



**Contribution of body-mounted
fins and tailplanes to
lateral derivatives due to
sideslip at subsonic speeds
for general body width to
height ratio**

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CONTRIBUTION OF BODY-MOUNTED FINS AND TAILPLANES TO LATERAL DERIVATIVES DUE TO SIDESLIP AT SUBSONIC SPEEDS FOR GENERAL BODY WIDTH TO HEIGHT RATIO

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CONTRIBUTION OF BODY-MOUNTED FINS AND TAILPLANES TO LATERAL DERIVATIVES DUE TO SIDESLIP AT SUBSONIC SPEEDS FOR GENERAL BODY WIDTH TO HEIGHT RATIO

1. NOTATION AND UNITS

The derivation notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes, with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivatives Y_v , N_v and L_v are often written as $\partial C_Y / \partial \beta$, $\partial C_n / \partial \beta$ and $\partial C_l / \partial \beta$ or $C_{Y\beta}$, $C_{n\beta}$ and $C_{l\beta}$ in other systems of notation, but attention must be paid to the reference dimensions used and it is to be noted that a constant datum value of V is employed in the Hopkin system.

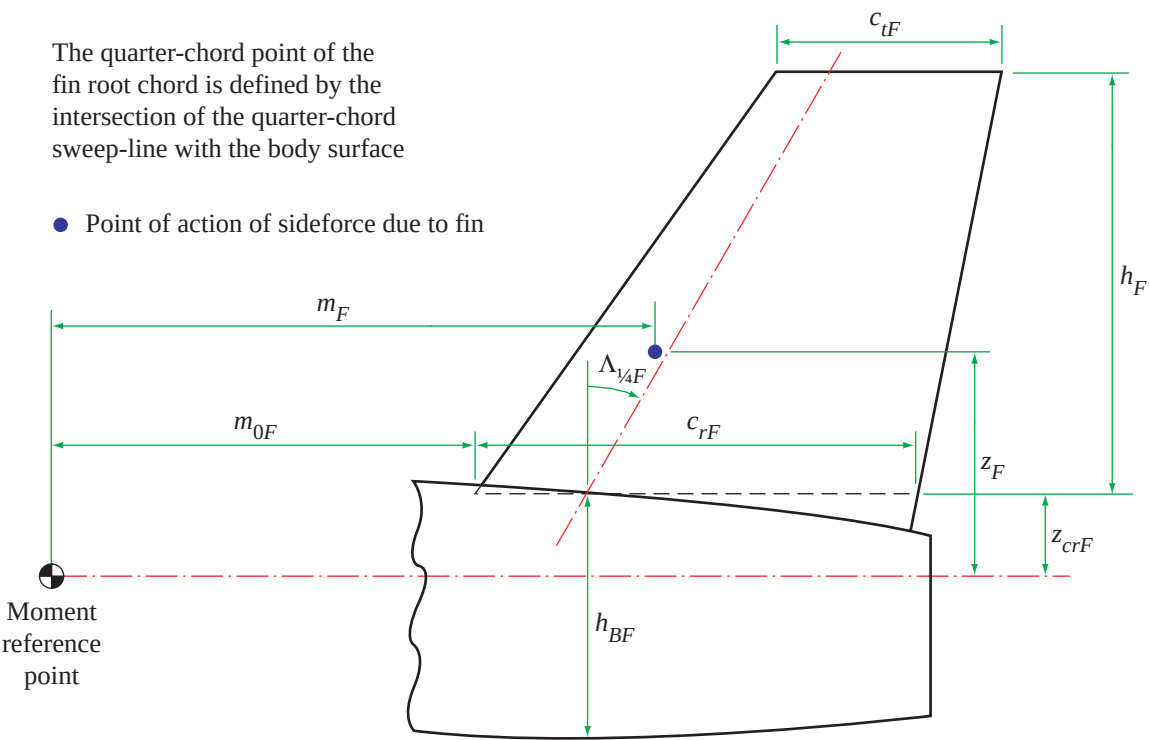
		SI	British
A_F	effective aspect ratio of fin, $2h_F^2/S_F$, see Section 3.1		
a_{1F}	lift-curve slope for exposed fin mounted on infinite reflection plane, see Section 3.1	rad^{-1}	rad^{-1}
b_T	tailplane span	m	ft
b_W	wing (reference) span	m	ft
c_{rF}	root chord of fin, see Sketch 1.1	m	ft
c_{rT}	root chord of tailplane	m	ft
c_{tF}	tip chord of fin, see Sketch 1.1	m	ft
h_{BF}	height of body cross-section at fin root quarter-chord station, see Sketches 1.1 and 1.2	m	ft
h_{BT}	height of body cross-section at tailplane root quarter-chord station, see Sketch 1.3	m	ft
h_{BW}	height of body cross-section at wing root quarter-chord station, see Sketch 1.3	m	ft
h_F	height of exposed fin, measured in plane of fin, see Sketches 1.1 and 1.2	m	ft
J_W	correction factor for effect of wing-body sidewash on fin derivatives for wing upstream of fin		
$K_{B(T)}$	interference factor for body in presence of tailplane		
$K_{B(T)max}$	value of $K_{B(T)}$ for tailplane mounted tangentially at top or bottom of axisymmetric body		

