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Equivalent Lengths of Struts

(for use in buckling calculations)

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EQUIVALENT LENGTHS OF STRUTS (FOR USE IN BUCKLING CALCULATIONS)

1. INTRODUCTION

The effects of end restraint conditions, an intermediate elastic support and type of loading on the buckling load of struts of uniform and tapered section can, for many practical struts, be conveniently taken into account by use of an 'equivalent length'. The 'equivalent length' is used in place of the true length in the Euler buckling equation for a strut of uniform section pinned at both ends. This Data Item sets out tables and figures that give equivalent lengths for various loadings, end restraint conditions, intermediate elastic supports placed at some point along the strut and forms of taper.

The equivalent lengths provided in the Item relate to theoretically perfect struts. These are normally used to account for the influence of end conditions and loading type in the assessment of the buckling load of practical struts containing initial bow and offset loading, see Data Items Nos Struct 01.01.01^{*}, 78021[†] and 90002[‡].

2. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

a	length of tapered section of strut, see Sketch 5.2	m	in
c	distance along strut axis to point through which line of loading passes, see Figure 13	m	in
d	length of portion of strut of uniform section, see Sketch 5.2	m	in
E	modulus of elasticity of strut material	N/m ²	lbf/in ²
I	value of I_x for portion of strut having uniform section	m ⁴	in ⁴
I_x	second moment of area of strut at point defined by x	m ⁴	in ⁴
j	distance from origin of variation of I_x to end of strut, see Sketch 5.2	m	in
k	distance from origin of variation of I_x to end of tapered portion of strut, see Sketch 5.2	m	in
l	actual length of strut	m	in
l'	equivalent length of strut	m	in
P	maximum axial compressive load along length of strut	N	lbf
P_A	axial load applied to one end of strut, see Figures 2a to 8c	N	lbf

* Data Item No. Struct 01.01.01 "The strength of struts".

† Data Item No. 78021 "Guide to Items on the strength and stability of struts".

‡ Data Item No. 90002 "Struts: strength under flexural, local and inter-rivet buckling".

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P_c	critical value of P at buckling, $P_c = \pi^2 EI/l'^2$	N	lbf
P_E	Euler buckling load for simply-supported strut of uniform section under constant load, $P_E = \pi^2 EI/l^2$	N	lbf
Q	spring reaction	N	lbf
q	distributed longitudinal load per unit length applied to strut (q constant unless otherwise stated)	N/m	lbf/in
q_x	value of varying q at point defined by x , see Equation (5.3)	N/m	lbf/in
r	index governing taper of strut section, see Section 5.3		
s	index governing value of q_x along strut, see Equation (5.3)		
x, y, z	co-ordinates, see Sketches 3.1, 3.2, 5.1 and 5.2		
α	load for unit compression of spring (spring stiffness)	N/m	lbf/in
δ	spring deflection	m	in
μ_0, μ_l	rotational restraints acting in xy -plane at ends $x = 0$ and $x = l$, respectively	(N m)/rad	(lbf in)/rad

2.1 Suffixes

$0, j, k, l$	substituted in place of suffix x in I_x and q_x to indicate value of x to which they apply
$1, 2$	used to label regions of strut having uniform flexural stiffness

2.2 Sign Convention

Compressive loads are positive, tensile loads are negative.

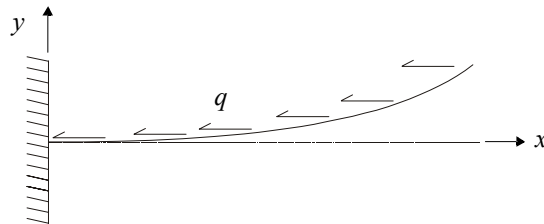
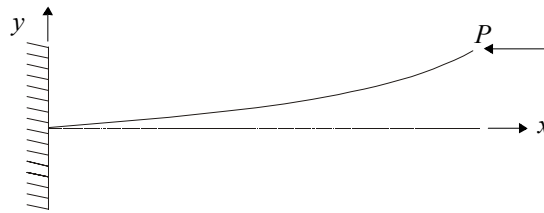
2.3 Representation of Strut Support Conditions

Throughout this Item the strut support conditions are represented as illustrated and defined in Table 9.1.

