

EXAMPLE OF STATISTICAL TECHNIQUES APPLIED TO ANALYSIS OF MEASUREMENTS OF THE LANDING AIRBORNE MANOEUVRE

(Multiple linear regression with two independent variables and one dependent variable)

1. NOTATION AND UNITS

a	regression relationship constant ("y" intercept)	*
B	matrix identifier for regression coefficients	
b	regression relationship constant (slope)	*
d	deceleration	ft/s ²
F	F Variance Ratio test statistic	
m	number of independent variables in regression relationship	
n	number of data points in sample	
RD	rate of descent	ft/s
r	correlation coefficient of sample	
SE	residual error sum of squares	*
SR	regression sum of squares	*
s	distance	ft
t	time	s
V	true airspeed	kn
VR	speed ratio, V_{screen}/V_{TD}	
X	matrix identifier for independent variables	*
x	statistical variable (independent)	*
y	statistical variable (dependent)	*
Z	matrix identifier for dependent variable	*
β	population regression coefficient	*
v	number of degrees of freedom	

* The quantities a , b , SE , SR , X , x , y , Z and β are dimensional but take units which depend on the particular parameters in each regression.

Subscripts

a	denotes landing airborne phase, between 50 ft screen height and touchdown
D	denotes denominator
E	denotes (residual) error
g	denotes landing ground run
I	denotes indicated value (as in indicated airspeed)
i	denotes “ith” data point
j	denotes “jth” independent variable
N	denotes numerator
R	denotes regression
$screen$	denotes conditions at 50 ft screen height
TD	denotes conditions at touchdown

Superscripts

$\wedge$	denotes best estimate
$-$	denotes mean
$'$	denotes transpose of matrix
-1	denotes inverse of matrix

2. INTRODUCTION

This Data Item uses the analysis of a set of landing airborne manoeuvre measurements to illustrate the following techniques.

The use of multiple linear regression with two independent variables and one dependent variable, the application of the “ F ” Variance Ratio test to test the significance of the derived regression relationships.

An introduction to the above techniques is given in Data Item No. 91017 (Reference 4).

3. USE OF MULTIPLE LINEAR REGRESSION IN THE ANALYSIS OF CERTIFICATION LANDING AIRBORNE MANOEUVRE MEASUREMENTS

When certificating a civil aircraft in accordance with the airworthiness requirements of Reference 1 or 3 it is necessary to schedule the required landing distance in the aircraft's flight manual. References 1, 2 and 3 specify the conditions to be tested and References 2 and 3 provide further guidance on test conditions and analysis methods.

The prescribed procedures for measuring and analysing the landing airborne manoeuvre involve carrying out a series of test measurements covering a range of rates of descent at the 50 ft screen and at touchdown, correlating airborne distance with the rates of descent at the screen and at touchdown (multiple linear regression) and scheduling an airborne distance calculated for standard values based on the test result correlation. In practice, rather than correlating airborne distance directly, it is usual to derive separate correlations* for t_a and V_{screen}/V_{TD} with the two rates of descent, RD_{screen} and RD_{TD} , and use these relationships to develop scheduled distance.

This Data Item uses the analysis of a set of transport aircraft landing measurements to illustrate the multiple linear regression techniques.

Throughout this Data Item quantities have been computed to a large number of significant figures so that no build up of rounding errors occurs. For convenience, however, values are *quoted*, after rounding, to fewer significant figures than were used in the computations.

* In order that Equation 3.4 (the application of Equation (3.1) to the correlation for t_a) should have a valid physical basis, $RD_a (= 50/t_a)$ is used as the dependent variable. The importance of so choosing correlating parameters and relationships is discussed in Reference 4.

3.1 The Measured Data

A sample of landing airborne manoeuvre measurements gave the results in Table 3.1.

