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Contact stress in disc cams with roller followers

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The Mechanisms Committee, which first met in 1975, is continuing the work of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

CONTACT STRESS IN DISC CAMS WITH ROLLER FOLLOWERS

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CONTACT STRESS IN DISC CAMS WITH ROLLER FOLLOWERS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI but the user is strongly recommended to check the units given in bearing manufacturers' catalogues and to make any necessary conversions. Units for the program input parameters are given in Sections 7.3.1 to 7.3.3.

		Units
A	body geometry parameter (see Equations (3.31) and (3.32))	m^{-1}
<i>a</i>	major semi-axis of contact ellipse	m
B	body geometry parameter (see Equations (3.31) and (3.32))	m^{-1}
<i>b</i>	minor semi-axis of contact ellipse	m
<i>D</i>	distance between cam rotational centre and follower pivot	m
<i>E</i>	follower offset	m
E_c	Young's modulus for cam material	N/m^2
E_r	Young's modulus for roller material	N/m^2
F_E	external force acting on moving follower (positive to increase contact force)	N
F_{xi}, F_{yi}	components of resultant of external forces acting on part <i>i</i> in <i>x</i> - and <i>y</i> -directions, respectively	N
$(f_z)_{\max}$	maximum compressive contact stress	N/m^2
<i>H</i>	follower lift corresponding to one segment	m
I_f	mass moment of inertia of follower system about follower pivot	kg m^2
k_c	material constant for cam (see Equation (3.33))	m^2/N
k_r	material constant for roller (see Equation (3.34))	m^2/N
k_{SC}	compression spring rate	N/m
k_{ST}	torsion spring rate	N m/rad
<i>L</i>	distance between follower pivot and roller centre	m
m_f	mass of follower system	kg
P_C	contact force normal to cam profile	N
P_{ij}	force exerted by part <i>i</i> on part <i>j</i>	N

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P_S	compression spring force	N
P_{SP}	compression spring preload	N
P_{xij}, P_{yij}	components of joint force P_{ij} in x - and y -directions, respectively	N
R_a	prime circle radius of cam	m
R_b	reference circle radius of cam	m
R_c	crown (or barrel) radius of roller	m
R_r	radius of roller	m
T_E	external moment acting on moving follower (positive to increase contact force)	N m
T_{ij}	moment exerted by part i on part j	N m
T_S	torsion spring moment	N m
T_{SP}	torsion spring preload	N m
W_c	width of cam	m
W_r	width of roller	m
w	normalised follower displacement	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
y	translational displacement of follower from start of segment	m
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
y_b	height of centre of roller above X_m -axis at inner dwell	m
y_m	instantaneous height of centre of roller above X_m -axis	m
y_0	height of centre of roller above X_m -axis at beginning of segment	m
Z_1, Z_2, Z_3	intermediate calculation parameters	—
z_1	force factor	—
z_2	preload factor	—
α	pressure angle	degree
β	semi-axis ratio of contact ellipse	—

δ	normal approach between cam centre and roller centre due to elastic deformation	m
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m
σ_c	Poisson's ratio for cam material	—
σ_r	Poisson's ratio for roller material	—
Φ	total angle of cam rotation corresponding to one segment	rad
ϕ	cam angle of rotation (from start of Segment 1)	degree
Ψ	follower swing corresponding to one segment	rad
ψ	rotational displacement of follower from start of segment	rad
$\dot{\psi}$	rotational velocity of follower	rad/s
$\ddot{\psi}$	rotational acceleration of follower	rad/s ²
ψ_b	angle between X_m -axis and line joining follower pivot to roller centre at inner dwell	rad
ψ_m	instantaneous angle between X_m -axis and line joining follower pivot to roller centre	rad
ψ_0	angle between X_m -axis and line joining follower pivot to roller centre at beginning of segment	rad
ω_c	angular velocity of cam	rad/s

Subscripts

<i>equiv</i>	refers to equivalent value
<i>e</i>	refers to end value of external load
<i>i</i>	refers to value at a particular calculation point
<i>max</i>	refers to maximum value
<i>min</i>	refers to minimum value
<i>q</i>	refers to value when separating force is a maximum
<i>s</i>	refers to start value of external load

2. INTRODUCTION

Cam/follower systems are liable to fail if the surface of the cam (or the roller) deteriorates under the influence of excessive load. These service failures can often be avoided if the system is designed such that stress in the contact region does not exceed an allowable value associated with the materials of the cam and the roller and the required life of the system. Therefore the prediction of contact stress under a variety of operating conditions is a vital step in the systematic design of a cam and follower.

This Data Item presents a procedure for calculating values of compressive stress in the contact region between a disc cam and a roller follower at regular intervals during one complete rotation of the cam. To determine these stress values it is necessary to find, for a series of angular positions of the cam, the radius of curvature of the cam profile and the contact force at the interface between the cam and the roller. This analysis is time consuming and is best carried out using the computer program (See Section 7) supplied with this Data Item. The program determines values of maximum compressive contact stress at intervals of one degree of cam rotation (assuming constant angular velocity of the cam) and finds the peak value of stress during the cycle. The dimensions of the contact zone and the normal approach between the centres of the cam and the roller due to elastic deformation in the contact zone are also calculated. The program can be used to determine suitable values of rate and preload for a return spring designed to ensure that the cam and roller remain in contact. Alternatively if a spring of known rate and preload is specified the program checks that contact is maintained. A check to ensure that undercutting does not occur is also included.

