

Stress analysis of single lap bonded joints (adhesive and adherends)

Associated software: ESDUpac A9241

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STRESS ANALYSIS OF SINGLE LAP BONDED JOINTS (ADHESIVE AND ADHERENDS)

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STRESS ANALYSIS OF SINGLE LAP BONDED JOINTS (ADHESIVE AND ADHERENDS)

1. INTRODUCTION, THEORY AND APPLICATION

1.1 INTRODUCTION

This Data Item, together with ESDUpac A9241, presents a Fortran computer program for the analysis of the distribution of the stresses and strains in the adhesive of a bonded single lap joint under an applied load, together with the nominal stresses in the adherends*.

The Data Item is provided in two main sections. Section 1 describes the theory and application of two, alternative, analyses of a single-step lap joint that are included in the program described in Section 2. The program options relating to the two analyses are summarised in Table 1.1 in Section 1.3.5.

The first analysis, the 'simple shear stress' analysis, may be used to analyse the shear stress distribution in the adhesive of a bonded lap joint loaded in either tension or shear. This analysis may be applied to either single or double lap joints, provided that the latter are symmetric, and can allow for the inelastic shear stress-strain properties of the adhesive.

The second analysis, the 'flexible' analysis, is applicable to single lap joints loaded in tension and takes account of bending of the joint in deriving the peel stresses resulting from that bending. This analysis also takes account of thermal stresses arising as a result of differences between joint working temperature and curing temperature but it does not allow for the inelastic shear stress-strain properties of the adhesive.

In both analyses it is assumed that the behaviour of the plate material, the adherend, is elastic and any stress concentrations are ignored.

Section 2 describes the Fortran program, its input, output and messages. Example calculations are also included. The program is available in two formats: (a) a Fortran source code and corresponding executable file, and (b) an executable program for use within ESDUview. Both versions can be found in ESDUscope (the CD version of the Series), on the internet version of the Series, and on disk in the Series Software Volume. Example input and output files are also provided.

The program, which presents the option to use either the 'simple shear stress' or the 'flexible' analysis, is intended for use in achieving a good, well-balanced design; to this end the program is equipped with a facility whereby it can find the overlap length and/or adhesive shear modulus that give the lowest maximum adhesive shear stress for a particular joint. It is emphasised, however, that the program can only accurately predict the behaviour of a joint if good material property data are available.

* This Data Item has been amended in order to incorporate a new Fortran program. The new program was written after it was found that under certain circumstances, particularly when analysing very long joints, the original program sometimes converged very slowly on the solution.

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

E_a	modulus of elasticity of adhesive	N/m ²	lbf/in ²
E_1, E_2	moduli of elasticity of materials of adherends 1 and 2 respectively	N/m ²	lbf/in ²
G_a	elastic shear modulus of adhesive material	N/m ²	lbf/in ²
G_1, G_2	elastic shear moduli of materials of adherends 1 and 2 respectively	N/m ²	lbf/in ²
l_1, l_3	lengths of zones 1 and 3 respectively of joint (see Sketch 1.4b)	m	in
l_2	length of overlap zone of joint (see Sketch 1.4b)	m	in
M	bending moment per unit width in adherends	N m/m	lbf in/in
N	number of intervals over each zone of joint for which output is required		
Q	shear load on joint per unit width, half load on double lap joint	N/m	lbf/in
T	tensile load on joint per unit width, half load on double lap joint	N/m	lbf/in
t_a	thickness of adhesive layer, assumed to be constant over length of step	m	in
t_1, t_2	thicknesses of adherends 1 and 2 respectively	m	in
x, y, z	in-plane co-ordinates	m	in
α_1, α_2	coefficients of thermal expansion of adherends 1 and 2 respectively	/°C	/°F
γ	shear strain in adhesive		
γ_{all}	selected maximum allowable shear strain in adhesive		
γ_F	shear strain at which cohesive failure occurred in adhesive under test		
$\delta\phi$	difference between joint working temperature and reference temperature*, $\delta\phi = \phi_w - \phi_{ref}$	°C	°F
ν_1, ν_2	Poisson's ratios of adherends 1 and 2 respectively		
σ_b	maximum bending stress in adherend	N/m ²	lbf/in ²
σ_z	direct (peel) stress in adhesive	N/m ²	lbf/in ²
τ	shear stress in adhesive	N/m ²	lbf/in ²

