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Blending profiles of disc cams with radial translating roller followers. Part 2: to incorporate an intermediate precision point or a constant velocity component

**Associated software: ESDUpac A9302
B9302
VIEWpac 9302A
9302B**

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BLENDING PROFILES OF DISC CAMS WITH RADIAL TRANSLATING ROLLER FOLLOWERS. Part 2: to incorporate an intermediate precision point or a constant velocity component

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BLENDING PROFILES OF DISC CAMS WITH RADIAL TRANSLATING ROLLER FOLLOWERS. Part 2: to incorporate an intermediate precision point or a constant velocity component

1. NOTATION

The units used in this Item are predominantly SI but any coherent system of units may be used. All angles are measured positively in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

		Units
H	total follower lift corresponding to one segment	m
k_B	blending ratio	—
O	cam centre of rotation and origin of co-ordinate axis system	—
R_a	prime circle radius	m
R_b	reference circle radius	m
R_g	cutter (or grinder) radius	m
R_r	roller radius	m
u	normalised cam angle	—
w	normalised follower displacement	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
X_c, Y_c	axes fixed in, and rotating with, cam with origin at point O	—
X_m, Y_m	axes fixed in machine, with origin at point O	—
y	translational displacement of follower	m
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
y_m	instantaneous height of centre of roller above X_m -axis	m
y_0	height of centre of roller above X_m -axis at inner dwell of segment	m

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Z	intermediate calculation parameter (see Equation (3.18))	–
α	pressure angle	degree
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m
Φ	total angle of cam rotation corresponding to one segment	degree
ϕ	intermediate angle of cam rotation from inner dwell	degree
ϕ_0	angle of cam rotation at inner dwell	degree
ω_c	angular velocity of cam	rad/s

Subscripts

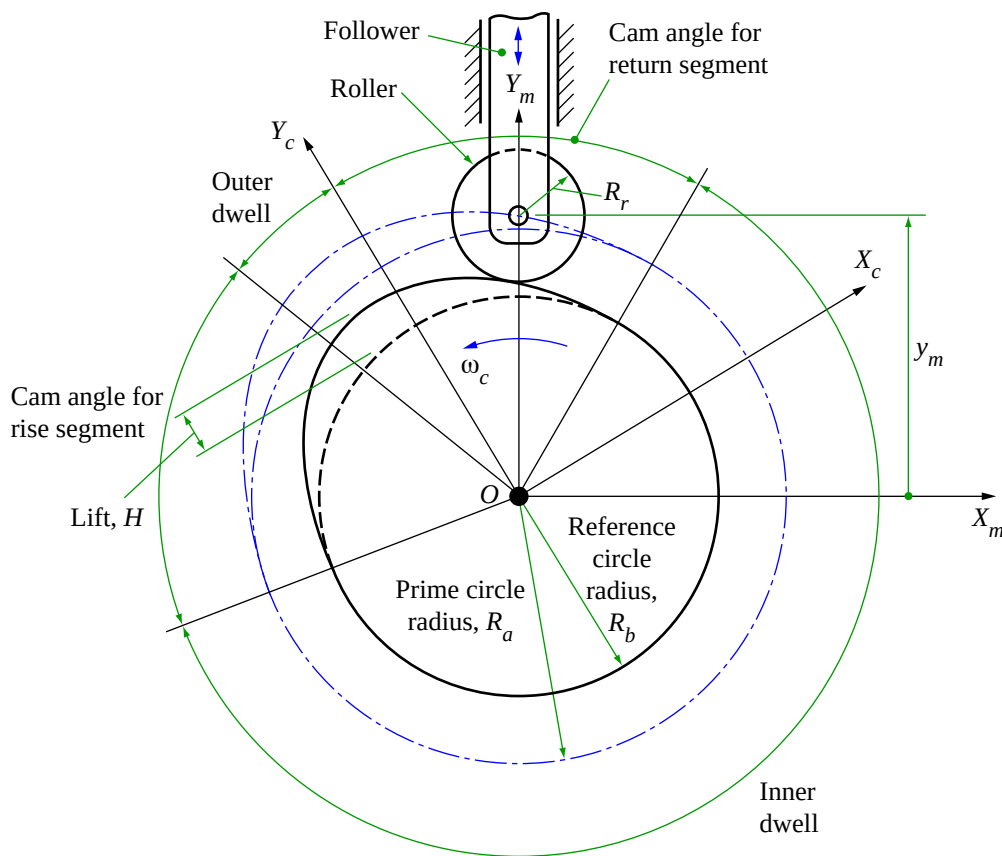
B	refers to blended segment
cv	refers to finish of constant velocity component
F	refers to finish of constant velocity component
max	indicates maximum magnitude
min	indicates minimum magnitude
$N1$	refers to notional dimension of inner component
$N2$	refers to notional dimension of outer component
pp	refers to precision point
S	refers to start of constant velocity component
$V1$	refers to normalised location of maximum follower velocity (w.r.t. inner component)
$V2$	refers to normalised location of maximum follower velocity (w.r.t. outer component)

2. INTRODUCTION

This Data Item presents procedures for designing blended cam profiles such that the resultant follower motion includes either an intermediate displacement precision point or an intermediate constant velocity component. Blended segments are formed by combining portions of commonly-used cam laws at the point of peak follower velocity such that there is continuity of follower displacement, velocity and acceleration. This principle is generally valid but the procedures presented here are directly applicable only to cams that drive a radial translating roller follower (see Sketch 2.1).

