

EQUIVALENT LENGTHS OF STRUTS (FOR USE IN BUCKLING CALCULATIONS)

1. INTRODUCTION

The effects of end restraint conditions and type of loading on the buckling load of uniform and tapered section struts can, for many practical struts, be conveniently taken into account by use of an 'equivalent length'. The 'equivalent length' is used in place of the true length in the Euler buckling equation for a uniform section strut pinned at both ends. This Data Item sets out tables and figures that give equivalent lengths for various loadings, end restraint conditions and forms of taper.

The equivalent lengths provided in the Item relate to theoretically perfect struts. These are normally used to account for the influence of end conditions and loading type in the assessment of the buckling load of practical struts containing initial bow and offset loading, see Data Item Nos. Struct 01.01.01, 78021 and 90002*.

2. NOTATION

Both SI and British units are quoted but any consistent system of units may be used.

a	length of tapered section of strut, see Sketch 5.1	m	in
c	distance along strut axis to point through which line of loading passes, see Figure 7	m	in
d	region of strut length of constant section, see Sketch 5.1	m	in
E	modulus of elasticity of strut material	N/m ²	lbf/in ²
j	distance from origin of variation of I_x to end of strut, see Sketch 5.1	m	in
k	distance from origin of variation of I_x to end of tapered portion of strut, see Sketch 5.1	m	in
l	actual length of strut	m	in
l'	equivalent length of strut	m	in
I	value of I_x for region of strut having uniform section	m ⁴	in ⁴
I_x	second moment of area of strut at point defined by x	m ⁴	in ⁴
P	maximum axial load along length of strut	N	lbf
P_A	axial load applied to one end of strut, see Figure 2	N	lbf
P_c	$\pi^2 EI/l'^2$, critical value of P at buckling	N	lbf

* Data Item No. Struct 01.01.01 "The strength of struts".

Data Item No. 78021 "Guide to Items on the strength and stability of struts".

Data Item No. 90002 "Struts: strength under flexural, local and inter-rivet buckling".

P_E	$\pi^2 EI/l^2$, Euler buckling load for uniform section simply-supported strut under constant load	N	lbf
q	distributed longitudinal loading per unit length applied to strut	N/m	lbf/in
q_x	value of varying q at point defined by x , see Equation (5.3)	N/m	lbf/in
r	index governing taper of strut section, see Section 5.2		
s	index governing value of q_x along strut, see Equation (5.3)		
x, y, z	coordinates, see Sketches 3.1, 3.2 and 5.1		
λ_0, λ_l	coefficient of rotational restraint acting in xy -plane at ends $x = 0$ and $x = l$, respectively	(N m)/rad	(lbf in)/rad

2.1 Suffixes

$0, j, k, l$	substituted in place of suffix x in I_x and q_x to indicate value of x to which they apply
1, 2	used to label regions of strut having uniform flexural stiffness

2.2 Sign Convention

Compressive loads positive, tensile loads negative.

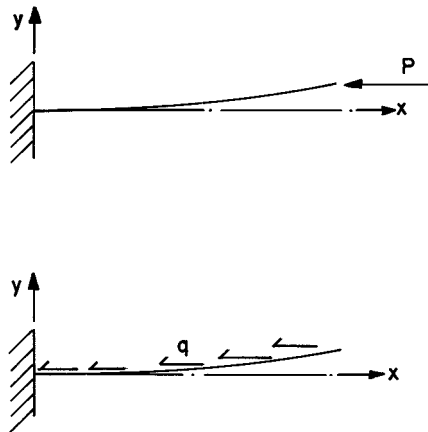
2.3 Representation of Support Conditions

Throughout this Item the strut support conditions are represented as illustrated and defined in Table 9.1.