τ_A	shear stress to which theoretical stress-strain curve of adhesive is asymptotic (see Sketch 1.3)	N/m ²	lbf/in ²
τ_E	shear stress in adhesive at elastic limit (see Sketch 1.3)	N/m ²	lbf/in ²
τ_F	shear stress at which cohesive failure occurred in adhesive under test	N/m ²	lbf/in ²
ϕ_{ref}	reference temperature [*] , that is, temperature at which adhesive is stress-free with joint unloaded	°C	°F
ϕ_w	joint working temperature	°C	°F

* In the absence of better information, this may be taken as the curing temperature.

1.2.1 Suffices

The suffices l and u refer to the lower and upper values respectively of ranges of l_2 , E_a and G_a in the program input.

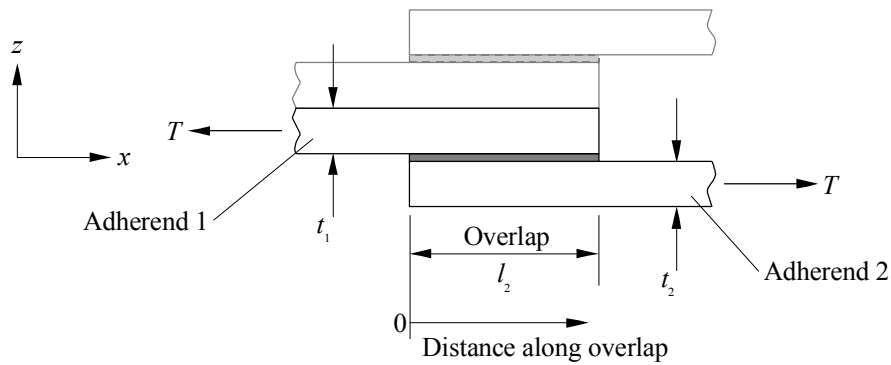
The suffix *max*, applied to σ_z and τ , indicates a maximum value.

1.3 OVERVIEW

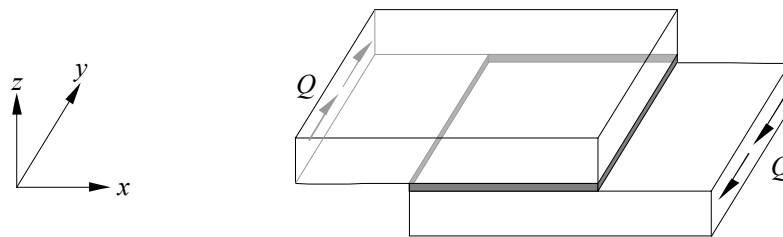
1.3.1 Mechanical Behaviour of Single Lap Joints

ESDUpac A9241 may be used to analyse single-step bonded lap joints loaded in either tension or shear and in which the bonded area is rectangular. Two analyses are available; they are denoted ‘simple shear stress’ analysis and ‘flexible’ analysis. Both analyses are based on conventional, though different, assumptions with the result that the stresses and strains calculated are engineering approximations in which local stress concentrations are neglected. Table 1.1 gives a summary of the capabilities of the program.

As the simple shear stress analysis does not take into account bending or peel effects, it is strictly relevant to joints where these effects are absent, for example, balanced double lap joints with adherends that cannot bend. In the simple analysis the adherends are subjected either to uniform tension, see Sketch 1.1, or to in-plane shear, see Sketch 1.2. Only the shear stresses and strains in the adhesive are considered, that is, stresses normal to the plane of the adhesive (peel stresses) are not included. However, the inelastic shear stress-strain properties of the adhesive may be taken into account, see Sketch 1.3 and Figure 1.