Occasionally, as part of a motion specification, a cam follower may be required to have a particular displacement at a specified intermediate cam angle (see Sketch 2.2(a)). It may be possible to satisfy such a displacement precision point to sufficient accuracy using a motion segment defined by a single cam law. However if this is not the case then the requirement can usually be met using a cam profile defined by blending together two unequal components of standard cam laws. The design procedure presented in this Item determines the blending ratio and the notional dimensions[†] of each component of the required blended segment.

Similarly the follower may be required to move with constant velocity during part of a rise or a return motion segment (see Sketch 2.2(b)) while the follower motion on each side of the constant velocity section is governed by components of standard cam laws. It is often advantageous for these components to be unequal and the design procedure described in this Item determines suitable notional dimensions for each component of the required blended segment.

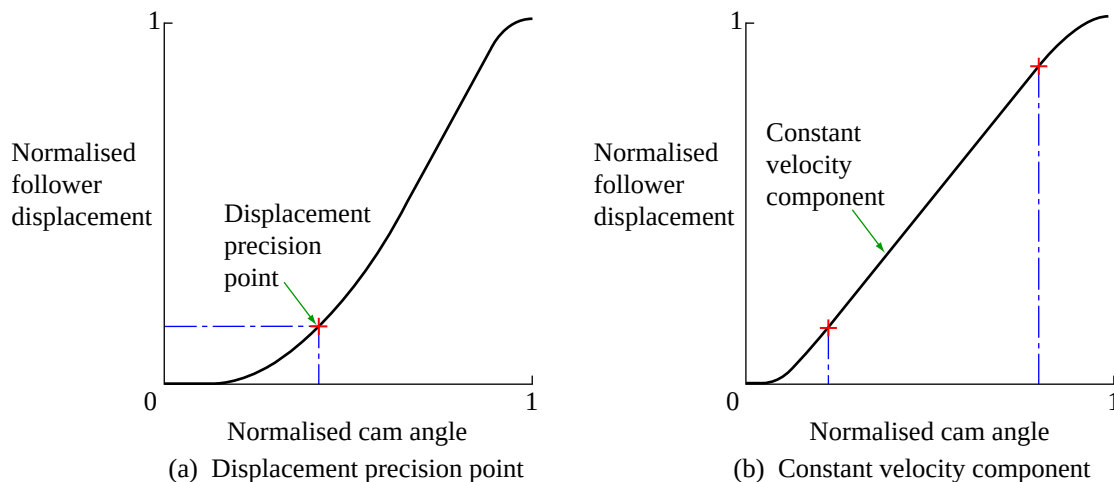


Sketch 2.1 Disc cam driving radial translating roller follower

[†] Notional dimensions are defined in Section 3.1.3.

One advantage of using portions of standard cam laws is that the minimum allowable size of the cam can be found without difficulty. The proposed methods may be implemented by hand, using design data contained in ESDU 92005 (Reference 8), but it is recommended that all calculations are carried out using one of the computer programs[†] supplied with the current Item, which can cover a wider range of possible designs. The programs include checks to ensure that the pressure angle of the resultant blended profile does not exceed a specified maximum value and that profile curvature is adequate for correct action of both the roller follower and the cutter used to manufacture the cam. The programs also calculate the follower motion (displacement, velocity, acceleration and jerk) throughout the blended segment at intervals of one degree of cam rotation.

ESDU 83027 (see Reference 5) presents alternative blending methods, based on hand calculation using a blend factor, for determining suitable profiles incorporating a precision point or a constant velocity component. These are applicable to cams driving translating or swinging followers but there is no direct means for finding the minimum allowable size of the resultant profiles. The computer-based methods presented in this Item are complementary to those shown in ESDU 83027 but are solely applicable to cams with radial translating roller followers.



Sketch 2.2 Special motion requirements

2.1 Reasons for Blending

It is not always possible to satisfy the follower motion requirements using single-law motion segments. The follower may be required to pass through a specified location at a given angle of cam rotation or a period of constant follower velocity may be required as an important function of the cam mechanism. In these cases it is worthwhile attempting to satisfy the specification using a blended profile.