Correct implementation of the calculation procedure suggested in this Item will result in a decrease in design time together with improvements in machine reliability.

2.1 Assumptions

The procedure is applicable to both translating and swinging crowned[†] roller followers driven by disc cams with spring force closure. During the derivation of contact force it is assumed that:

- (a) contact force at the cam/roller interface is due only to follower system inertia, action of the return spring and externally-applied load acting on the follower system (friction in the follower system and gravitational force may be modelled as part of the external load if required),
 - (b) friction torque at the roller pivot is negligible,
 - (c) slip between the cam and roller is negligible,
 - (d) the cam and roller are designed and manufactured so that the roller can accurately follow the cam profile at all times, *i.e.* undercutting does not occur,
 - (e) the axes of rotation of both the cam and roller are aligned such that the shape of the contact zone is not modified by edge effects,
 - (f) the widths of the cam and roller are chosen such that the shape of the contact zone is not modified by edge effects,
 - (g) the cam and roller remain in contact throughout the operating cycle (see Section 3.4)
- and (h) the follower system is sufficiently stiff to be considered rigid.

[†] The calculation procedure presented in this Data Item is strictly applicable only to "crowned" followers (see Sketch 4.5) in which some misalignment of the cam and follower axes is tolerated.

A number of further assumptions are implicit in the method adopted for calculation of contact stress (see Section 3.3). These are:

- (i) the cam and roller materials are isotropic and linearly elastic,
 - (j) the dimensions of the contact area are very small in comparison with the radii of the cam profile and roller in the vicinity of the contact,
 - (k) the shape of the deformed zone adjacent to the contact zone can be described with sufficient accuracy by a second order equation, *i.e.* surfaces are smooth,
 - (l) the contact force acts normal to the tangent to the cam surface at the contact point,
 - (m) the surfaces are clean and unlubricated (see Section 2.2)
- and (n) surface shear stresses resulting from traction are ignored.

Discussion of the implications of Assumptions (i) to (n) is presented in full in ESDU 78035 (Derivation 1) and is not repeated here. Particular comments regarding the suitability of the algorithm for calculation of contact stress at the cam/roller interface are given in Section 2.2.

2.2 Scope

2.2.1 Follower type

Cams with roller followers (translating or swinging) can be analysed. The algorithm used to calculate the compressive contact stress at the surface (taken from ESDU 78035, Derivation 1) is strictly applicable only to static contacts where no traction forces act. In the contact zone between a cam and a crowned roller follower traction forces (due to friction) may be present. However, if the roller does not slip, the coefficient of friction is likely to be very low (less than 0.1) and the values of compressive contact stress predicted by the algorithm will be sufficiently accurate.

2.2.2 Follower motion

The method presented in this Item can be used to analyse cams with a wide range of profiles. In cam design the relationship between the motion of the cam and the motion of the follower, during a dwell-rise-dwell (DRD) or a dwell-return-dwell (DRD) motion, is usually governed by a mathematical equation known as a cam law (see ESDU 82006, Reference 5).

Although the theory (presented in Section 3 of this Item) is general, in that any mathematical cam law can be specified the computer program contains only six commonly-used cam laws (identified in Entry 11, Section 7.3.1). If the required cam law for a particular motion segment is not included in the current version of ESDUpac A9301 then the data can be entered as a motion data file. This motion data file must contain values of follower displacement, follower velocity, follower acceleration and follower jerk at intervals of one degree of cam rotation. The required format of the file is described in Section 7.3.2 and the reasons for specifying motion in this way are explained in Section 4.1.1.

2.3 Use of This Data Item

2.3.1 General guidance

The theoretical background is presented in Section 3 of this Data Item. It is not necessary to have a deep understanding of this theory to carry out the required calculation using the accompanying computer program described in Section 7. A guide to the choice of input values for this program and a discussion of the program output, including design guidance, is presented in Section 4. The examples in Section 6 also provide valuable hints on the effective use of the program.

Calculation of contact stress may be useful:

- (a) during a feasibility study,
 - (b) as part of a detail design, especially during the selection of a suitable cam material (see ESDU 94017, Reference 24),
- or
- (c) to investigate the cause of deterioration of the surface of a cam or a roller (see Tallian, Reference 35).

It is likely that the nature of the information available in these cases will vary. For instance during a feasibility study the mass properties of the follower and the externally-applied forces may not be known accurately. The analytical procedure is applicable when uncertainty surrounds values of inputs but the accuracy of the results is entirely dependent on the quality of the input information.

2.3.2 Sources of information regarding values of allowable contact stress

Values of compressive contact stress obtained using the procedure in this Item should be used in conjunction with a corresponding “allowable” value of contact stress. However, there are a number of difficulties in obtaining reliable values of this limiting stress (or surface strength).