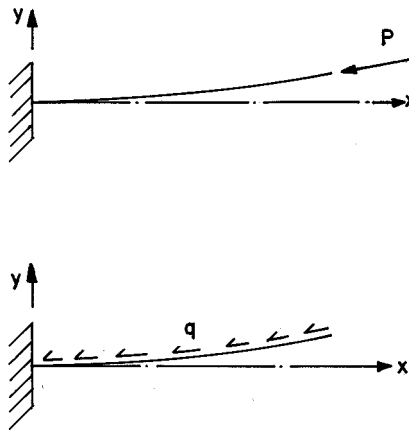
3. LOADING ACTIONS ON STRUTS

Sketches 3.1 and 3.2 each show two loading actions applied to two uniform section struts. The struts all have one end fixed and one end free. In Sketch 3.1 the presumption for stability considerations is that the loading always remains parallel to the original, undeflected, x -axis of the strut. In Sketch 3.2 the loading remains tangential to the deflected shape of the strut centre-line. Throughout this Item these loading cases are referred to as fixed-direction loading, Sketch 3.1, and tangential loading, Sketch 3.2.

The terms 'conservative loading' and 'non-conservative loading' are employed in the literature. 'Conservative loading' is used for fixed-direction loadings, where the work done by the loading is path-independent so that the buckling load is derivable from a load potential. 'Non-conservative loading' is used for tangential loadings, where the work done by the loading is path-dependent so that the buckling load is not derivable from a load potential. Loading actions exist in which the load is neither fixed in direction nor tangential to the deflected shape of the strut. Section 6 deals with such a case.



Sketch 3.1 Fixed direction loading



Sketch 3.2 Tangential loading

All struts subject to a fixed-direction loading buckle if the load is sufficiently large. Within this Item buckling refers to the theoretical buckling load of a perfect strut. Prior to buckling a small lateral y -direction displacement of the strut will be resisted and, on removal of the displacing force, the strut will return to be straight. The theoretical buckling load is that load at which, when a small displacement is applied, the strut remains stable but displaced. Beyond the theoretical buckling load any attempted lateral displacement of the perfect strut gives rise to rapidly growing displacement and to collapse.

The determination of the buckling load for struts under tangential loading is more intricate than for cases under fixed-direction loading. To ensure that the lowest buckling load is found the possibility of dynamic buckling, as well as the possibility of static buckling as described above, must be investigated. As with static buckling dynamic buckling here relates to a perfect strut but with the additional assumption that there is zero damping. Prior to dynamic buckling any lateral disturbance will lead to vibration of the strut about its neutral axis and the amplitude of this will decay with time until it vanishes. At the theoretical buckling load any disturbance will cause a vibration of unchanging amplitude; that is, it does not decay. Beyond the dynamic buckling load any disturbance will initiate a vibration that will grow with each oscillation.

For a strut with a particular combination of end support conditions and subject to a form of tangential loading, no simple engineering statement can be provided to indicate which type of buckling will dominate. An analysis that takes no account of the strut dynamics may suggest that the strut remains stable or gives a higher buckling load than that which causes dynamic buckling. It is therefore necessary to derive and

examine both the governing stability equations. Derivation 5 gives an exposition of the general forms of stability equations that treat the different types of buckling. Derivation 4 reviews the subject of loss of stability under tangential (non-conservative) forces and provides additional background material.

In some structural members fixed-direction and tangential loadings can be applied simultaneously. An example is a strut that carries a fixed-direction load along the line of its undeflected axis and also carries a distributed longitudinal loading entering the strut from the shear in an attached sheet. In such a case the distributed loading will tend to follow the deflected form of the strut.

The exact line of action of a distributed loading is, usually, difficult to define. Therefore, application of values of l' for tangential loading should be used with caution when they are less than the corresponding value for fixed direction loading.