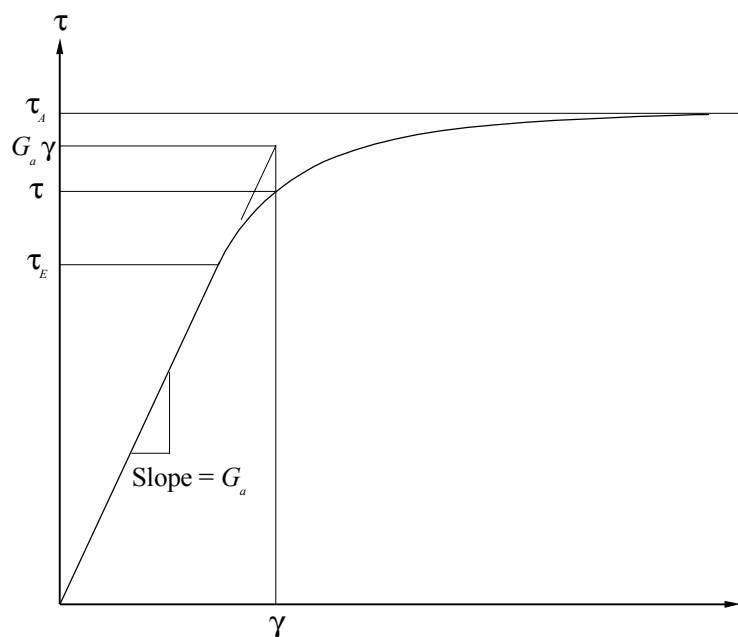


Sketch 1.1 Single lap joint modelled as half of a symmetrical double lap joint for the simple shear stress analysis (the analysis only takes account of the behaviour over the length l_2 and assumes there is no bending)



Complementary shear forces omitted for clarity

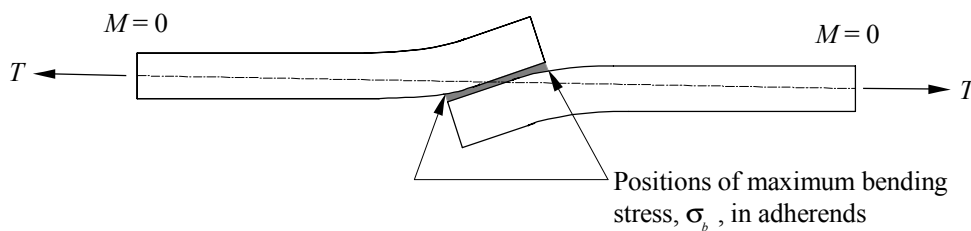
Sketch 1.2 Single lap joint loaded in shear. Simple shear stress analysis only



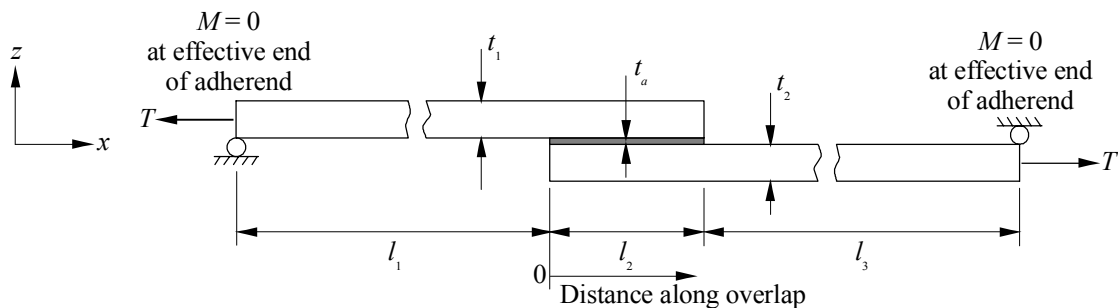
Sketch 1.3 Example of adhesive shear stress-strain curve. G_a relates to the elastic region

In the flexible analysis, bending of the joint is taken into account, see Sketch 1.4a, and both shear and peel stresses in the adhesive are considered, but only for tensile loading and for linear-elastic stress-strain properties of the adhesive. If the non-linear shear stress-strain properties of the adhesive are to be taken into account, the simple shear stress analysis must be used.

