

Equations (4.1b) and (4.2) represent upper limits on roller and cutter radii. For guidance on roller selection see Reference 11.