It may also be advantageous to blend cam profiles for the following reasons (not dealt with directly in this Item).

- (a) An asymmetric motion segment may be necessary to reduce the peak follower acceleration towards the cam and hence to reduce the spring force required to maintain contact. In this case the cam must have a larger reference circle radius than that required for the corresponding symmetrical motion.

[†] ESDUpac A9302 relates to the calculation of notional dimensions of a blended profile with an intermediate displacement precision point and ESDUpac B9302 relates to the calculation of notional dimensions of a blended profile with an intermediate constant velocity component.

- (b) The segment angle required for the follower to reach a given lift (for a cam with specified values of reference circle radius and maximum allowable pressure angle) can be minimised by blending (see ESDU 92014 – Reference 9).
- (c) The minimum allowable reference circle radius required for a cam with specified segment angle, follower lift and maximum allowable pressure angle can be reduced by blending (see ESDU 92014 – Reference 9).
- (d) The maximum allowable follower lift that can be obtained with specified segment angle, reference circle radius and maximum allowable pressure angle can be increased by blending (see ESDU 92014 – Reference 9).

2.2 Use of this Data Item

The theoretical background to the blending methods, including definitions and terminology, is explained in Section 3. Calculation procedures are described in Section 4 and their use is illustrated by a series of examples in Section 7. It is strongly recommended that the calculations are performed using the computer programs (ESDUpacs A9302 and B9302) supplied with this Item. See Section 5 and Section 8 for information regarding the use of these programs, which includes construction of program input, interpretation of program output and guidance on the design of blended profiles.

3. THEORETICAL BACKGROUND

3.1 Definitions and Terminology

3.1.1 Motion segments and single-law motion

For the purposes of this Data Item a motion segment is defined as that part of the cam motion cycle during which the follower moves from a period of zero velocity (possibly a dwell) to another period of zero velocity. A motion segment may be described as dwell-rise-dwell (DRD), dwell-return-dwell (DRD) or dwell-rise-return-dwell (DRRD). A DRRD motion segment can only be dealt with directly, using this Item, if it can be treated as two DRD motion segments joined together such that there is continuity of follower displacement, velocity and acceleration. The most common way to specify follower motion, and hence determine the cam profile, during rise or return motion is to use one of a recommended series of equations, known as cam laws, relating cam angle of rotation to follower motion (see ESDU 82006 – Reference 4). To define a single-law DRD motion segment, that is, one governed by a single cam law, it is only necessary to specify the cam law, the total follower lift and the total segment angle. A typical non-dimensional follower motion (displacement, velocity and acceleration) corresponding to a symmetrical single-law motion segment is shown in Sketch 3.1.

3.1.2 Dimensionless form of displacement, velocity and acceleration

Sketch 2.1 shows two pairs of co-ordinate axes having origins coincident with the centre of cam rotation. The X_m -, Y_m -axes are fixed to the machine frame whilst the X_c -, Y_c -axes are fixed to the cam and rotate with it. This Item is concerned only with the common case in which the follower path coincides with the Y_m -axis, i.e. there is no offset.

A cam law is the mathematical function defining the normalised follower displacement, w , during a single motion segment in terms of normalised cam rotation, u . It is conveniently expressed as

$$W = f(u). \quad (3.1)$$

This function is evaluated readily by substituting the equation for the relevant cam law. In this Item the cam laws are identified by their generic names, for example Cycloidal or Modified Sinusoidal Acceleration. The equations of motion for commonly-used cam laws are given as formulae and graphically in ESDU 82006 (see Reference 4). For a disc cam driving a radial translating follower (see Sketch 2.1), the normalised cam rotation and normalised follower displacement for each motion segment are given by