Although flexure of the adherends is taken into account, the theory assumes that there are no moments at the remote ends of the joint. If l_1 and l_3 are not explicitly known, this condition of zero moment may be used to define the effective lengths of the adherends, see Sketch 1.4b. The lengths l_1 and l_3 are discussed further in Section 1.3.3. The stresses arising as a result of using the joint at a temperature other than that at which it was cured may also be accounted for in this analysis.



(a) Joint loaded in tension showing exaggerated deformations



(b) Treatment of single lap joint for the flexible joint analysis of shear and peel stresses, taking account of bending (analysis includes l_1 , l_2 and l_3 in all of which zones flexure may occur)

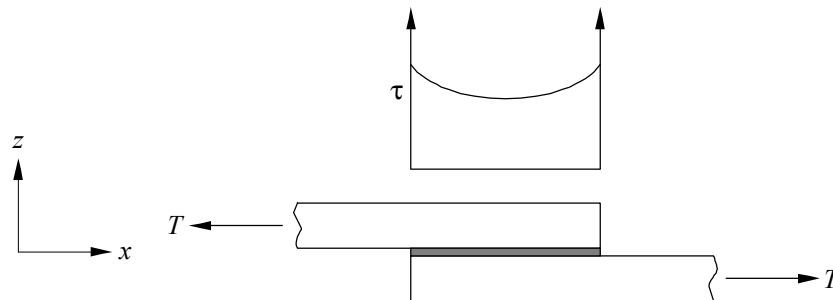
Sketch 1.4 Single lap joint loaded in tension noting assumptions made for flexible joint analysis

The analyses are described in Sections 1.4.2 and 1.4.3 respectively and full details can be found in Derivation 3. Details of the program options available regarding the type of applied load, adhesive properties and the form of the output are given in Section 1.3.5.

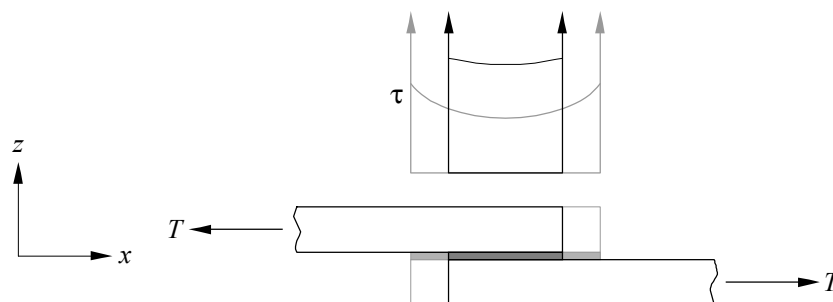
1.3.2 Simple Analysis – Distribution of Shear Stress

The distribution of shear stress in a joint with a given type of loading is directly related to the adhesive and adherend properties, the joint geometry and the adhesive thickness, all factors that can usually be controlled to some extent in joint design. The properties of adhesives vary widely, and can be time-dependent, but the inelastic properties may be represented adequately for the simple analysis by appropriate choice of the two constants, τ_E and τ_A . For the region of the adhesive stress-strain curve between those two constants, the program assumes a simple explicit form for the curve. Further details of the way in which the program treats the inelastic shear stress-strain behaviour of the adhesive are given in Derivation 3.

