

ESTIMATION OF RATE OF CLIMB

1. NOTATION AND UNITS^{*†}

		<i>SI</i>	<i>British</i>
A	wing aspect ratio ($= b^2/S$)		
b	wing span	m	ft
C_D	drag coefficient ($= D/(\rho V^2 S/2)$)		
C_L	lift coefficient ($= (L/\rho V^2 S/2)$)		
D	airframe drag	N	lbf
D_M	intake momentum drag (per aircraft)	N	lbf
F	net jet thrust along flight path (= component of F_G along flight path minus D_M , see Data Item No. 69006 (Reference 1) for definitions)	N	lbf
F_G	engine gross thrust (per aircraft)	N	lbf
F_N	engine net thrust (per aircraft)	N	lbf
F_s	value of F_G for $V = 0$ at climb rating for appropriate height	N	lbf
g	acceleration due to gravity	m/s ²	ft/s ²
g_0	standard sea-level value [†] of g	m/s ²	ft/s ²
H	geopotential height [†]	m	ft
K_1, K_2	coefficients in drag equation $C_D = K_1 + K_2 C_L^2$		
k	induced drag factor ($K_2 = k/\pi A$)		
L	airframe lift	N	lbf
m	mass of aircraft	kg	slug
n	load factor, (aircraft acceleration in lift direction)/ g_0		
P	total engine power supplied to propeller(s) [*]	kW	hp
P_s	value of P for $V = 0$, at climb rating for appropriate height [*]	kW	hp

* The equations in this Data Item are written for coherent units. Section 1.1 gives conversion factors used in this Item. Data Item No. 80032 (Reference 5) gives further conversion factors for use with various common mixed units.

† See Data Item No. 77022 (Reference 3) for definitions.

r_v	ratio of speed to minimum drag speed, V/V_{md}		
S	wing area	m^2	ft^2
t	time	s	s
V	true airspeed (air-path velocity or resultant velocity relative to air)	m/s	ft/s
$V_{max\ roc}$	value of V for maximum rate of climb	m/s	ft/s
v_c	rate of climb*	m/s, m/min	ft/s, ft/min
$v_{c_{max}}$	maximum value of v_c *	m/s, m/min	ft/s, ft/min
W	aircraft weight ($= mg_0$)	N	lbf
α	angle of attack, <i>i.e.</i> angle between projection of true airspeed vector on to aircraft plane of symmetry and longitudinal datum or body axis	rad	rad
γ_a	air-path angle of climb (air-path inclination angle)	rad	rad
η	propeller efficiency		
ρ	density of air	kg/m^3	slug/ft ³
τ	angle between projection of engine gross thrust vector on to aircraft plane of symmetry and longitudinal datum or body axis; positive when F_G augments positive lift	rad	rad

Subscript

md denotes conditions of minimum drag (with respect to speed) in 1g level flight

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1.1 Conversion Factors Used

Quantity	SI units	British units
v_c	1 m/s $\equiv$ 60 m/min	1 ft/s $\equiv$ 60 ft/min
P	1 kW $\equiv$ 1000 mN/s	1 hp $\equiv$ 550 ft lbf/s
P/W	1 kW/N $\equiv$ 1000 m/s = 60 000 m/min	1 hp/lbf $\equiv$ 550 ft/s = 33 000 ft/min
V	1 knot $\equiv$ 0.5144 m/s	1 knot $\equiv$ 1.688 ft/s

2. INTRODUCTION

This Data Item presents methods for the calculation of rate of climb, illustrated by an example for a jet-powered aircraft. A method for the approximate estimation of maximum rate of climb for propeller-driven aircraft is included.

3. GENERAL THEORETICAL CONSIDERATIONS

The calculation of the rate of climb of an aircraft is based on the fact that the rate at which useful work is done by the powerplant(s) is equal to the rate at which work is done on the aircraft. For the case of constant wind, this is expressed in the (scalar) performance equation* (see Reference 5).

$$\eta P + FV = DV + mg_0 v_c + mV \frac{dV}{dt} \quad (3.1)$$

Rate of climb, v_c , can then be expressed in the alternative forms,

$$v_c = \frac{1}{W} \left(\eta P + FV - DV - mV \frac{dV}{dt} \right) \quad (3.2)$$

$$= \frac{(\eta P + FV - DV)}{W} \times \frac{1}{\left[1 + \frac{V}{g_0} \frac{dV}{dH} \right]} \quad (3.3)$$

The quantity in square brackets in Equation (3.3) is referred to as the acceleration factor and its magnitude depends on the manner in which the aircraft climbs or descends. Values of acceleration factor for several common climb techniques are given in Data Item No. 81046 (Reference 7).