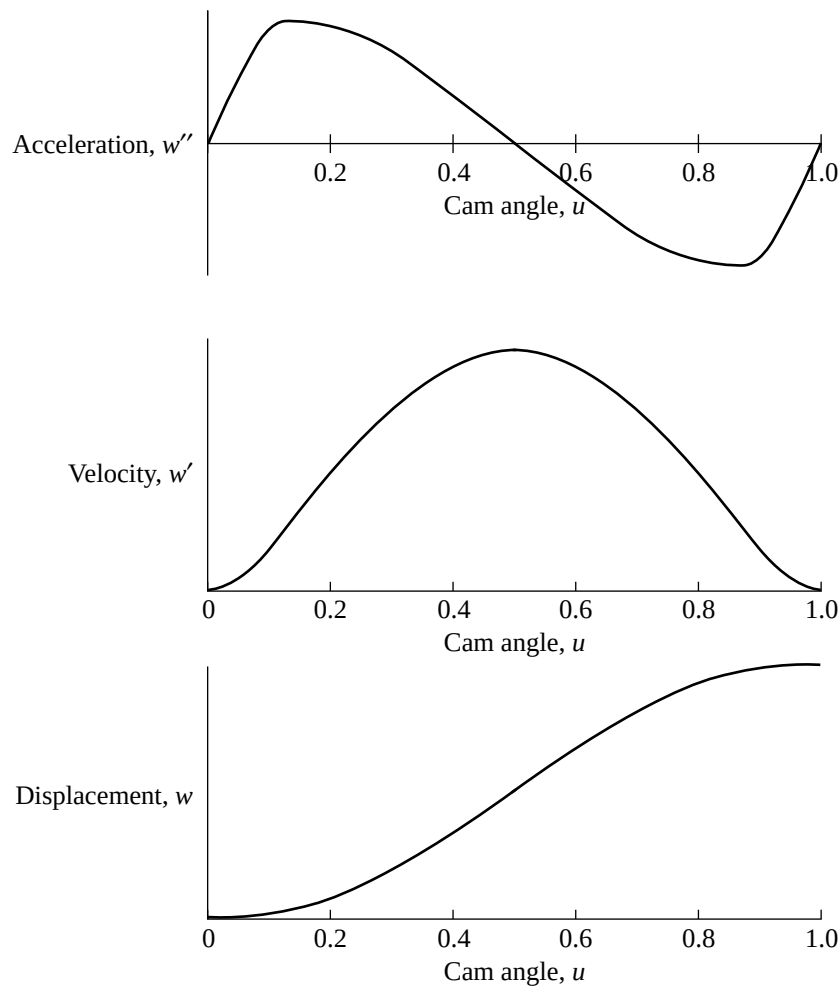
$$u = (\phi - \phi_0)/\Phi \quad (3.2)$$

$$\text{and} \quad w = (y_m - y_0)/H. \quad (3.3)$$

Making the usual assumption that the cam rotates at constant angular velocity, the dimensionless forms of the follower velocity and acceleration are defined as

$$w' = \frac{\dot{y}}{\omega_c H} \left\{ \frac{\pi \Phi}{180} \right\} \quad (3.4)$$

$$\text{and} \quad w'' = \frac{\ddot{y}}{\omega_c^2 H} \left\{ \frac{\pi \Phi}{180} \right\}^2. \quad (3.5)$$



Sketch 3.1 Non-dimensional follower motion for a typical single-law DRD segment

3.1.3 Blended motion and notional dimensions

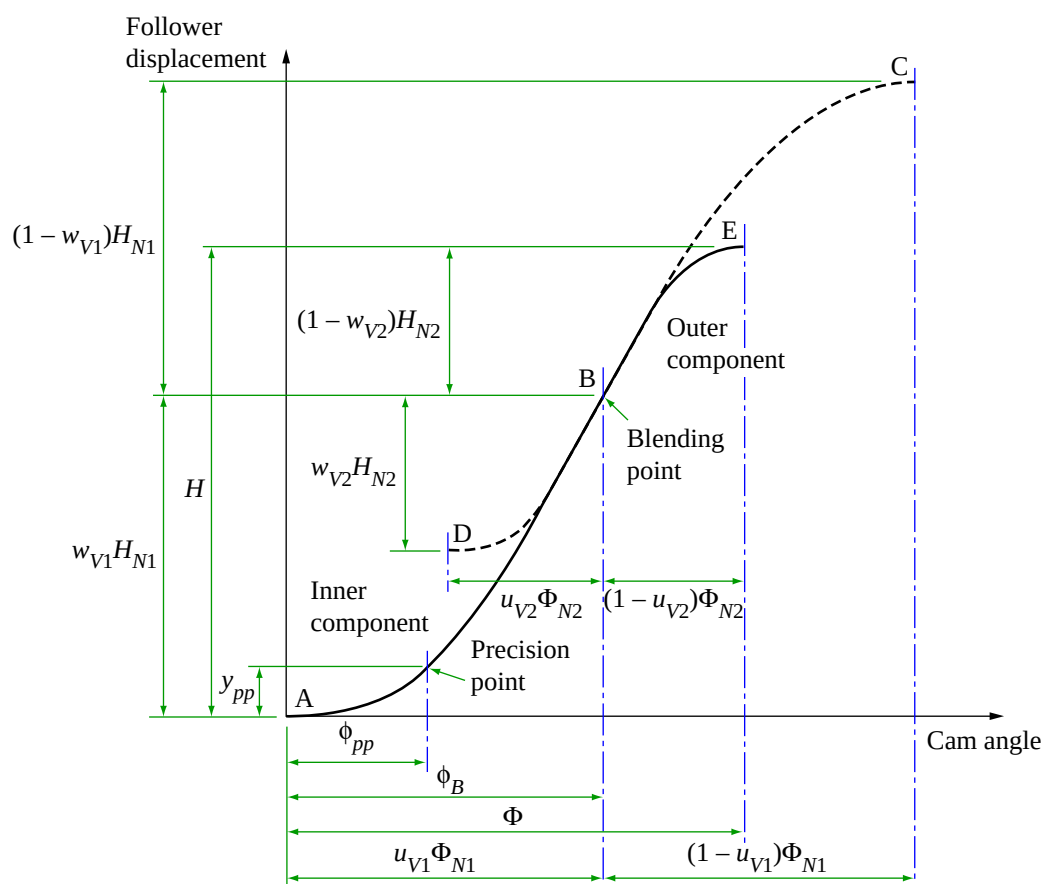
This Item considers the use of motion segments formed by blending the inner component of one single-law segment[†] with the outer component of another single-law segment at the point of maximum follower velocity (see Sketches 3.2 and 3.4 for rise motion and Sketches 3.3 and 3.5 for return motion). These components are characterised by notional dimensions that are distinguished by the suffixes $N1$ and $N2$. Component AB is the inner part of a single-law motion, ABC, having a notional segment angle, Φ_{N1} , and notional follower lift, H_{N1} , between the limits $0 \leq u_{N1} \leq u_{V1}$, where u_{V1} is the normalised cam displacement at the position of maximum follower velocity. Component BE is the outer part of a different single-law motion (not necessarily the same cam law), DBE, having a notional segment angle, Φ_{N2} , and notional follower lift, H_{N2} , between the limits $u_{V2} \leq u_{N2} \leq 1$, where u_{V2} is the normalised cam displacement at the position of maximum follower velocity. The notional prime circle radius corresponding