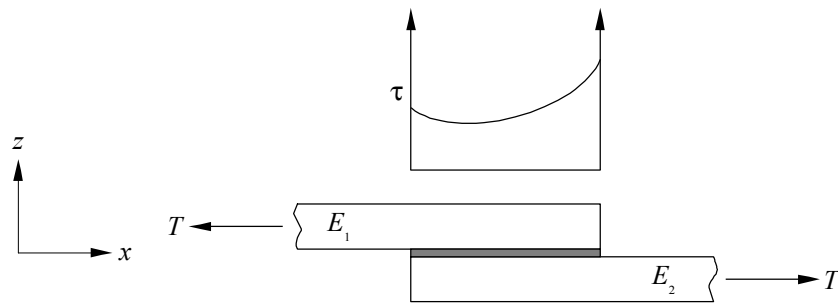
As a consequence of the different stiffnesses of the adhesive and the adherends, the distribution of stress and strain in the adhesive in a single lap joint loaded in tension or shear will almost always be of a U-shaped form along the joint. The highest shear stresses in the adhesive occur very close to the ends of the overlap and are assumed to occur at the ends in both of the analyses presented in this Data Item. If a joint is made of adherends of the same material and thickness (referred to in this Data Item as a uniform joint), the stress distribution will be symmetrical about the centre of the overlap, see Sketch 1.5. A short overlap will give a higher shear stress level than an otherwise similar longer overlap, see Sketch 1.6. For a joint made with adherends of dissimilar materials of the same thickness, such as that shown in Sketch 1.7, the higher adhesive stress occurs at the end of the adherend with the higher modulus. In a joint consisting of adherends of the same material but of different thicknesses, such as that in Sketch 1.8, the higher adhesive shear stress occurs at the end of the thicker adherend.



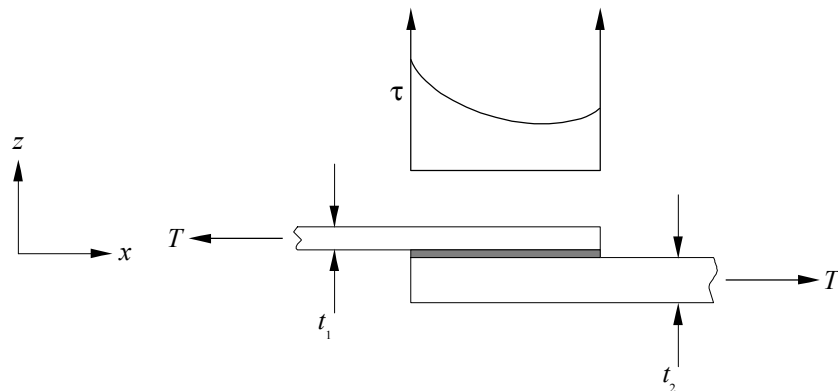
Sketch 1.5 Uniform joint ($E_1 = E_2, t_1 = t_2$) loaded in tension giving a symmetrical adhesive stress distribution



Sketch 1.6 Reduced overlap length giving higher adhesive stress levels



Sketch 1.7 Joint of dissimilar materials ($E_1 > E_2, t_1 = t_2$) loaded in tension giving an asymmetrical stress distribution



Sketch 1.8 Joint with different adherend thicknesses ($E_1 = E_2, t_1 < t_2$) loaded in tension giving an asymmetrical stress distribution

1.3.3 Flexible Analysis – Distributions of Shear and Peel Stresses

The shear and peel stresses in the adhesive depend, in different ways, on the joint geometry, the adherend and adhesive properties and the adhesive thickness. In general, if the total joint length ($l_1 + l_2 + l_3$, see Sketch 1.4b) is kept constant, varying the overlap length, l_2 , will affect the relative magnitudes of the shear and peel stresses and will usually produce different maximum peel stresses at the ends of the adhesive.

As illustrated by Figures 5b and 6b, as the length of the overlap is increased from zero, the value of the maximum peel stress rises to a peak and then decreases gradually. Although very small overlap lengths lead to reduced values of maximum peel stress in the adhesive, as already stated, and as can be seen clearly in Figures 5a and 6a, such small overlap lengths result in very severe levels of shear stress in the adhesive and should therefore be avoided.

As shown in Figures 5b and 6b, the ‘critical’ overlap length at which the peak in the value of the maximum peel stress occurs varies with joint geometry. The ‘critical’ length is influenced significantly by adherend thickness; the greater is the thickness, the larger is the overlap length at which the peak in the maximum adhesive peel stress occurs. As can be seen in Figure 6b, for the range of joints represented by the plots on that figure, the ‘critical’ overlap length occurs between values of $l_2/t^{1/2}$ of about 4.5 and 7.

In the light of the points discussed above and Figures 5a to 6b, it is clear that a longer overlap is preferable as, beyond the 'critical' length, it results in both lower shear and lower peel stresses in the adhesive.