Firstly, there are several different modes by which a surface can fail and each mode has different criteria associated with it. Surfaces can deteriorate as a result of plastic deformation, brittle cracking, fatigue cracking or wear-related processes such as scuffing and polishing. ESDU 84017 (Reference 14) includes a detailed discussion of the failure criteria usually associated with surfaces in contact. For cams with roller followers the most likely cause of surface failure is a fatigue phenomenon caused by the cyclical nature of the load and known as “pitting”. Initiation of fatigue cracks below the surface is believed to be associated with cyclic variation of the orthogonal shear stress. The magnitude and location of this quantity could be found as described in ESDU 85007 (Reference 15). However limiting values of the range of orthogonal shear stress are not readily available.

A more pragmatic approach is to make use of allowable values of compressive contact stress that have been obtained by experiment. Such values are given in References 3, 24 and 32 for a range of commonly-used cam materials. The reliability and generality of these sources is unknown and in practice there is no substitute for data obtained in service using identical materials in a similar application.

The allowable stress is heavily dependent on the hardness of the surface which can usually be increased by surface treatment. Advice regarding the improvement of resistance to pitting is given in ESDU 86040 and ESDU 94017 (References 17 and 24). As fatigue cracks are often initiated below the surface it is advisable to ensure that surface hardness is maintained to an appropriate depth.

3. THEORETICAL BACKGROUND

This section presents details of the mathematical model on which the calculation procedure is based. It is strongly recommended that the calculations are carried out using the computer program described in Section 7. The equations used to evaluate contact force and cam profile radius of curvature in this program are derived in Section 3.1, for the case of a translating follower, and in Section 3.2, for that of a swinging follower. Section 3.3 introduces the algorithm for calculating contact stress and Section 3.4 presents the formulae used for initial selection of the spring. Recommended values of the quantities needed as inputs to the computer program are given in Section 4.1. The convention adopted for forces and moments in this Item is such that P_{x23} is the component of force in the x -direction exerted by Part 2 on Part 3. Thus

$$P_{x23} = -P_{x32} .$$

The magnitude of the force exerted by Part 2 on Part 3 is given by

$$P_{23} = \left(P_{x23}^2 + P_{y23}^2 \right)^{1/2} .$$

Similarly, T_{45} is the moment (positive in the anticlockwise sense) exerted by Part 4 on Part 5. Thus

$$T_{45} = -T_{54} .$$

3.1 Translating Roller Follower

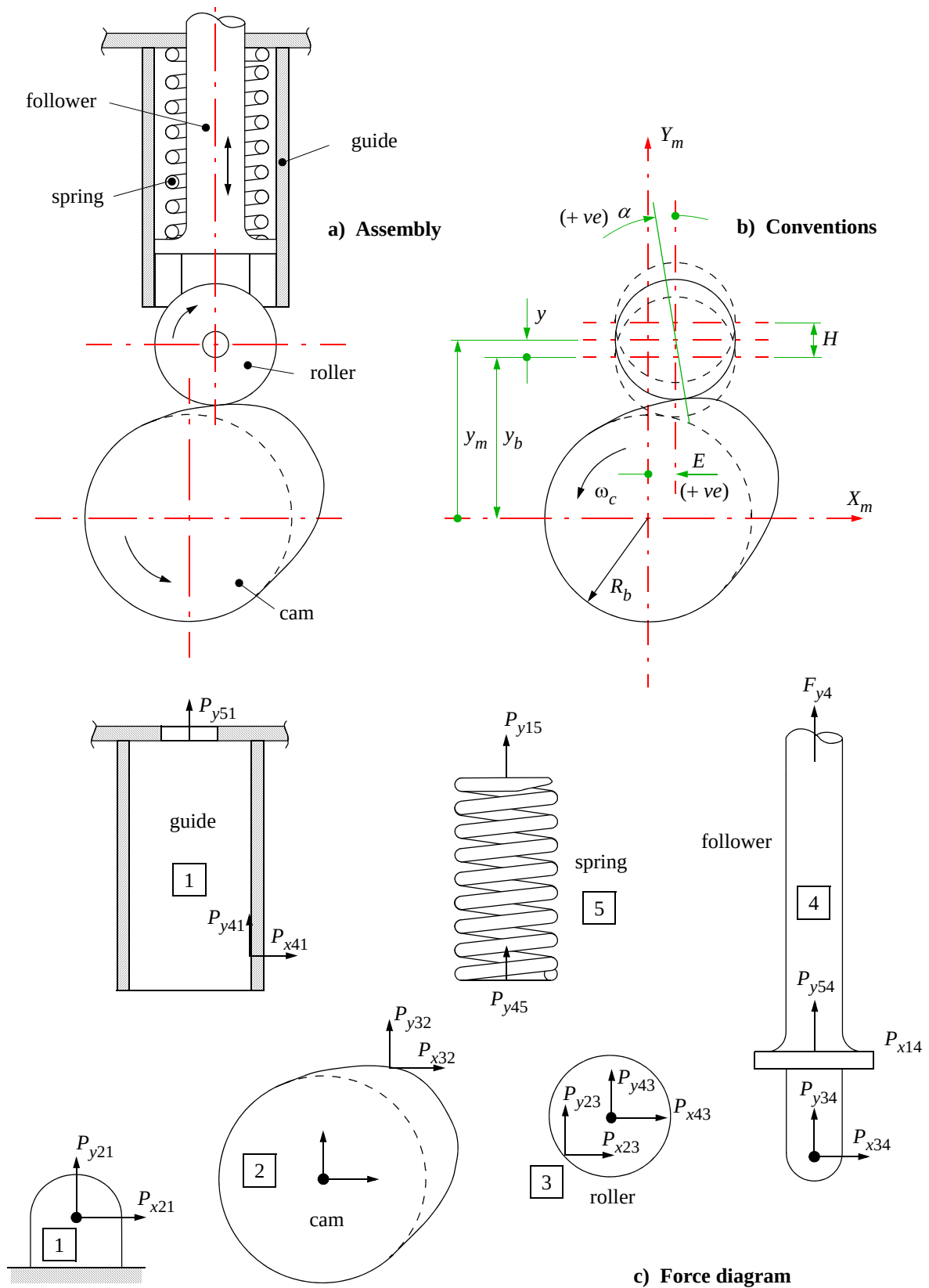
Sketch 3.1(a) shows a typical sub-system composed of a disc cam with an offset translating roller follower. A compression spring is used to provide force closure. The convention used is shown in Sketch 3.1(b). Diagrams showing the forces acting on each part are shown in Sketch 3.1(c).