[†] The equations presented in this section are general in the sense that the inner and/or outer component of a blended segment can be formed using an asymmetrical cam law. However the computer programs (ESDUpac A9302 and B9302) cater only for segments composed from the commonly-used symmetrical cam laws in which the maximum velocity occurs at mid-segment.

to Component AB is identical to that of the actual cam but the notional prime circle radius Component BE is given by

$$(R_a)_{N2} = (R_a)_{N1} + H - H_{N2}. \quad (3.6)$$

Appropriate notional dimensions must be used for all calculations involving Component BE.



Sketch 3.2 General form of follower motion for rise segment formed by blending two single-law motion segments

3.1.4 Proportions of blended profiles (with intermediate displacement precision point)

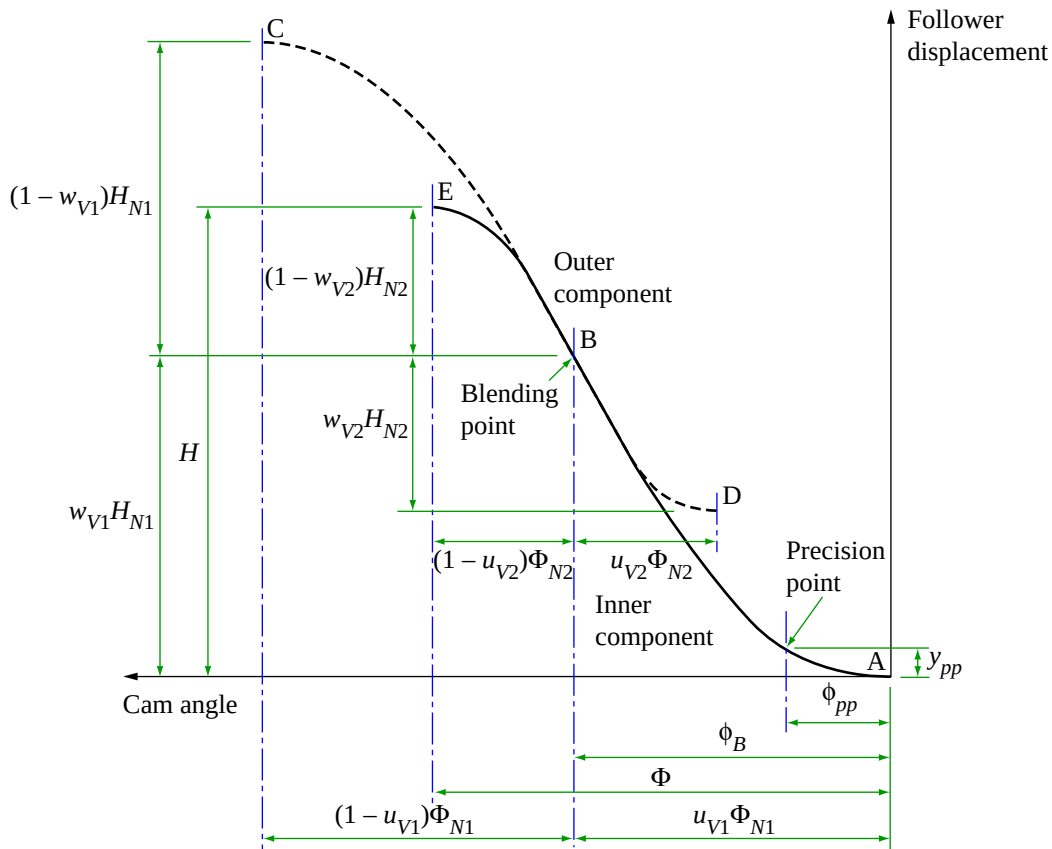
The procedure described in Section 4.1 of this Item requires that the blending point always coincides with the position of maximum follower velocity to ensure continuity of follower velocity and acceleration. The proportions of blended profiles in this Item are defined by a blending ratio given by