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K_F	overall interference factor for single fin in plane of symmetry		
$K_{F(B)}$	interference factor for single fin in plane of symmetry in presence of body		
$K_{F(BT)}$	interference factor for single fin in plane of symmetry in presence of body and tailplane		
$K_{F(BW)}$	interference factor for single fin in plane of symmetry in presence of body and wing with root chord that extends below fin		
k_{2F}	correction factor applied to K_F for twin fins		
$\mathcal{L}$	rolling moment	N m	lbf ft
L_v	rolling moment derivative due to sideslip, $(\partial \mathcal{L} / \partial v) / (\frac{1}{2} \rho_\infty V_\infty S_W b_W)$		
ΔL_v	increment in L_v due to empennage		
$(\Delta L_v)_F$	increment in L_v due to single fin		
$(\Delta L_v)_{2F}$	increment in L_v due to twin fins		
$(\Delta L_v)_T$	increment in L_v due to fin interference on tailplane		
M_∞	free-stream Mach number		
m_F	longitudinal distance from moment reference point to point of action of sideforce increment due to fin, see Equation (3.3)	m	ft
m_{0F}	longitudinal distance from moment reference point to leading-edge of fin root chord, see Sketch 1.1	m	ft
m_T	longitudinal distance from moment reference point to midpoint of tailplane root chord	m	ft
$\mathcal{N}$	yawing moment	N m	lbf ft
N_v	yawing moment derivative due to sideslip, $(\partial \mathcal{N} / \partial v) / (\frac{1}{2} \rho_\infty V_\infty S_W b_W)$		
ΔN_v	increment in N_v due to empennage		
$(\Delta N_v)_F$	increment in N_v due to single fin		
$(\Delta N_v)_{2F}$	increment in N_v due to twin fins		
$(\Delta N_v)_T$	increment in N_v due to tailplane interference on body		