3.1.1 Derivation of contact force

It is required to derive an expression for the contact force, P_C , between the roller and the cam due to the action of the spring, the inertia of the follower system and the external force acting on the follower. The follower is constrained to move only in the y -direction and it is assumed that friction force, P_{y14} , between the follower and the guide is zero. However, in practice, friction can be included by considering it as part of the external force, F_{y4} . The sum of the components of force acting on the follower in the y -direction is equal to the product of the mass of the follower (see Section 4.1.1) and the acceleration of its centre of mass in the y -direction. Thus for equilibrium of the follower

$$P_{y34} + P_{y54} + F_{y4} = m_f \ddot{y} , \quad (3.1)$$

where P_{y34} is the force exerted by the roller on the follower,
 P_{y54} is the force exerted by the spring on the follower,
 F_{y4} is the external force exerted on the follower
 and m_f is the mass of the follower system.



Sketch 3.1 Cam and follower system

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The instantaneous motion (displacement, y , velocity, $\dot{y}$, and acceleration, $\ddot{y}$) of the roller centre during a rise or return segment is required. These quantities are given by:

$$y = wH, \quad (3.2a)$$

$$y_m = y_0 + wH, \quad (3.2b)$$

$$\dot{y} = \frac{w' \omega_c H}{\Phi} \quad (3.3)$$

$$\text{and} \quad \ddot{y} = \frac{w'' \omega_c^2 H}{\Phi^2}, \quad (3.4)$$

where y_0 is the height of the roller centre above the X_m -axis at the start of the motion segment (rise or return),

H is the total follower lift corresponding to one motion segment (in this Data Item it is taken to be a positive scalar quantity for both rise and return motion segments),

Φ is the total cam angle corresponding to one motion segment (note that units are radians),

ω_c is the angular velocity of the cam

and w, w', w'' are the dimensionless follower displacement, velocity and acceleration, respectively (expressions for these parameters corresponding to commonly-used cam laws are given in ESDU 82006 (Reference 5)).

Note that

(a) w and w' are positive during a rise motion and negative during a return motion

and (b) w'' changes sign during every motion segment (it is positive during the first part of a rise segment and negative during the first part of a return segment).

The spring force is composed of the preload, P_{SP} , with the product of the spring rate, k_{SC} , and the follower displacement from inner dwell ($y_m - y_b$), and is given by

$$P_S = P_{SP} + k_{SC} (y_m - y_b), \quad (3.5)$$

$$\text{where} \quad y_b = \left((R_b + R_r)^2 - E^2 \right)^{1/2}. \quad (3.6)$$

It is assumed that the mass-moment of inertia of the rotating parts of the roller is negligible which implies that the torque to accelerate the outer race and the rolling elements is negligible, and so only two forces act on the roller. These forces must be of equal magnitude and opposite in direction. Therefore, for equilibrium of the roller

$$P_{23} + P_{43} = 0 \quad (3.7)$$

and the force exerted on the roller by the cam is given by

$$\begin{aligned} P_{23} &= -P_{43} \\ &= \frac{P_{y34}}{\cos \alpha} , \end{aligned} \quad (3.8)$$

where α is the pressure angle (see ESDU ME2, Derivation 2) given by

$$\alpha = \tan^{-1} \left[\frac{\left(\frac{w'H}{\Phi} \right) - E}{y_m} \right] . \quad (3.9)$$

Note that for a rise segment w' is positive and for a return segment w' is negative (see ESDU ME2, Derivation 2).

Substituting Equation (3.1) into Equation (3.8) gives

$$P_{23} = \frac{(-P_{y54} - F_{y4} + m_f \ddot{y})}{\cos \alpha} . \quad (3.10)$$

The external force, F_E , and the spring force, P_S , are considered as positive (by convention) when they act to increase the contact force on the cam. So

$$F_E = -F_{y4} \quad (3.11)$$

and
$$P_S = -P_{y54} . \quad (3.12)$$

Thus the expression for contact force, as used in the program, is obtained by first substituting Equations (3.11) and (3.12) in Equation (3.10) and then substituting Equations (3.5) into the result to give

$$P_C = P_{23} = \frac{\left(P_{SP} + k_{SC} (y_m - y_b) \right) + F_E + m_f \ddot{y}}{\cos \alpha} . \quad (3.13)$$

Contact force is calculated in the computer program (see Section 7) at intervals of one degree of cam angle during one full rotation of the cam.