4. ASSUMPTIONS USED IN CALCULATING ALL EQUIVALENT LENGTHS

The buckling loads used to derive values of l' have been calculated using engineer's theory of bending and assuming that the strut behaves elastically. Transverse shear deflections in the xy -plane have been neglected. The data are therefore only applicable if deflections resulting from transverse shear are negligible in relation to those arising from flexure. The struts were assumed to be geometrically perfect with no initial eccentricities. Also, all loads were applied at the neutral axis. Buckling was assumed to occur in the xy -plane only; no account was taken of torsional effects. In cases of dynamic instability it has also been assumed that the mass is uniformly distributed along the length of the strut and that there is zero damping.

5. STRUTS WITH FIXED-DIRECTION LOADING

5.1 Uniform Section Struts

Table 9.2 provides values of l' for struts subject to load on the ends, to distributed axial loading or a combination of these loadings for a range of support conditions. Figure 1 provides values of l'/l for struts loaded on the ends only and in which the ends are elastically restrained against rotation. The restraint is expressed by the coefficients λ_0 and λ_l which can differ, that is $\lambda_0 \neq \lambda_l$. Figure 2 provides curves of l'/l for struts subject to different axial loads applied at the ends and a distributed axial load for two sets of support conditions.

5.2 Struts Continuous Over an Intermediate Support

Figure 3 provides curves for equivalent lengths for struts that are pinned at both ends and at another intermediate point along their length. The distances between the supports can be different, $l_1 \neq l_2$, and the values of EI between each support can also differ, $(EI)_1 \neq (EI)_2$. The entire strut is subjected to the same, constant, axial load applied at the ends so that l'_2 is related to l'_1 as shown by the equation on Figure 3. Both values of l' will be required when assessing the strength of a practical strut that has initial bow, offset loading and is subject to material inelasticity effects (see Section 1).

5.3 Tapered Struts

Figures 4 and 5 provide values of l'/l for tapered struts that are either pinned at both ends or fixed at one end and free at the other and are subject to fixed-direction loading. Note that, as shown on both figures, P_c

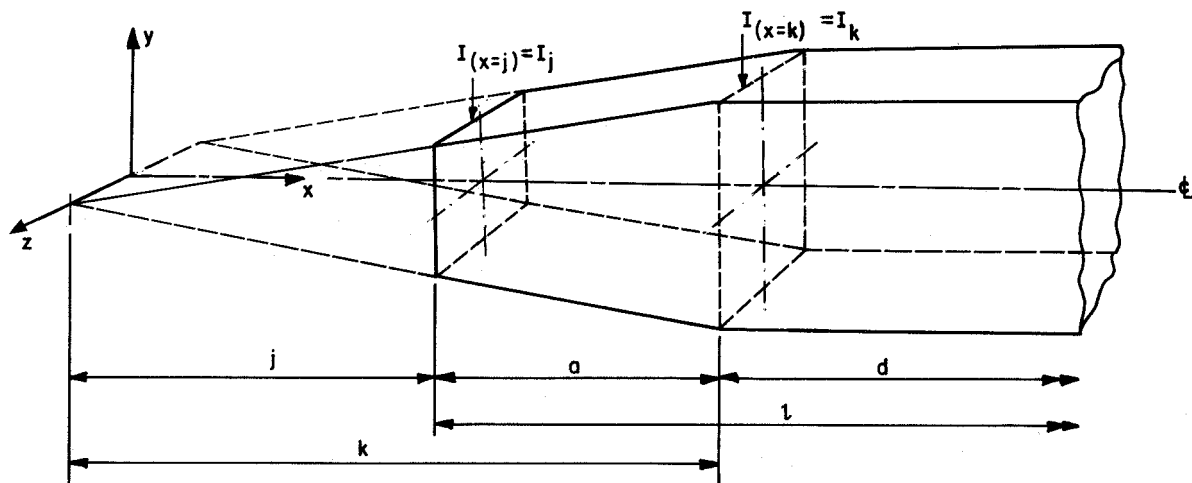
is related to I' through I_k . The form of the taper of the section is defined by the relationship

$$(I_x)_{(j \leq x \leq k)} = I_k (x/K)^r, \quad (5.1)$$

in which

$$k = \frac{a}{1 - (I_j/I_k)^{1/r}}. \quad (5.2)$$

Figures 4 and 5 provide curves for $r = 1, 2, 3$ or 4 and Sketch 5.1 shows a tapered strut for which $r = 3$.