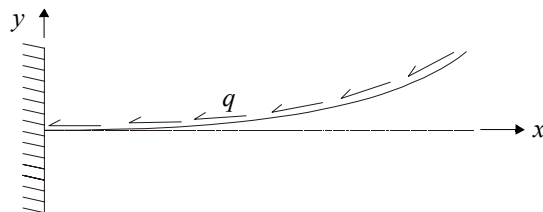
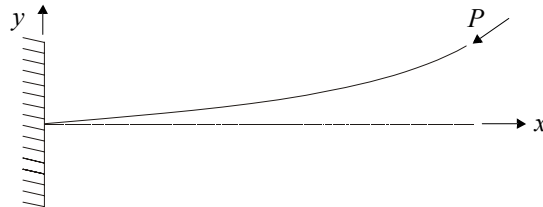
3. LOADING ACTIONS ON STRUTS

Sketches 3.1 and 3.2 each show two loading actions applied to two struts of uniform section. Both struts have one end fixed and one end free. In Sketch 3.1 the presumption for stability considerations is that the loading always remains parallel to the original, undeflected, x -axis of the strut. In Sketch 3.2 the loading remains tangential to the deflected shape of the strut centre-line. Throughout this Item these loading cases are referred to as fixed-direction loading, Sketch 3.1, and tangential loading, Sketch 3.2.

The terms 'conservative loading' and 'non-conservative loading' are employed in the literature. 'Conservative loading' is used for fixed-direction loading, where the work done by the load is path-independent so that the buckling load can be derived from a load potential. 'Non-conservative loading' is used for tangential loading, where the work done by the load is path-dependent so that the buckling load cannot be derived from a load potential. Loading actions exist in which the load is neither fixed in direction nor tangential to the deflected shape of the strut. Section 6 deals with such a case.



Sketch 3.1 Fixed-direction loading



Sketch 3.2 Tangential loading

All slender struts subjected to a fixed-direction load buckle if the load is sufficiently large. Within this Item the term buckling load refers to the theoretical buckling load of a perfect strut. Prior to buckling, a small lateral y -direction displacement of the strut will be resisted and, on removal of the displacing force, the strut will return to being straight. The theoretical buckling load is that load at which, when a small displacement is applied, the strut remains stable but displaced. Beyond the theoretical buckling load any attempted lateral displacement of the perfect strut gives rise to rapidly growing displacement and to collapse.

The determination of the buckling load for a strut under tangential loading is more intricate than for cases under fixed-direction loading. To ensure that the lowest buckling load is found, the possibility of dynamic buckling, as well as the possibility of static buckling as described above, must be investigated. As with static buckling, dynamic buckling here relates to a perfect strut but with the additional assumption that there is zero damping. Prior to dynamic buckling any lateral disturbance will lead to vibration of the strut about its neutral axis and the amplitude of this will decay with time until it vanishes. At the theoretical buckling load any disturbance will cause a vibration of unchanging amplitude; that is, it does not decay. Beyond the dynamic buckling load any disturbance will initiate a vibration the amplitude of which will grow with each oscillation.

For a strut with a particular combination of end support conditions and subjected to a form of tangential loading, no simple engineering statement can be provided to indicate which type of buckling will dominate. An analysis that takes no account of the strut dynamics may suggest that the strut remains stable or gives a higher buckling load than that which causes dynamic buckling. It is therefore necessary to derive and examine both the governing stability equations. Derivation 5 gives an exposition of the general forms of stability equations that treat the different types of buckling. Derivation 4 reviews the subject of loss of stability under tangential (non-conservative) forces and provides additional background material.

In some structural members fixed-direction and tangential loads can be applied simultaneously. An example is a strut that carries a fixed-direction load along the line of its undeflected axis and also carries a distributed longitudinal load entering the strut from the shear in an attached sheet. In such a case the distributed load will tend to follow the deflected form of the strut.