$$k_B = \frac{\phi_B}{\Phi}, \quad (3.7)$$

where ϕ_B is the cam angle at the blending point (measured from the beginning of a rise segment and from the end of a return segment as shown on Sketches 3.2 and 3.3).

For the purpose of blending, a return motion segment is considered as a mirror-image of a rise segment.

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Sketch 3.3 General form of follower motion for return segment formed by blending two single-law motion segments

Expressions for the notional dimensions of a blended motion segment, in terms of known parameters, are derived in this section. The two components of motion are blended together at the point of maximum follower velocity and so, equating the expressions (obtained by rearranging Equation (3.4)) for follower velocity at the blending point gives

$$\dot{y} = \frac{180(w'_{N1})_{max} \omega_c H_{N1}}{\pi \Phi_{N1}} = \frac{180(w'_{N2})_{max} \omega_c H_{N2}}{\pi \Phi_{N2}}, \quad (3.8)$$

which leads to

$$\frac{\Phi_{N1}}{\Phi_{N2}} = \frac{(w'_{N1})_{max} H_{N1}}{(w'_{N2})_{max} H_{N2}}. \quad (3.9)$$

Values of maximum non-dimensional follower velocity are given in Table 4.1 for commonly-used cam laws. By inspection of Sketch 3.2 or Sketch 3.3 it can be seen that

$$\Phi = u_{V1}\Phi_{N1} + (1 - u_{V2})\Phi_{N2} \quad (3.10)$$

$$\text{and} \quad H = w_{V1}H_{N1} + (1 - w_{V2})H_{N2}. \quad (3.11)$$

Note that for commonly used cam laws (including those give in the computer program)

$$u_{V1} = u_{V2} = w_{V1} = w_{V2} = 0.5. \quad (3.12)$$

Also, since

$$u_{V1}\Phi_{N1} = \phi_B, \quad (3.13)$$

the notational segment angle of the inner component is given, in terms of the blending ratio, by

$$\Phi_{N1} = \frac{\phi_B}{u_{V1}} = \frac{k_B\Phi}{u_{V1}}. \quad (3.14)$$

Similarly the notational segment angle of the outer component is given by

$$\Phi_{N2} = \frac{(1 - k_B)\Phi}{1 - u_{V2}}. \quad (3.15)$$

Also, by inspection of Sketches 3.2 and 3.3, the notional segment angle of the outer component is given by

$$\Phi_{N2} = \frac{\Phi - u_{V1}\Phi_{N1}}{1 - u_{V2}}. \quad (3.16)$$

Equation (3.9) can be rearranged to

$$H_{N1} = \frac{\Phi_{N1}(w'_{N2})_{max}H_{N2}}{\Phi_{N2}(w'_{N1})_{max}} = Z H_{N2}, \quad (3.17)$$

$$\text{where} \quad Z = \frac{\Phi_{N1}(w'_{N2})_{max}}{\Phi_{N2}(w'_{N1})_{max}} = \frac{k_B(1 - u_{V2})(w'_{N2})_{max}}{u_{V1}(1 - k_B)(w'_{N1})_{max}}. \quad (3.18)$$

Also, by inspection of Sketches 3.2 and 3.3, the notional lift of the outer component is given by

$$H_{N2} = \frac{H - w_{V1}H_{N1}}{(1 - w_{V2})}. \quad (3.19)$$

Substituting Equation (3.19) into Equation (3.17) gives

$$H_{N1} = \frac{Z(H - w_{V1}H_{N1})}{(1 - w_{V2})} \quad (3.20)$$

and so rearranging gives

$$H_{N1} = \frac{ZH}{(1 - w_{V2}) + Zw_{V1}} \quad (3.21)$$

In the design procedure (see Section 4.1) it is necessary to evaluate the follower displacement corresponding to a specified value of intermediate cam angle given the blending ratio, the notional dimensions and the cam law. From Sketches 3.2 and 3.3 it can be seen that if $\phi_{pp} < \phi_B$ the precision point is in the inner component and the normalised cam angle at that point, relative to the inner component, is given by

$$u_{pp} = \frac{\phi_{pp}}{\Phi_{N1}} \quad (3.22)$$

Using this value of normalised cam angle with the cam law for the inner component (see Reference 5) the normalised follower displacement at the precision point ($w_{pp} = f(u_{pp})$) can be evaluated. By inspection of Sketches 3.2 and 3.3 the follower displacement at the precision point is given by

$$y_{pp} = w_{pp}H_{N1} \quad (3.23)$$

However if $\phi_{pp} > \phi_B$ the precision point is in the outer component and the normalised cam angle at that point, relative to the start of the outer component, is given by

$$u_{pp} = \frac{u_{V2}\Phi_{N2} - u_{V1}\Phi_{N1} + \phi_{pp}}{\Phi_{N2}} \quad (3.24)$$