If the overlap length is kept constant, varying l_1 and l_3 will affect the adhesive peel stresses and, to a lesser extent, the shear stresses. If l_1 and l_3 are different, the distribution of peel stress in the adhesive will be asymmetric along the joint with the maximum adhesive peel stress occurring at the end of the shorter adherend.

(Note that, because of the non-linear distribution of the adhesive stresses over the length of the overlap, the strength of a joint is not proportional to the overlap length; that is, doubling the overlap length will not double the strength of a joint.)

As can be seen in Figure 7, the lengths l_1 and l_3 also have a significant effect on the bending stresses in the adherends. The maximum bending stresses in the adherends occur at the ends of the overlap, see Sketch 1.4a. As mentioned in Section 1.3.1, the analysis assumes that the moments at the remote ends of the adherends are zero and, if l_1 and/or l_3 are not known exactly, this condition may be used to define them. In some cases it might not be possible to determine the lengths in this way and in such cases it is suggested, as in Data Item No. 67008, that they be taken as the distances to the nearest lateral supports. For cases where it is not possible to determine the lengths according to the above conditions, a facility is provided within the program to use default values for l_1 and l_3 of $6l_2$. As indicated by Figure 7, using these default values, or larger values, should in most cases result in a conservatively high value of maximum bending stress in the adherends.

In general, decreasing the thickness of the adhesive layer will have a beneficial effect on the shear stress distribution in the adhesive but a detrimental effect on the peel stress distribution. Therefore, in cases where shear stress dominates, good practice in joint design will usually indicate an adhesive layer as thin as practicable. Increasing the adhesive thickness normally reduces the value of the maximum peel stress and practical tests confirm that this is beneficial in situations where peel stress dominates. It should be noted that this effect of increasing the adhesive thickness will not continue indefinitely: increasing the thickness of the adhesive beyond a certain limit will lead to yielding of the adhesive.

1.3.4 Modes of Failure for Single Lap Bonded Joints

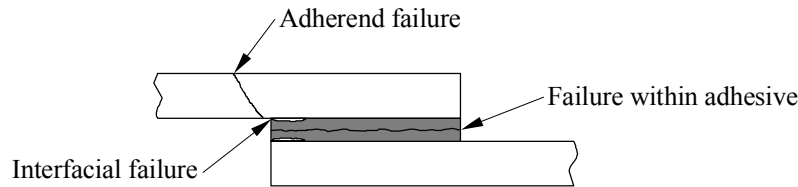
While bonded joints can fail in a number of ways, the most common modes of failure in statically loaded bonded joints are as follows (see also Sketch 1.9).

- (i) Adherend failure at the joint extremities.
- (ii) Failure within the adhesive related to the shear and peel stresses.
- (iii) Adhesion failure at the interface of the adherend and the adhesive.

This Data Item may be used to assess the stresses relevant to mode (ii). Consideration of those stresses with reference to data obtained from the manufacturer of the adhesive and/or adhesive test data can then be used to assess the strength of a joint. This Data Item may also be used to assess the possibility of failure by mode (iii), though this mode of failure is greatly influenced by such factors as surface preparation and bonding techniques.

When selecting an adhesive, it should be noted that, because of the differences in the stress-strain behaviour of different adhesives, the adhesive with the highest failure stress is not necessarily the best choice. The allowable stress values supplied by manufacturers are also a function of the thickness of the adhesive layer and will usually have been derived from a standard test, for example, that described by ASTM D1002

(Reference 14), using a specific adhesive thickness (usually as thin as practicable) and thick adherends. This form of test emphasises shear stress capability rather than peel stress failure; ASTM D1876 (Reference 15) describes a method of assessing the peel resistance of adhesives.



Sketch 1.9 Different modes of failure in a single lap bonded joint

1.3.5 Program Options

Three program options are available and these are set out in Table 1.1. The option chosen will depend on the type of analysis required, the nature of the applied load and whether or not the inelastic shear stress-strain properties of the adhesive are to be taken into account.