The exact line of action of a distributed load is, usually, difficult to define. Therefore, application of values of l' for tangential loading should be used with caution when they are less than the corresponding values for fixed-direction loading.

4. ASSUMPTIONS USED IN CALCULATING ALL EQUIVALENT LENGTHS

The buckling loads used to derive values of l' have been calculated using engineer's theory of bending and assuming that the strut behaves elastically. Transverse shear deflections in the xy -plane have been neglected. The data are therefore only applicable if deflections resulting from transverse shear are negligible in relation to those arising from flexure. The struts were assumed to be geometrically perfect with no initial eccentricities. Also, all loads were applied at the neutral axis. Buckling was assumed to occur in the xy -plane only; no account was taken of torsional effects. In cases of dynamic instability it has also been assumed that the mass is uniformly distributed along the length of the strut and that there is zero damping.

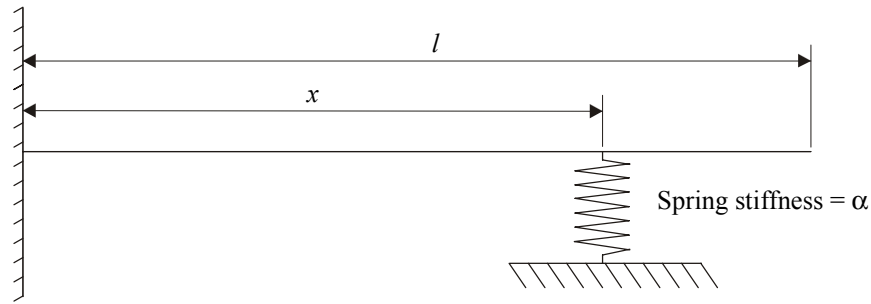
5. STRUTS WITH FIXED-DIRECTION LOADING

5.1 Struts of Uniform Section ($r = 0$)

Table 9.2a provides values of l'/l for struts subjected to load on the ends, to distributed axial load or a combination of these loads for a range of support conditions. Figure 1 provides values of l'/l for struts loaded on the ends only and in which the ends are elastically restrained against rotation. The restraint is expressed by μ_0 and μ_l which can differ. Figures 2a and 2b provide curves of l'/l for struts subjected to different axial loads applied at the ends and a distributed axial load for five combinations of support conditions.

5.1.1 Struts of uniform section with an intermediate elastic support

Tables 9.2b to 9.2f provide values of l'/l for struts subjected to load on the ends or to distributed axial load or a combination of these loads for six combinations of end support conditions and with an elastic support at location $x/l = 0.25, 0.50$ or 0.75 .



Sketch 5.1 Strut of uniform section with an intermediate elastic support

Figures 3a to 8c provide curves of l'/l for struts subjected to different axial loads applied at the ends and a distributed axial load for six combinations of support conditions and with an elastic support at location $x/l = 0.25, 0.50$ or 0.75 . Note that for the cases in Figures 6a to 8c no buckling occurs when P is positive (*i.e.* when the strut is in tension).

The load for unit compression of the spring (spring stiffness) is defined in terms of the spring deflection, δ , under a load Q as $\alpha = Q/\delta$.

Where a kink appears in any of the curves in Figures 3a to 8c it results from a change in the buckling mode which leads to a discontinuous change in the equivalent length.

5.2 Struts Continuous Over an Intermediate Support

Figure 9 provides curves for equivalent lengths of struts that are pinned at both ends and at another intermediate point along their length. The distances between the supports, l_1 and l_2 , can be different and the values of EI between each support, $(EI)_1$ and $(EI)_2$, can also differ. The entire strut is subjected to the same, constant, axial load applied at the ends so that l'_2 is related to l'_1 as shown by the equation on Figure 9. Both values of l' will be required when assessing the strength of a practical strut that has initial bow, offset loading or is subject to material inelasticity effects (see Section 1).