Using this value of normalised cam angle with the cam law for the outer component (see Reference 5) the normalised follower displacement at the precision point ($w_{pp} = f(u_{pp})$) can be evaluated. By observation of Sketches 3.2 and 3.3 the follower displacement at the precision point is given by

$$y_{pp} = w_{V1}H_{N1} - w_{V2}H_{N2} + w_{pp}H_{N2} \quad (3.25)$$

3.1.5 Proportions of blended profiles (with intermediate constant velocity component)

The design procedure described in this Item (see Section 4.2) requires that, during the constant velocity component, the follower velocity shall be the maximum thereby ensuring continuity of follower displacement, velocity and acceleration. A return motion is considered as the mirror-image of a rise motion

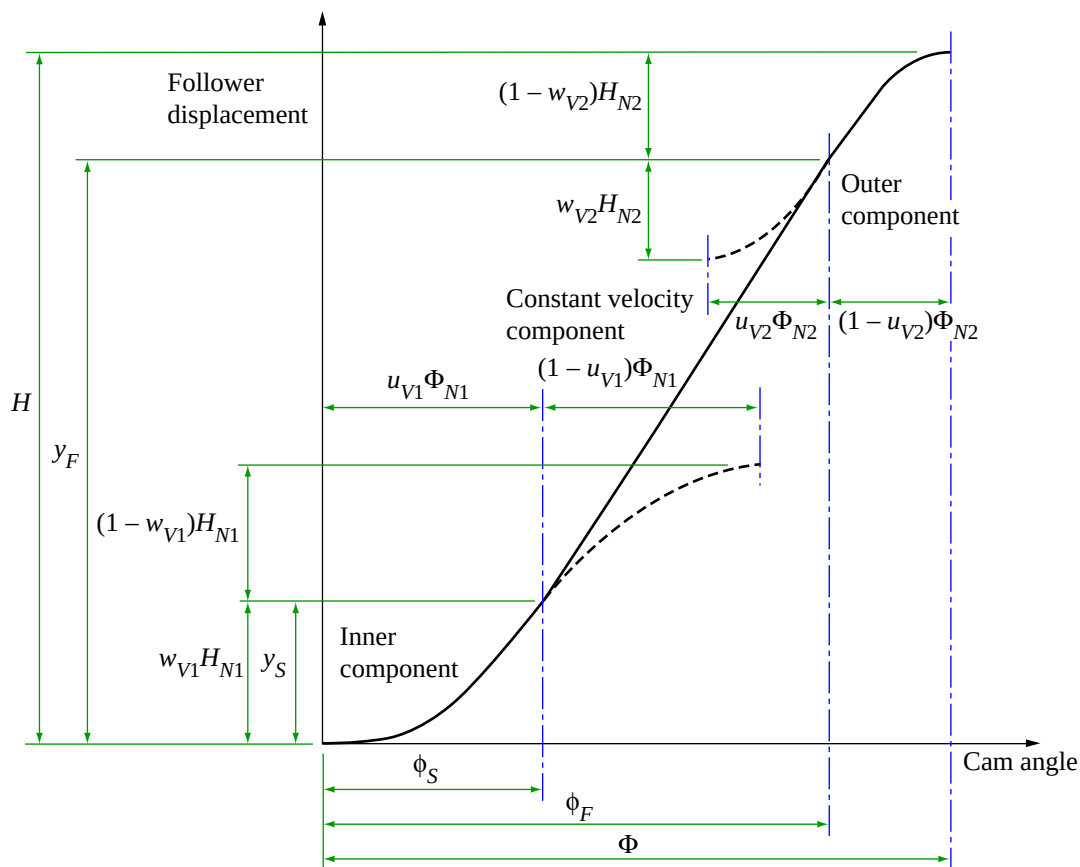
(see Sketches 3.4 and 3.5) and values of non-dimensional follower velocity are given in Table 4.1 for commonly-used cam laws. By inspection of Sketch 3.4 or Sketch 3.5 it can be seen that, if angles (Φ , ϕ_S and ϕ_F) are known, the notional cam angles for the inner and outer components are given by

$$\Phi_{N1} \left\{ \begin{array}{ll} = \frac{\phi_S}{u_{V1}} & \text{for a rise motion} \\ = \frac{(\Phi - \phi_F)}{u_{V1}} & \text{for a return motion} \end{array} \right. \quad (3.26a)$$

$$\Phi_{N2} \left\{ \begin{array}{ll} = \frac{(\Phi - \phi_F)}{(1 - u_{V2})} & \text{for a rise motion} \\ = \frac{\phi_S}{(1 - u_{V2})} & \text{for a return motion.} \end{array} \right. \quad (3.27a)$$

and

$$\Phi_{N2} \left\{ \begin{array}{ll} = \frac{\phi_S}{(1 - u_{V2})} & \text{for a return motion.} \end{array} \right. \quad (3.27b)$$