4. STEADY STATE CLIMBS

Steady State Climbs, *i.e.* climbs at constant true airspeed and $n = 1$, while unlikely to be sustained in practice, nevertheless provide a useful guide to instantaneous climb performance – and one which is independent of any particular speed/height schedule. For such climbs Equation (3.3) may be simplified to

$$v_c = \frac{\eta P + FV - DV}{W} \quad (4.1)$$

The calculation of rate of climb from Equation (4.1) may be simplified in cases where it is permissible to use a parabolic expression to represent aircraft drag coefficient,

$$C_D = K_1 + K_2 C_L^2 \quad (4.2)$$

Equation (4.2) is one of several such expressions given in Data Item No. 81026 (Reference 6); the coefficients K_1 , K_2 are constants (at least for the range of speed or Mach number considered) and K_2 may

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be expressed as

$$K_2 = k/\pi A. \quad (4.3)$$

Further simplification is possible if it is assumed that, for small angles of climb, $L = W$ and, therefore, that $C_L = W/(\rho V^2 S/2)$. Drag may then be expressed as

$$D = \frac{1}{2} \rho V^2 S \left[K_1 + K_2 \left(\frac{2W}{\rho V^2 S} \right)^2 \right]. \quad (4.4)$$

From Equation (4.4) it may be deduced (see Reference 6) that drag is a minimum at speed V_{md} , given by

$$V_{md} = \left(\frac{2W}{\rho S} \right)^{1/2} \left(\frac{K_2}{K_1} \right)^{1/4} \quad (4.5)$$

and that the corresponding minimum value of drag/weight is

$$D_{md}/W = 2(K_1 K_2)^{1/2}. \quad (4.6)$$

The rate of climb, $v_{c_{md}}$, at the minimum drag speed is

$$v_{c_{md}} = \frac{\eta P}{W} + \frac{F V_{md}}{W} - 2 \left(\frac{2}{\rho} \right)^{1/2} K_1^{1/4} K_2^{3/4} \left(\frac{W}{S} \right)^{1/2}. \quad (4.7)$$

At speeds other than the minimum drag speed, *i.e.* where

$$V = r_v V_{md}, \quad (4.8)$$

it may be shown that

$$D = \frac{D_{md}}{2} \left(r_v^2 + \frac{1}{r_v^2} \right). \quad (4.9)$$

The rate of climb then becomes

$$v_c = \frac{\eta P}{W} + \frac{F r_v V_{md}}{W} - \left(r_v^3 + \frac{1}{r_v} \right) \left(\frac{2}{\rho} \right)^{1/2} K_1^{1/4} K_2^{3/4} \left(\frac{W}{S} \right)^{1/2}. \quad (4.10)$$

The first two terms on the right-hand side of Equation (4.10) cover the possibility of both jet thrust and shaft power. In general, only one of these two terms is necessary, the first in the case of shaft power (provided that the power of turbo-prop engines is replaced by an equivalent output-shaft power which takes account of the residual exhaust thrust, see Reference 5), the second term in the case of jet thrust.

The maximum rate of climb, and the corresponding airspeed, may be obtained from Equation (4.10) by optimising v_c with respect to r_v .

4.1 Jet-Powered Aircraft

When the power term in Equation (4.10) is set to zero

$$v_c = \frac{Fr_v V_{md}}{W} - \left(r_v^3 + \frac{1}{r_v}\right) \left(\frac{2}{\rho}\right)^{1/2} K_1^{1/4} K_2^{3/4} \left(\frac{W}{S}\right)^{1/2} \quad (4.11)$$

The calculation of the maximum rate of climb, and the corresponding airspeed, from Equation (4.11) is a straightforward process using standard computational techniques.

For jet-powered aircraft, a study of the range of values of rate of climb resulting from a large selection of the aircraft design parameters and engine thrust versus speed characteristics shows that a reasonable correlation of results is only possible if they are plotted against a “thrust power”/weight ratio ($V_{max\,roc} F_s / W$). However, since $V_{max\,roc}$ can only be determined by a calculation that would also provide a value for the maximum rate of climb, a simple method based on such a correlation is of no value in the case of turbo-jet and turbo-fan engines. (For propeller-driven aircraft the situation is quite different – see Section 4.2.)

In addition, turbo-jet and turbo-fan engines are generally used on high performance aircraft where

- (a) compressibility drag rise effects may be present at speeds for maximum rate of climb so that more detailed consideration is required than is possible in a simplified method,
- (b) the performance capability is such that consideration should be given to flight path optimisation procedures (Data Items Nos 89015 and 90012, References 8 and 9 respectively), which show that, for instance, the best time between two altitude/speed conditions is not normally achieved by use of the maximum rate of climb at any individual altitude.

An example illustrating the complete calculation of rate of climb for a turbo-jet aircraft is given in Section 6.1.