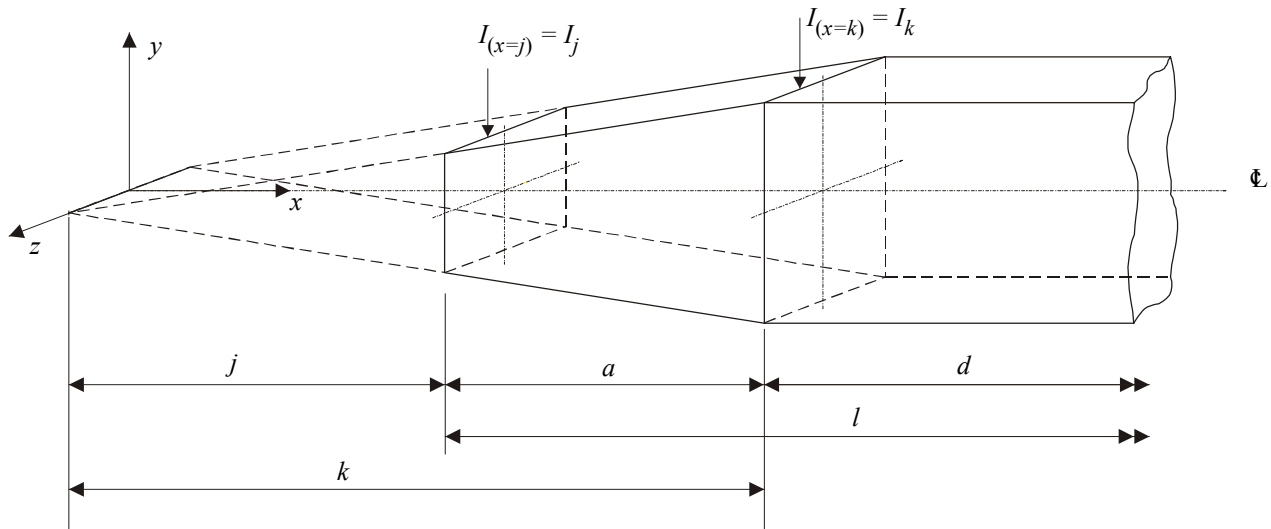
5.3 Tapered Struts

Figures 10 and 11 provide values of l'/l for tapered struts that are either pinned at both ends or fixed at one end and free at the other and are subjected to fixed-direction, constant axial load. Note that, as shown on both figures, P_c is related to l' through I_k . The form of the taper of the section is defined by the relationship

$$(I_x)_{(j \leq x \leq k)} = I_k(x/k)^r, \quad (5.1)$$

where

$$k = \frac{a}{1 - (I_j/I_k)^{1/r}}. \quad (5.2)$$



Sketch 5.2 Rectangular section solid strut with tapered section for which $r = 3$

Figures 10 and 11 provide curves for $r = 2$ or 4 and $r = 1, 2, 3$ or 4 respectively. Sketch 5.2 shows a solid tapered strut for which $r = 3$.

In practical cases it will sometimes be necessary to calculate or estimate an appropriate value of r and then to interpolate between the curves given in the figures. Substituting different values of r into Equation (5.1) results in various forms of taper, of which the following are examples.

- (i) When $r = 0$ then $I_x = I_j = I_k$ and the strut is of uniform section over length a (see Section 5.1).
- (ii) An example of a case where $r = 1$ is a strut of solid rectangular section whose z -direction dimension increases linearly with increasing x .
- (iii) An example of a case where $r = 2$ is a strut constructed from booms of identical unchanging section whose separation in y varies linearly with x . The booms must be connected by a structure capable of forming a stiff shear connection but which makes negligible contribution to I_x .
- (iv) An example of a case where $r = 3$ is a strut of solid rectangular section whose y -direction dimension increases linearly with increasing x . Another example is a linearly tapered circular tube of thin and constant wall thickness.
- (v) An example of a case where $r = 4$ is a strut of solid rectangular section whose y - and z -direction dimensions increase linearly and simultaneously with increasing x . Note that the tapers for this case must decrease to a common point at the origin.