Sketch 3.4 General form of motion for rise segment incorporating constant velocity component

Note that for commonly-used cam laws (including those used in ESDUpac B9302)

$$u_{V1} = u_{V2} = w_{V1} = w_{V2} = 0.5.$$

At the blending point between the inner component and the constant velocity component the maximum non-dimensional follower velocity is given, by substituting the notional values in Equation (3.4), as

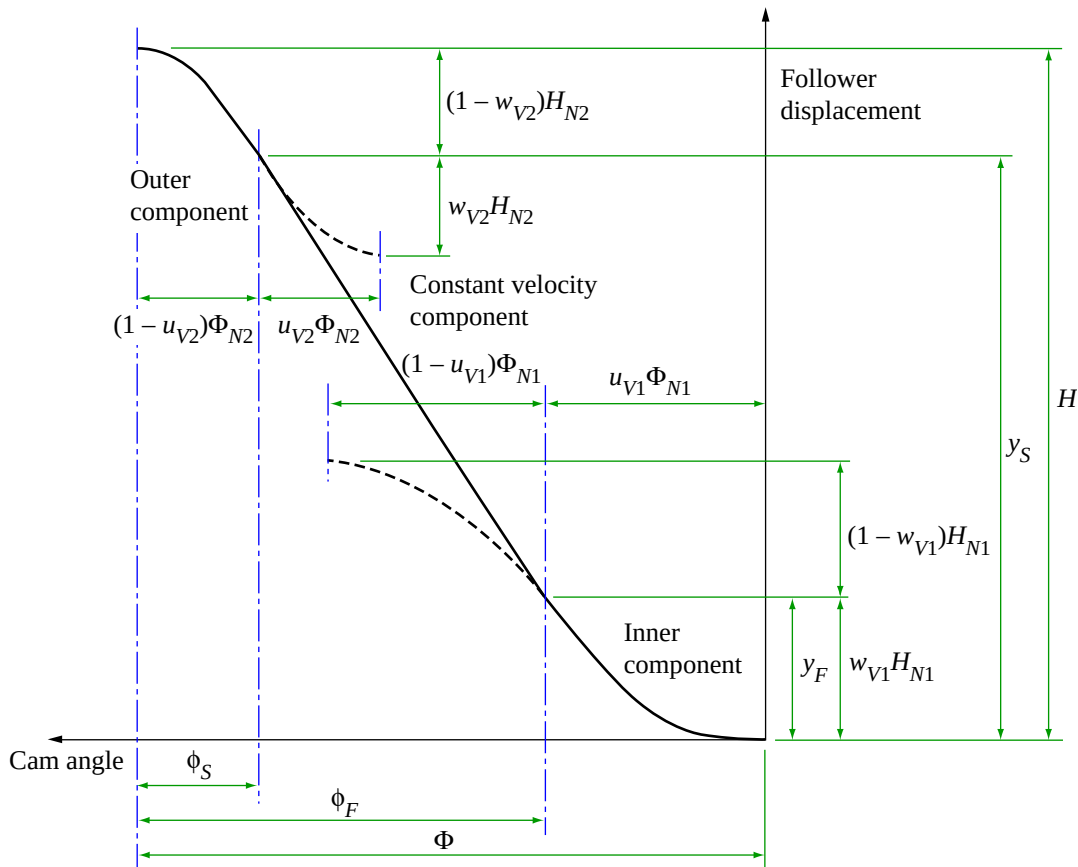
$$(w'_{N1})_{max} = \frac{|\dot{y}_{cv}| \pi \Phi_{N1}}{180 \omega_c H_{N1}}. \quad (3.28)$$

Therefore the notional follower lift of the inner component is given by

$$H_{N1} = \frac{|\dot{y}_{cv}| \pi \Phi_{N1}}{180 \omega_c (w'_{N1})_{max}} \quad (3.29)$$

and, similarly, the notional follower lift of the outer component is given by

$$H_{N2} = \frac{|\dot{y}_{cv}| \pi \Phi_{N2}}{180 \omega_c (w'_{N2})_{max}}. \quad (3.30)$$



Sketch 3.5 General form of motion for return segment incorporating constant velocity component