Sketch 5.1 Rectangular section strut with tapered section for which $r = 3$

The integer values of r imply the following types of taper.

- (i) When $r = 0$ then $I_x = I_j = I_k$ and the strut is of uniform section over length a (see Section 5.1).
- (ii) The case where $r = 1$ corresponds to a linear increase only in the z -direction dimensions of the strut section with increasing x .
- (iii) The case where $r = 2$ may be used to represent a strut constructed from constant section booms of identical unchanging section whose separation in y varies linearly with x . The booms must be connected by a structure capable of forming a stiff shear connection but which makes negligible contribution to I_x .
- (iv) The case where $r = 3$ corresponds to a linear increase only in the y -direction dimensions.
- (v) The case where $r = 4$ corresponds to a simultaneous linear increase in both the y - and z -direction dimensions of the strut section with increasing x . Note that the tapers for this case must decrease to a single point at the origin.

5.3.1 Struts tapered to a point

Figure 6 gives l'/l for the special case of a tapered section strut that is fixed at the large end and free at the pointed end and is subject to a fixed-direction varying distributed longitudinal load. For this particular case $j = 0$ and $a = k = l$. Equation (5.1) describes the form of the taper with l replacing k . The varying distributed longitudinal loading is described by the relationship

$$q_x = q_l(x/l)^s \quad (5.3)$$

Curves are provided on Figure 6 for $r = 0, 1, 2$ or 3 over the range $0 \leq s \leq 5$. When $s = 0$ the distributed longitudinal loading is uniform along the strut and $q_x = q_l$. For all other values of s , $q_x = 0$ at $x = 0$ and

$$P = \int_{x=0}^{x=l} q_x \, dx = \frac{q_l l}{(s+1)} \quad (5.4)$$

As stated on Figure 6 the value of P_c is obtained using I_l with l' .

6. STRUTS LOADED THROUGH A FIXED POINT

Figure 7 provides a curve of l'/l against c/l for a strut fixed at one end and loaded through a fixed point located on the x -axis that is c distant from the loaded, free, end. Reference 2 shows that, although P is not fixed in direction, this case may be analysed without reference to the path taken by the load as the strut is loaded. This is therefore a “conservative loading” case similar to those of fixed-direction loading in which the work done by the loading is path-independent. Static buckling therefore occurs and is derivable from the load potential.

7. STRUTS WITH TANGENTIAL LOADING

Table 9.3 provides values of l' for struts subject to load on the ends only or to distributed longitudinal tangential loading for a range of end conditions. Where the value of l' given relates to dynamic buckling this is noted in the table. Note that some values of l' given on Table 9.3 have been obtained from one source and have not been independently verified. These are identified on the table and should be used with caution. For Cases 3.2, 3.3 and 3.5 to 3.9 the distributed longitudinal loading is tangential but the load P is in the x -direction. For loading on the ends of Cases 3.1, Case A is the only case where tangential loading can occur. In all other cases of loading on the ends the support conditions are either constrained so as to load the strut along the x -direction or the support reaction takes the y -direction component of the loading leaving the strut subject to a fixed x -direction loading.

Because of the assumptions listed in Section 4, values of l' for dynamic buckling cannot be applied to struts in which the mass distribution is not uniform, even though the stiffness is constant along the length. Also, the values cannot be applied to struts where significant damping will occur.

Where dynamic buckling is indicated the value of l' accounts for the effects of end conditions and this value may be used when making an assessment of the strength of a practical strut. This is because the value of l' relates to a perfect strut and the introduction of allowances for imperfections will reduce the allowable strut load below that for dynamic buckling of the perfect strut.