3.1.2 Radius of cam profile curvature

The radius of curvature of the cam profile is required for the calculation of contact stress. The derivation of an expression for this quantity is given in ESDU ME2 (Derivation 2) where the radius of curvature of the pitch curve for a cam driving a translating roller follower is given by

$$\rho = \frac{y_m^2}{\frac{H}{\Phi} \cos^2 \alpha \left(w' \sin \alpha - \frac{w''}{\Phi} \cos \alpha \right) + y_m \cos \alpha} \quad (3.14)$$

and the radius of curvature of the cam profile is given by

$$\rho_p = \rho - R_r . \quad (3.15)$$

This quantity is determined in the computer program at intervals of one degree of cam angle during one full rotation of the cam. The determination of contact stress is presented in Section 3.3.

3.2 Swinging Roller Follower

Sketch 3.2(a) shows a typical sub-system composed of a disc cam driving a swinging roller follower. A torsion spring is used to provide force closure. The convention used is shown in Sketch 3.2(b). The equations in this section are obtained assuming that length D is positive and angle ψ_m does not exceed 180 degrees. When D is negative some sign changes are necessary and these are accounted for in the computer program. Diagrams showing the forces and moments acting on each part of the assembly are shown in Sketch 3.2(c).

3.2.1 Derivation of contact force

It is required to derive an expression for the contact force, P_C , between the roller and the cam due to the action of the spring, the inertia of the follower system and the external moment acting on the follower. The follower system is constrained to rotate about a pivot and it is assumed that friction torque at the pivot is zero. However, in practice friction can be included by considering it as part of the external moment, T_4 . The sum of the moments (anticlockwise positive) acting on the follower about the pivot is equal to the product of the mass-moment of inertia of the follower (including the roller) and the angular acceleration of the follower about the pivot. Thus for equilibrium of the follower

$$- P_{34}L \cos \alpha + T_{54} + T_4 = I_f \ddot{\psi} , \quad (3.16)$$

where P_{34} is the force exerted by the roller on the follower in the direction of the contact normal,
 T_{54} is the moment exerted by the spring on the follower,
 T_4 is the external moment exerted on the follower,
 I_f is the mass-moment of inertia of the follower system about the follower pivot
 and L is the distance between the follower pivot and the roller centre.

The instantaneous motion (rotational displacement, ψ , rotational velocity, $\dot{\psi}$ and rotational acceleration, $\ddot{\psi}$) of the follower during a rise or return segment is required.

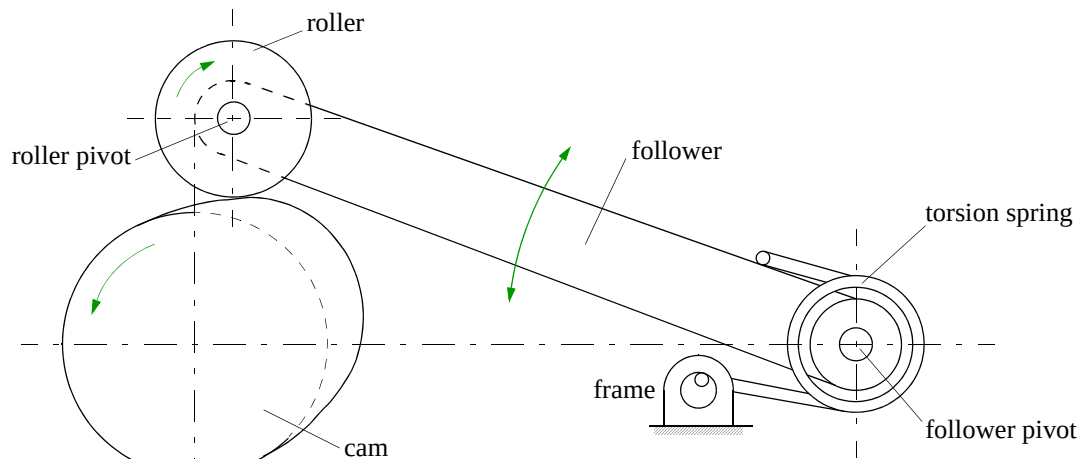
These quantities are given (for the configuration in Sketch 3.2(b)) by