Program options 1 and 2 use the simple analysis to calculate the shear stresses and strains in the adhesive and the nominal stresses in the adherends. Both options allow the inelastic shear stress-strain properties of the adhesive to be taken into account. Option 1 applies to a joint loaded in tension, see Sketch 1.1; option 2 applies to a joint loaded in shear, see Sketch 1.2. This is the only difference between options 1 and 2.

Program option 3 uses the flexible analysis to calculate the shear and peel stresses in the adhesive, the rotations and displacements and the nominal tensile and bending stresses in the adherends of a joint loaded in tension. This option does not consider the inelastic shear stress-strain properties of the adhesive, nor is it applicable to joints loaded in shear.

The simple shear stress analysis (options 1 and 2) assumes that the adherends carry no bending moment resulting either from the transmission of the load from one adherend to the other or from external bending moments and does not therefore consider the effect on the adhesive or the adherends of through-the-thickness (peel) stresses.

The flexible joint shear and peel stress analysis (option 3) evaluates the adhesive shear and peel stresses, taking account of bending stresses arising from deformation of the adherends, but not of those caused by externally applied bending moments. This analysis also takes account of the stresses arising as a result of using the joint at a temperature other than that at which it was cured. Examples of stress distribution curves obtained using the two analyses are shown in Figures 2 to 4.

Various factors that affect the design and performance of bonded joints are discussed in Data Items Nos 67008, 80039 and 81022.

Data Item No. 67008, "Elastic stresses in single lap joints under tension or compression", gives the maximum total elastic stress resulting from the combination of the bending stress and the applied direct stress in a single lap joint between two flat or curved plates of the same thickness and material. The plates are assumed to be rigidly joined.

Data Item No. 80039, “Elastic adhesive stresses in multistep lap joints loaded in tension”, presents a program similar to the one that accompanies this Item, the main difference being that Data Item No. 80039 deals with multi-step joints.

Data Item No. 81022, “Guide to the use of Data Items in the design of bonded joints”, provides guidance on the use of the analytical procedures presented in other Data Items on bonded joints. Examples of the adhesive material properties required for use with the data are given together with recommendations on how to obtain such properties. Attention is drawn to important factors affecting joint strength not covered by the analytical procedures and references are provided as a source of further information. Notes are also provided on the principal complementary or alternative methods currently available.

The availability of adhesive and adherend properties data and/or the loading applied to the joint might well define which program option can be used. However, in the absence of those restrictions, the following general guidelines will assist in choosing a program option. The simple shear stress analysis (options 1 and 2) is most applicable to the cases where the displacements and the effects of bending are at a minimum. For joints with very thin adherends, large out-of-plane displacements will occur on loading and the flexible joint analysis (option 3) will give more accurate results.

TABLE 1.1 PROGRAM OPTIONS

<i>Theoretical procedure</i>	<i>Loading</i>	<i>Program option</i>	<i>Adherend stresses considered</i>	<i>Adhesive stresses considered</i>	<i>Results output</i>
Simple shear stress analysis	Tensile	1	Axial stress (elastic) corresponding to T	Shear stress (elastic or inelastic, see Sketch 1.3)	Nominal adherend axial stress and adhesive shear stresses and strains
Simple shear stress analysis	Shear	2	Shear stress (elastic) corresponding to Q	Shear stress (elastic or inelastic, see Sketch 1.3)	Nominal adherend shear stress and adhesive shear stresses and strains
Flexible joint shear and through-the-thickness (peel) stress analysis	Tensile and thermal	3	Axial stress (elastic) corresponding to T and $\delta\phi$ and induced bending stresses	Shear stress (elastic) and peel stress (elastic)	Shear and peel stresses in adhesive and adherend axial and bending stresses and displacements and rotations

1.4 THEORY

1.4.1 Introduction

The following two sections describe the analyses used by the program for evaluating the shear stress and strain and peel stress distributions along the adhesive. Derivation 3 describes the theory in greater detail.

1.4.2 Simple Shear Stress Analysis

The simple shear stress analysis (sometimes referred to as ‘shear lag analysis’) is taken from Derivation 3 and is used for program options 1 and 2, described in Section 1.3.5.