4.2 Propeller-Driven Aircraft

When the thrust term in Equation (4.10) is set to zero

$$v_c = \frac{\eta P}{W} - \left(r_v^3 + \frac{1}{r_v}\right) \left(\frac{2}{\rho}\right)^{1/2} K_1^{1/4} K_2^{3/4} \left(\frac{W}{S}\right)^{1/2} \quad (4.12)$$

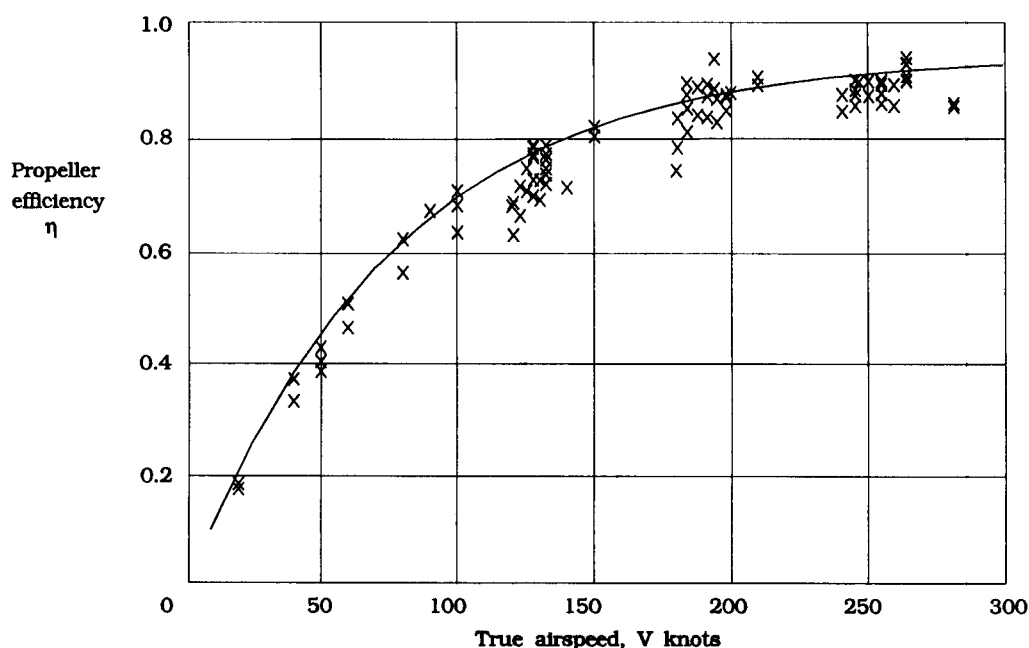
where the propeller efficiency, η , is a function of forward speed and propeller rotational speed, and power variation with speed depends on the type of engine.

Calculation of the maximum rate of climb, and the corresponding airspeed, from Equation (4.12) is again a straightforward process using standard computational techniques. Figure 1 shows the range of results obtained for over 250 000 combinations of values of K_1 , K_2 , wing loading, and power/weight ratio as follows.

K_1	0.0105 to 0.045,	
k	1.05 to 1.45,	$(K_2 = k/\pi A)$,
A	5 to 12.5,	
W/S	10 to 70 lbf/ft ²	(approx. 480 to 3350 N/m ²)
P/W	200 to 10 000 ft/min	(approx. 0.006 to 0.3 hp/lbf; 61 to 3048 m/min; 0.001 to 0.05 kW/N).

Each combination of these parameters was evaluated at five heights from sea level to 20 000 ft. A variation of propeller efficiency with forward speed was used (see Sketch 4.1) which represents a value (including installation effects, *etc.*) achievable by typical propellers at each speed.

The average of the calculated values of maximum rate of climb at each value of P/W is shown in Figure 1, together with their greatest and least values. Note that P/W is expressed in the same units as rate of climb, thereby giving a simple indication of the efficiency of conversion of shaft power into rate of climb. It was found that the “least” values obtained varied markedly with the value of wing loading and, consequently, separate “least” curves are shown for each value of W/S .



Sketch 4.1 Propeller efficiency variation with airspeed used in derivation of Figure 1

4.2.1 Piston-engined aircraft

For piston engines, power supplied to the propeller(s) is virtually independent of forward speed.

The curves from Figure 1 are compared with published values of maximum rate of climb at sea level in Figure 2 and at both sea level and altitude (from operations data for a more limited range of aircraft) in Figure 3. Two “least” curves are shown in Figures 2 and 3, corresponding to the spread of values of wing loading in the respective published data used. The following points may be made as a result of those comparisons.

- (a) The “greatest” curve is unlikely to be exceeded in practice, although it should be noted that this is not impossible, given a combination of even more favourable design parameters. Higher aspect ratio, lower K_1 and lower induced drag factor all lead to higher rates of climb, whilst higher wing loading leads to higher rates of climb at the higher values of power/weight ratio and lower at the lower values. Furthermore, higher propeller efficiency, possibly by the use of more advanced propeller design, could improve rates of climb.
- (b) The “average” curve indicates a general level of likely performance.
- (c) There are several points below the “least” curves. This situation can arise either because the quoted performance is not, in fact, the maximum achievable by the aircraft, or because some or all of the