$$\psi = - w \Psi , \quad (3.17a)$$

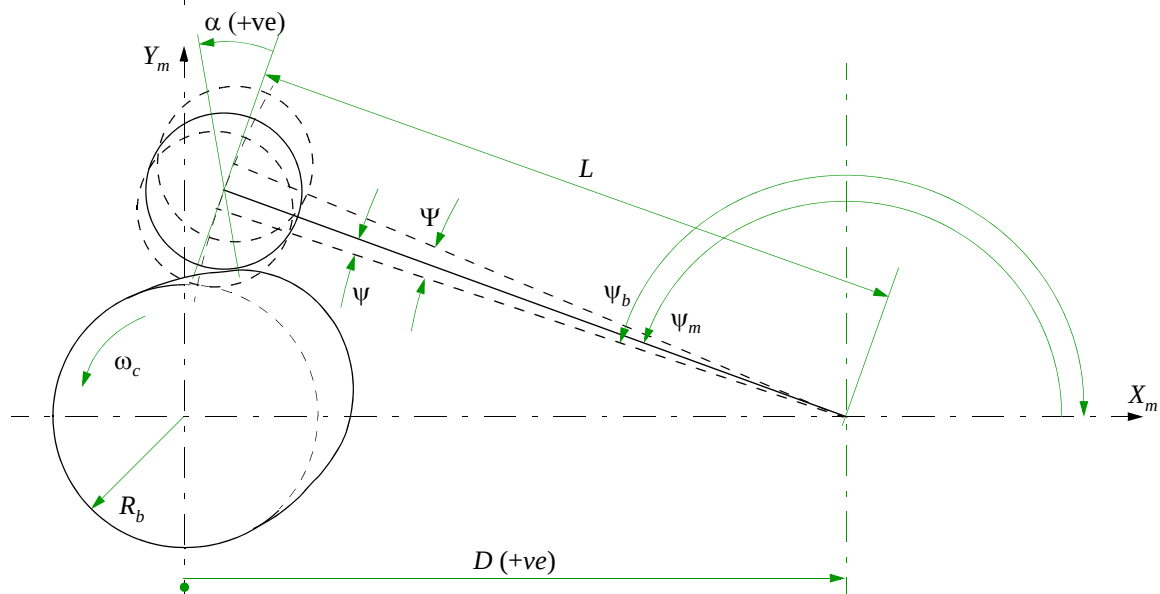
$$\psi_m = \psi_0 - w \Psi , \quad (3.17b)$$

$$\dot{\psi} = \frac{w' \omega_c \Psi}{\Phi} \quad (3.18)$$

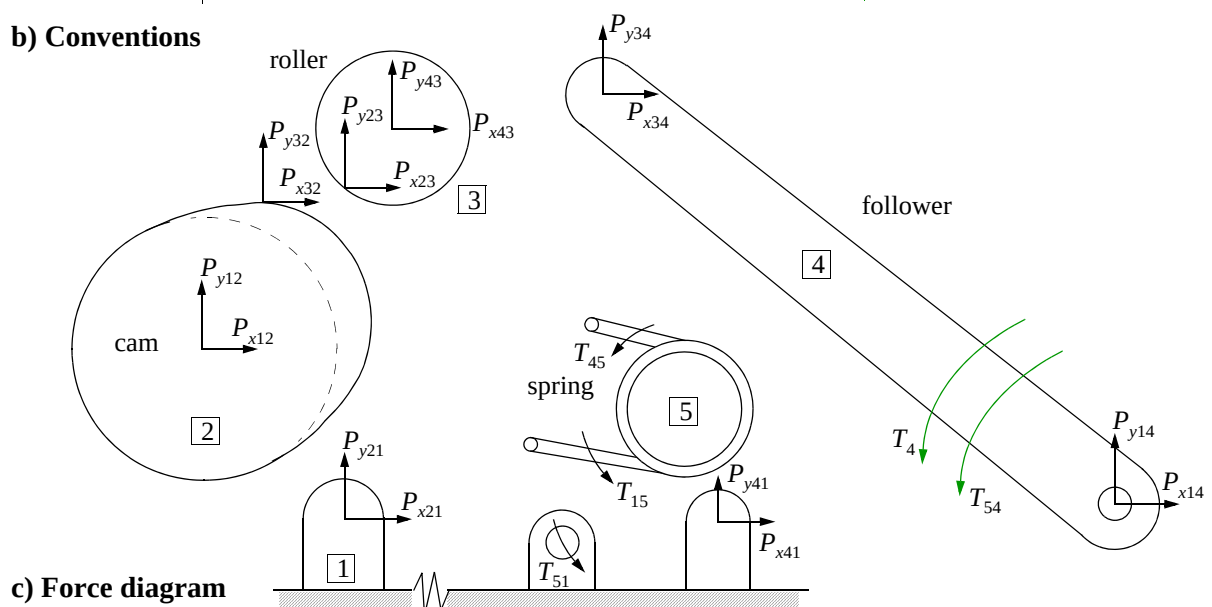
$$\text{and} \quad \ddot{\psi} = \frac{w'' \omega_c^2 \Psi}{\Phi^2} , \quad (3.19)$$



a) Assembly



b) Conventions



c) Force diagram

Sketch 3.2 Cam and follower system

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where	ψ_o	is the angle between the X_m -axis and the line joining the follower pivot to the roller centre at the beginning of a segment,
	Ψ	is the total follower swing corresponding to one motion segment (in this Data Item it is taken to be a positive scalar quantity for both rise and return motion segments),
	Φ	is the total cam angle corresponding to one motion segment (note that units are in radians),
	ω_c	is the angular velocity of the cam
and	w, w', w''	are the dimensionless follower displacement, velocity and acceleration, respectively (expressions for these parameters corresponding to commonly-used cam laws are given in ESDU 82006 (Reference 5)).