5.3.1 Tapered struts with varying shear flow q

Figure 12 gives l'/l for the special case of a tapered section strut that is fixed at the large end and free at the pointed end and is subjected to a fixed-direction longitudinally-varying axial load. For this particular case $j = 0$ and $a = k = l$. Equation (5.1) describes the form of the taper with l replacing k . The varying distributed longitudinal load is described by the relationship

$$q_x = q_l(x/l)^s. \quad (5.3)$$

Curves are provided on Figure 12 for $r = 0, 1, 2$ and 3 over the range $0 \leq s \leq 5$. When $s = 0$ the distributed longitudinal load is uniform along the strut and $q_x = q_l$. For all other values of s , $q_x = 0$ at $x = 0$ and

$$P = \int_{x=0}^{x=l} q_x \, dx = \frac{q_l l}{s+1}. \quad (5.4)$$

As stated on Figure 12, the value of P_c is obtained using I_l with l' .

6. STRUTS LOADED THROUGH A FIXED POINT

Figure 13 provides a curve of l'/l against c/l for a uniform section strut fixed at one end and loaded through a fixed point located on the x -axis that is distance c from the loaded, free, end. Reference 2 shows that, although P is not fixed in direction, this case may be analysed without reference to the path taken by the load as the strut is loaded. This is therefore a 'conservative loading' case similar to those of fixed-direction loading in which the work done by the load is path-independent. Static buckling therefore occurs and the buckling load can be derived from the load potential, see Section 3.

7. STRUTS WITH TANGENTIAL LOADING

Table 9.3 provides values of l' for struts of uniform section subjected to fixed-direction concentrated axial load and/or distributed longitudinal tangential load for a range of end conditions. Where the value of l' given relates to dynamic buckling this is noted in the table. Note that some values of l' given in Table 9.3 have been obtained from only one source and have not been independently verified. These are identified on the table and should be used with caution. For Cases 9.3.1 to 9.3.9 the distributed longitudinal load is tangential but the load P is in the x -direction. For a strut loaded tangentially at the ends only, Case A is the only case that is theoretically possible; in all the other cases of a strut loaded tangentially at the ends only, the support conditions are either constrained so as to load the strut along the x -direction or the support reaction takes the y -direction component of the load leaving the strut subjected to a fixed, x -direction, load. Although, as stated, Case A is the only theoretically possible case for a strut loaded tangentially at the ends only, such a system is not practically possible and therefore the value of l' for that case is given in a footnote to the table.

Because of the assumptions listed in Section 4, values of l' for dynamic buckling cannot be applied to struts in which the mass distribution is not uniform, even though the stiffness is constant along the length. Also, the values cannot be applied to struts where significant damping will occur.

Where dynamic buckling is indicated, the value of l' accounts for the effects of end conditions and this value may be used when making an assessment of the strength of a practical strut. This is because the value of l' relates to a perfect strut and the introduction of allowances for imperfections will reduce the allowable strut load below that for dynamic buckling of the perfect strut.

8. DERIVATION

This section lists selected sources that have assisted in the preparation of this Item.

1. METTAM, G.M. Uniform and tapered struts with various end conditions and loadings. Stress Note No. S.N.47, Handley Page Ltd, March 1955.
2. TIMOSHENKO, S.P. GERE, J.M. *Theory of elastic stability*. Chapter 2, Second edition, McGraw-Hill, New York, 1961.
3. – Equivalent lengths of variously loaded struts with different end conditions but constant M. of I. SDS 249, Handley Page Ltd, July 1964.
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5. LEIPHOLZ, H. *Stability theory. An introduction to the stability of dynamic systems and rigid bodies*. Academic Press, New York, 1970.
6. GRIMSHAW, H.C. Private communication from British Aerospace, Warton, UK, September 1992.
7. VAN DER SLOOT, J.H. Private communication, April 1999.