TABLE 3.1

Test point No.	V_{screen} kn	V_{TD} kn	V_{screen}/V_{TD}	t_a s	RD_{screen} ft/s	RD_{TD} ft/s	RD_a ft/s
1	143.0	138.2	1.0347	5.72	11.56	4.1	8.74
2	139.8	138.4	1.0101	5.44	10.93	5.3	9.19
3	136.9	134.1	1.0209	6.30	12.66	4.6	7.94
4	139.1	134.6	1.0334	7.95	9.97	2.7	6.29
5	139.5	136.3	1.0235	5.97	9.84	4.1	8.38
6	131.6	129.2	1.0186	5.73	10.69	6.3	8.73
7	131.0	128.6	1.0187	5.81	10.03	4.8	8.61
8	132.1	128.8	1.0256	6.19	10.21	6.6	8.08
9	130.8	129.9	1.0069	6.96	10.35	3.9	7.18
10	131.3	125.1	1.0496	11.14	3.47	2.6	4.49
11	133.8	131.1	1.0206	5.56	9.56	4.9	8.99
12	137.8	136.8	1.0073	4.76	13.02	6.3	10.50
13	137.4	135.1	1.0170	5.06	11.95	5.3	9.88
14	139.5	138.8	1.0050	6.12	10.29	4.3	8.17
15	138.6	135.7	1.0214	5.72	10.42	4.5	8.74
16	136.0	132.5	1.0264	5.34	11.02	3.9	9.36
17	135.8	132.7	1.0234	5.95	9.48	6.3	8.40
18	136.7	133.3	1.0255	6.36	8.82	2.8	7.86
19	132.6	133.5	0.9933	6.57	8.38	4.4	7.61
20	130.2	129.9	1.0023	5.48	10.24	6.5	9.12
21	139.5	139.0	1.0036	5.41	10.75	7.3	9.24
22	133.5	132.1	1.0106	6.52	10.16	3.9	7.67
23	134.9	133.4	1.0112	7.70	12.45	2.9	6.49
24	130.8	131.4	0.9954	5.41	11.42	5.7	9.24
25	129.9	134.1	0.9687	6.49	12.26	2.5	7.70
26	131.1	130.3	1.0061	6.41	10.56	3.9	7.80
27	139.0	136.6	1.0176	6.15	9.39	6.0	8.13
28	138.0	135.7	1.0169	5.58	11.09	6.1	8.96
29	142.6	138.9	1.0266	6.01	10.66	4.9	8.32
30	137.2	132.7	1.0339	5.79	10.50	4.5	8.64
31	131.7	131.4	1.0023	5.43	10.49	5.1	9.21
32	127.3	124.9	1.0192	7.57	9.04	4.6	6.61
33	126.1	124.5	1.0129	7.39	8.62	3.6	6.77
Mean			1.015	6.24	10.31	4.7	8.213

3.2 Regression Analysis of t_a with RD_{screen} and RD_{TD}

Section 8 of Reference 4 describes a method to calculate the values of the coefficients a , b_1 and b_2 where the best estimate for the i th value of a dependent variable, y , expressed in terms of two independent variables x_1 and x_2 , is given by

$$\hat{y}_i = a + b_1(x_{1_i} - \bar{x}_1) + b_2(x_{2_i} - \bar{x}_2). \quad (3.1)$$

The solution is provided by the relationship,

$$a = \bar{y} \quad (3.2)$$

and the matrix equation,

$$\mathbf{B} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z} \quad (3.3)$$

where

$$\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} y_1 - \bar{y} \\ " \\ " \\ " \\ y_n - \bar{y} \end{bmatrix},$$

and

$$\mathbf{X} = \begin{bmatrix} x_{1_1} - \bar{x}_1 & x_{2_1} - \bar{x}_2 \\ " & " \\ " & " \\ " & " \\ x_{1_n} - \bar{x}_1 & x_{2_n} - \bar{x}_2 \end{bmatrix}.$$

In the context of this example, the dependent variable is the mean airborne rate of descent, RD_a , and the independent variables are the two rates of descent RD_{screen} and RD_{TD} . Consequently, Equation (3.1) can be rewritten,

$$\hat{RD}_a = a + b_1(RD_{screen_i} - \overline{RD}_{screen}) + b_2(RD_{TD_i} - \overline{RD}_{TD}). \quad (3.4)$$

Equation (3.2) can be rewritten

$$a = \overline{RD}_a. \quad (3.5)$$

In Equation (3.3) the terms Z , X , X' are

$$Z = \begin{bmatrix} RD_{a_1} - \overline{RD}_a \\ " \\ " \\ " \\ RD_{a_{33}} - \overline{RD}_a \end{bmatrix} \quad (3.6)$$

$$X = \begin{bmatrix} (RD_{screen_1} - \overline{RD}_{screen}) & (RD_{TD_1} - \overline{RD}_{TD}) \\ " & " \\ " & " \\ " & " \\ (RD_{screen_{33}} - \overline{RD}_{screen}) & (RD_{TD_{33}} - \overline{RD}_{TD}) \end{bmatrix} \quad (3.7)$$

and

$$X' = \begin{bmatrix} (RD_{screen_1} - \overline{RD}_{screen}) \dots \dots \dots (RD_{screen_{33}} - \overline{RD}_{screen}) \\ (RD_{TD_1} - \overline{RD}_{TD}) \dots \dots \dots (RD_{TD_{33}} - \overline{RD}_{TD}) \end{bmatrix} . \quad (3.8)$$

From Equation (3.5)

$$a = \overline{RD}_a = 8.21 \text{ ft/s} .$$

The solution of Equation (3.3) (see Appendix A, Section A1) gives

$$B = \begin{bmatrix} 0.3514 \\ 0.4902 \end{bmatrix}$$

that is, the coefficients b_1 and b_2 of Equation (3.4) are, $b_1 = 0.3514$ and $b_2 = 0.4902$.

Substituting in Equation (3.4) for a , b_1 , b_2 , $\overline{RD}_{screen}$ and $\overline{RD}_{TD}$ gives,

$$\hat{RD}_{a_i} = 8.21 + 0.3514(RD_{screen_i} - 10.31) + 0.4902(RD_{TD_i} - 4.7) \text{ ft/s} . \quad (3.9)$$

Equation (3.9) permits values of $\hat{RD}_a$ to be calculated for each of the original test points, as shown in the following Table, and plotted on the scatter diagram of Sketch 3.1.