Note that:

- (a) w and w' are positive during a rise motion and negative during a return motion
- and (b) w'' changes sign during every motion segment (it is positive during the first part of a rise segment and negative during the first part of a return segment).

Pressure angle is given by (see ESDU ME2, Derivation 2)

$$\alpha = \tan^{-1} \left[\frac{L \left(1 + \frac{w' \Psi}{\Phi} \right) + D \cos \psi_m}{D \sin \psi_m} \right]. \quad (3.20)$$

The spring moment is composed of the preload, T_{SP} , with the product of the spring rate, k_{ST} , and the follower displacement from inner dwell, and is given by

$$T_S = T_{SP} + k_{ST} (\psi_b - \psi_m), \quad (3.21)$$

$$\text{where} \quad \psi_b = \pi - \cos^{-1} \left(\frac{L^2 + D^2 - (R_b + R_r)^2}{2LD} \right). \quad (3.22)$$

It is assumed that the mass-moment of inertia of the rotating parts of the roller about its centre is negligible which implies that the torque to accelerate the outer race and the rolling elements is negligible, and so only two forces are considered to act on the roller. These forces must be of equal magnitude and opposite in direction.

Therefore, for equilibrium of the roller

$$P_{23} + P_{43} = 0 \quad (3.23)$$

and the force exerted on the roller by the cam is given by

$$P_{23} = -P_{43} = P_{34}. \quad (3.24)$$

Substituting Equation (3.16) into Equation (3.24) gives

$$P_{23} = \frac{T_{54} + T_4 - I_f \ddot{\Psi}}{L \cos \alpha}. \quad (3.25)$$

The external moment, T_E , and the spring moment, T_S , are considered as positive (by convention) when they act to increase contact force on the cam. So

$$T_E = T_4 \quad (3.26)$$

and
$$T_S = T_{54}. \quad (3.27)$$

Thus the expression for contact force, as used in the program, is obtained by substituting Equations (3.21) and (3.26) in Equation (3.25) to give

$$P_C = P_{23} = \frac{(T_{SP} + k_{ST}(\psi_b - \psi_m)) + T_E - I_f \ddot{\Psi}}{L \cos \alpha}. \quad (3.28)$$

Contact force is calculated in the computer program at intervals of one degree of cam angle during one full rotation of the cam.

3.2.2 Radius of cam profile curvature

An expression for the radius of curvature of the cam profile is required for the calculation of contact stress. The derivation of an expression for this quantity (assuming D is positive) is given in ESDU ME2 (Derivation 2) where the radius of curvature of the pitch curve for a cam driving a swinging roller follower is given by

$$\rho = \frac{-D^2 \left(1 - w' \frac{\Psi}{\Phi}\right) \frac{\sin^2 \psi}{\cos^2 \alpha}}{w' \frac{\Psi}{\Phi} L \left(1 - w' \frac{\Psi}{\Phi}\right)^2 \sin \alpha + D \frac{\Psi}{\Phi^2} w'' \cos(\alpha + \psi) - D \left(1 - w' \frac{\Psi}{\Phi}\right) \frac{\sin \psi}{\cos \alpha}} \quad (3.29)$$

and the radius of curvature of the cam profile is given by

$$\rho_p = \rho - R_r. \quad (3.30)$$

This quantity is determined in the computer program at intervals of one degree of cam angle during one full rotation of the cam. The determination of contact stress is presented in Section 3.3.

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3.3 Evaluation of Contact Stress

This section comprises a statement of the equations used to calculate values of maximum compressive contact stress in the computer program (see Section 7). The approach used to determine these values for an elliptical contact zone is that described in Table 9.4 of ESDU 78035 (Derivation 1) while for a rectangular contact zone the relevant equations are given in Table 9.5 of the same Data Item. The equations are simplified because of the relatively simple geometry of the contact zone between a cam and roller and the notation used is that of this Item. Derivations of the following equations are not given in this Item.

The first step is to evaluate the body geometry parameters, **A** and **B**

$$\text{If } \frac{1}{2R_c} < \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_r} \right) \text{ then } \mathbf{A} = \frac{1}{2R_c} \text{ and } \mathbf{B} = \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_r} \right) \quad (3.31)$$

$$\text{or if } \frac{1}{2R_c} \geq \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_r} \right) \text{ then } \mathbf{A} = \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_r} \right) \text{ and } \mathbf{B} = \frac{1}{2R_c} . \quad (3.32)$$

The material constants, k_c and k_r , for the cam and roller respectively, are given by

$$k_c = \frac{(1 - \sigma_c^2)}{\pi E_c} \quad (3.33)$$

$$\text{and } k_r = \frac{(1 - \sigma_r^2)}{\pi E_r} . \quad (3.34)$$

For an elliptical contact region (where $\mathbf{A/B} \geq 2.0 \times 10^{-6}$) the semi-axis ratio of the contact zone is given by

$$\beta \left\{ \begin{array}{ll} = \left(\frac{\mathbf{A}}{\mathbf{B}} \right)^{1/2} \left(0.555 \log_e \left(\frac{\mathbf{B}}{\mathbf{A}} \right) + 0.755 \right)^{-1/2} & \text{if } \mathbf{A/B} < 0.155 \\ = \left(\frac{\mathbf{A}}{\mathbf{B}} \right)^{2/3} \left(1 + \frac{1}{324} \left(\log_e \left(\frac{\mathbf{B}}{\mathbf{A}} \right) \right)^3 \right) & \text{if } \mathbf{A/B} \geq 0.155 \end{array} \right. \quad (3.35a)$$

$$(3.35b)$$

and the intermediate calculation parameters (corresponding to the expressions for elliptical integrals in ESDU 78035, Derivation 1) are given by

$$Z_1 = 1 + \left(\frac{\mathbf{A}\pi}{\mathbf{B}2} - 1 \right) \quad (3.36)$$

$$\text{and } Z_2 = \frac{\pi}{2} \left(1 + 0.36 \log_e \left(\frac{\mathbf{B}}{\mathbf{A}} \right) \right) . \quad (3.37)$$