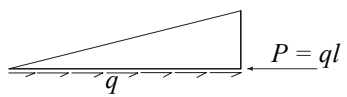
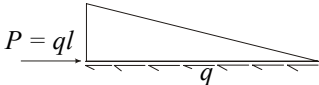
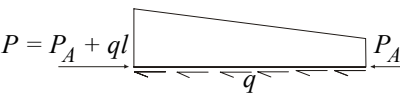
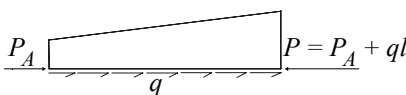
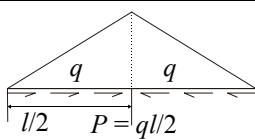
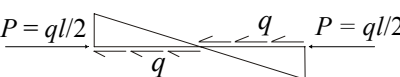
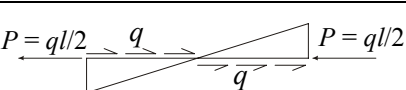
9. TABLES

TABLE 9.1 REPRESENTATION OF STRUT SUPPORT CONDITIONS

The symbols illustrated below are used throughout this Data Item to represent strut end support conditions.

End support condition	Representation
Pinned	
Free	
Fixed	
Fixed against rotation but free in translation	

TABLE 9.2a EQUIVALENT LENGTHS FOR STRUTS OF UNIFORM SECTION SUBJECTED TO FIXED-DIRECTION LOADING ($P_c = \pi^2 EI/l'^2$)

Loading (Showing internal axial load distribution)	End conditions					Case No.
	A	B	C	D	E	
	Equivalent length/actual length, l'/l					
Loaded at ends only (see also Figure 1)						
	2.000	1.000	1.000	0.699	0.500	9.2.1
Linearly varying axial load (q constant)						
	1.685	0.722	0.729	0.573	0.364	9.2.2
	1.122	0.722	0.729	0.434	0.364	9.2.3
	See Fig. 2a	See Fig. 2a	See Fig. 2a	See Fig. 2a	See Fig. 2a	9.2.4
	See Fig. 2b	See Fig. 2b	See Fig. 2b	See Fig. 2b	See Fig. 2b	9.2.5
	1.433	0.842	0.561	0.443	0.361	9.2.6
	0.440	0.439	0.487	0.236	0.236	9.2.7
	1.338	0.439	0.487	0.449	0.236	9.2.8

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**TABLE 9.2b EQUIVALENT LENGTHS FOR STRUTS OF UNIFORM SECTION WITH AN INTERMEDIATE ELASTIC SUPPORT
SUBJECTED TO FIXED-DIRECTION LOAD**

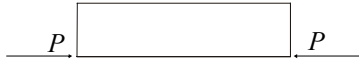
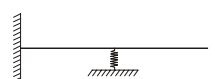
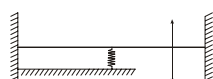
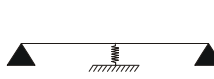
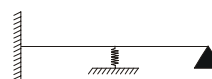
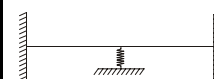
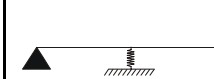
<i>Loading (Showing internal axial load distribution)</i>	<i>End Conditions</i>																	
																		
$\alpha l/P_E$	<i>Spring location from left-hand end along the strut length, x/l</i>																	
	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	<i>Equivalent length/actual length, l'/l</i>																	
2	1.966	1.676	1.212	0.984	0.861	0.688	0.915	0.844	0.915	0.691	0.649	0.657	0.494	0.476	0.494	3.169	1.787	1.215
5	1.925	1.505	1.039	0.964	0.768	0.559	0.829	0.708	0.829	0.679	0.595	0.608	0.486	0.447	0.486	2.329	1.505	1.122
10	1.874	1.402	0.973	0.939	0.709	0.528	0.743	0.583	0.743	0.664	0.538	0.549	0.475	0.408	0.475	2.001	1.421	1.103
20	1.811	1.334	0.941	0.908	0.673	0.520	0.667	0.500	0.667	0.643	0.485	0.483	0.459	0.354	0.459	1.832	1.383	1.096
40	1.748	1.297	0.926	0.876	0.654	0.518	0.620	0.500	0.620	0.619	0.458	0.442	0.441	0.350	0.441	1.748	1.365	1.092
∞	1.625	1.255	0.911	0.813	0.636	0.516	0.580	0.500	0.580	0.566	0.439	0.422	0.408	0.350	0.408	1.668	1.348	1.089