A further intermediate calculation parameter is given by

$$Z_3 = \left(\frac{3P_C \pi (k_c + k_r) (\mathbf{A} + \mathbf{B})^2}{4} \right)^{1/3} \quad (3.38)$$

The maximum compressive surface contact stress is given by

$$(f_z)_{max} = - \left(\frac{\pi^2 \beta}{4Z_1^2} \right)^{1/3} \left(\frac{2Z_3}{\pi^2 (k_c + k_r)} \right) \quad (3.39)$$

and the normal approach between the cam and roller is given by

$$\delta = Z_2 \left(\frac{4\beta^2}{\pi^2 Z_1^2} \right)^{1/3} \frac{Z_3^2}{(\mathbf{A} + \mathbf{B})} \quad (3.40)$$

The contact ellipse major semi-axis is given by

$$a = \left(\frac{2Z_1}{\pi\beta^2} \right)^{1/3} \frac{Z_3}{(\mathbf{A} + \mathbf{B})} \quad (3.41)$$

and the minor semi-axis is given by

$$b = a \beta \quad (3.42)$$

If $\mathbf{A}/\mathbf{B} < 2.0 \times 10^{-6}$ then the contact zone is regarded as rectangular and Equations (3.43) to (3.45) are used. The maximum compressive contact stress is given by

$$(f_z)_{max} = - \left(\frac{P_C}{W \pi^2 (k_c + k_r)} \left(\frac{\rho_p + R_r}{\rho_p R_r} \right) \right)^{1/2} \quad (3.43)$$

where W is the smaller of the cam facewidth W_c and the effective width of the roller W_r . The normal approach between the cam and roller is given by

$$\delta = \frac{2P_C k_c}{W} \left(\log_e \frac{4\rho_p}{b} - 0.5 \right) + \frac{2P_C k_r}{W} \left(\log_e \frac{4R_r}{b} - 0.5 \right) \quad (3.44)$$

where half of the width of the contact zone is given by

$$b = \left(\frac{4P_C(k_c + k_r)}{W} \left(\frac{\rho_p + R_r}{\rho_p R_r} \right) \right)^{1/2} . \quad (3.45)$$

The computer program implements an algorithm using the above equations at intervals of one degree of cam angle and the maximum value of contact stress during one complete rotation of the cam is determined.

3.4 Determination of Suitable Spring Rate and Preload

The prime function of the spring is to maintain contact between cam and roller throughout the motion. For the purposes of this Item it is sufficient to characterise a spring by values of spring rate and preload that ensure contact throughout one rotation of the cam under the worst combination of operating conditions. In many practical design situations these appropriate values of spring rate and preload are unknown. The program contains an option that allows the determination of values of spring rate and preload that will ensure positive contact force throughout the cycle. The way that these values are determined is described in this section.

To ensure contact for both translating followers and swinging followers, the condition

$$P_C > 0 \quad (3.46)$$

must hold at every calculated cam position throughout the cycle. For translating followers substituting Equation (3.13) into Expression (3.46) gives

$$\frac{(P_{SP} + k_{SC}(y_m - y_b)) + F_E + m_f \ddot{y}}{\cos \alpha} > 0 . \quad (3.47)$$

Since $\cos \alpha$ is always positive this condition becomes

$$(P_{SP} + k_{SC}(y_m - y_b)) + F_E + m_f \ddot{y} > 0 . \quad (3.48)$$

The spring force $(P_{SP} + k_{SC}(y_m - y_b))$ must exceed the separating force $(-F_E - m_f \ddot{y})$. The program finds the cam angle, ϕ_q , at which the maximum separating force occurs and the corresponding follower motion at this position is defined by y_q , $\dot{y}_q$ and $\ddot{y}_q$ while the value of the external force is F_{Eq} . A user-specified factor of safety known as the force factor, z_1 , is then introduced and the inequality of Expression (3.48) becomes

$$(P_{SP} + k_{SC}(y_{mq} - y_b)) + z_1(F_{Eq} + m_f \ddot{y}_q) = 0 . \quad (3.49)$$

The force factor is then the ratio of spring force to separating force defined at the cam angle ϕ_q , which is the point on the cam when the resultant force is a minimum. Since the spring force must be greater than the separating force for the follower to remain in contact with the cam, z_1 must be chosen to be greater than unity (see Section 4.1.2). Often the worst case of this minimum occurs when the cam is running under no external load conditions and the follower reaches a value of maximum acceleration (see example shown in Sketch 3.3).