**TABLE 9.2c EQUIVALENT LENGTHS FOR STRUTS OF UNIFORM SECTION WITH AN INTERMEDIATE ELASTIC SUPPORT
SUBJECTED TO FIXED-DIRECTION LOAD**

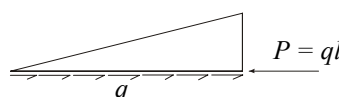
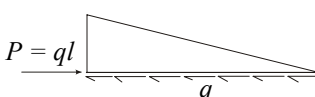
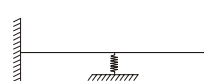
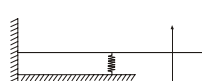
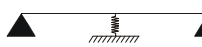
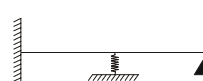
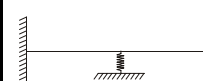
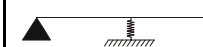
Loading (Showing internal axial load distribution)	End Conditions																	
	A			B			C			D			E			F		
$\alpha l/P_E$	Spring location from left-hand end along the strut length, x/l																	
	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	Equivalent length/actual Length, l'/l																	
2	1.661	1.450	1.085	0.714	0.649	0.534	0.683	0.626	0.658	0.568	0.539	0.537	0.361	0.348	0.358	2.431	1.510	1.089
5	1.633	1.324	0.948	0.705	0.601	0.464	0.637	0.541	0.583	0.561	0.502	0.494	0.357	0.330	0.350	1.890	1.324	0.996
10	1.597	1.247	0.891	0.694	0.570	0.435	0.594	0.474	0.502	0.552	0.465	0.441	0.352	0.307	0.338	1.679	1.261	0.967
20	1.553	1.195	0.860	0.679	0.549	0.422	0.554	0.428	0.414	0.539	0.428	0.373	0.345	0.283	0.321	1.567	1.229	0.953
40	1.509	1.165	0.844	0.664	0.536	0.416	0.525	0.406	0.358	0.523	0.402	0.302	0.337	0.270	0.301	1.509	1.213	0.946
∞	1.419	1.131	0.828	0.632	0.523	0.410	0.489	0.388	0.344	0.481	0.374	0.297	0.316	0.262	0.273	1.450	1.198	0.939

**TABLE 9.2d EQUIVALENT LENGTHS FOR STRUTS OF UNIFORM SECTION WITH AN INTERMEDIATE ELASTIC SUPPORT
SUBJECTED TO FIXED-DIRECTION LOAD**

Loading (Showing internal axial load distribution)	End Conditions																	
	A 			B 			C 			D 			E 			F 		
$\alpha l/P_E$	Spring location from left-hand end along the strut length, x/l																	
	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	Equivalent length/actual length, l'/l																	
2	1.095	0.882	0.588	0.705	0.598	0.489	0.658	0.626	0.683	0.426	0.400	0.415	0.358	0.348	0.361	2.095	1.043	0.722
5	1.062	0.747	0.451	0.685	0.506	0.376	0.583	0.541	0.637	0.415	0.362	0.395	0.350	0.330	0.357	1.427	0.749	0.647
10	1.021	0.666	0.433	0.654	0.441	0.353	0.502	0.474	0.594	0.400	0.319	0.374	0.338	0.307	0.352	1.141	0.685	0.640
20	0.970	0.618	0.430	0.626	0.405	0.351	0.414	0.428	0.554	0.380	0.284	0.352	0.321	0.283	0.345	0.990	0.668	0.638
40	0.919	0.593	0.429	0.593	0.391	0.350	0.358	0.406	0.525	0.358	0.280	0.337	0.301	0.270	0.337	0.919	0.662	0.637
∞	0.823	0.569	0.428	0.530	0.381	0.350	0.344	0.388	0.489	0.322	0.278	0.321	0.273	0.262	0.316	0.857	0.657	0.636