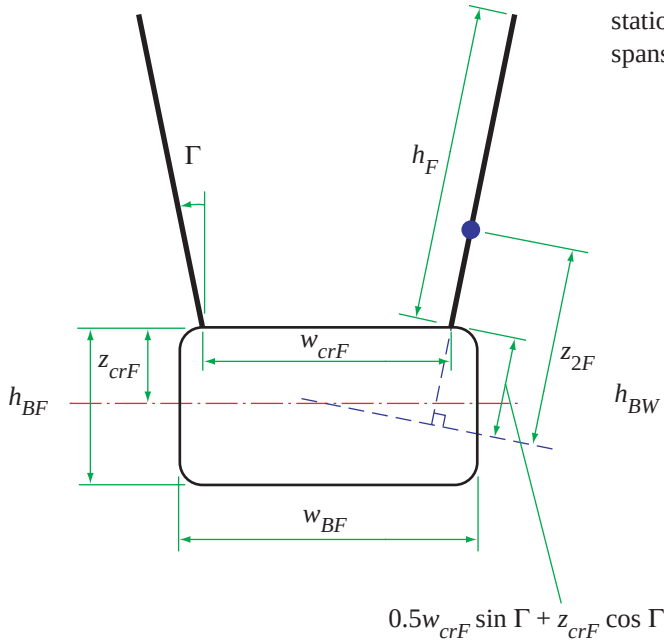
S_F	area of fin, $\frac{1}{2}(c_{rF} + c_{tF})h_F$	m ²	ft ²
S_W	wing (reference) area	m ²	ft ²
V_∞	free-stream velocity	m/s	ft/s
v	sideslip velocity	m/s	ft/s
w_{BF}	width of body cross-section at fin root quarter-chord station, see Sketch 1.2	m	ft
w_{BT}	width of body cross-section at tailplane root quarter-chord station, see Sketch 1.3	m	ft
w_{BW}	width of body cross-section at wing root quarter-chord station, see Sketch 1.3	m	ft
w_{crF}	width between root quarter-chord stations of twin fins, see Sketch 1.2	m	ft
Y	sideforce	N	lbf
Y_v	sideforce derivative due to sideslip, $(\partial Y/\partial v)/(\frac{1}{2}\rho_\infty V_\infty S_W)$		
ΔY_v	increment in Y_v due to empennage		
$(Y_v)_B$	body-alone sideforce derivative due to sideslip		
$(\Delta Y_v)_F$	increment in Y_v due to single fin		
$(\Delta Y_v)_{2F}$	increment in Y_v due to twin fins		
$(\Delta Y_v)_T$	increment in Y_v due to tailplane interference on body		
z_{crF}	height of fin root chord above longitudinal axis through moment reference point, see Sketch 1.1	m	ft
z_F	moment arm for calculating rolling moment derivative for single fin, see Section 3.2	m	ft
z_{2F}	moment arm for calculating the rolling moment derivative for twin fins, see Equation (3.7) and Sketch 1.2	m	ft
z_T	height of tailplane root quarter-chord point above local body mid-point, see Sketch 1.3	m	ft
z_W	height of wing root quarter-chord point above local body mid-point, see Sketch 1.3	m	ft
α	angle of attack	deg	deg
β	angle of sideslip	deg	deg

Γ	fin cant angle, see Sketch 1.2	deg	deg
$\Lambda_{1/4F}$	quarter-chord sweep of fin	deg	deg
$\Lambda_{1/2F}$	half-chord sweep of fin	deg	deg
$\Lambda_{1/4T}$	quarter-chord sweep of tailplane	deg	deg
$\Lambda_{1/4W}$	quarter-chord sweep of wing	deg	deg
λ_F	taper ratio of fin, c_{tF}/c_{rF}		
ρ_∞	free-stream density	kg/m ³	slug/ft ³

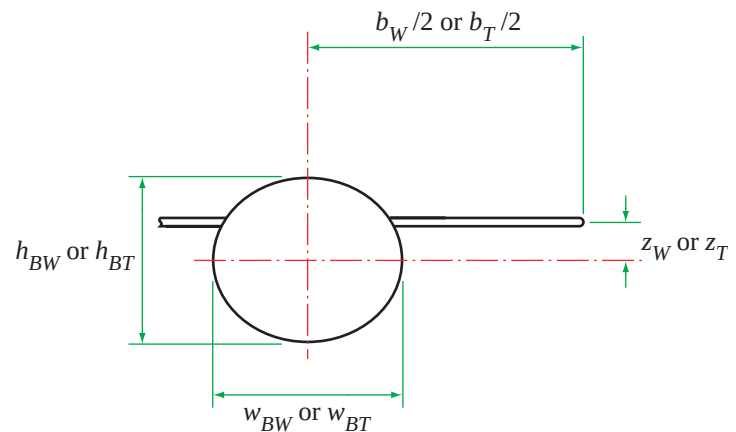


Sketch 1.1 Single fin in plane of symmetry

Body heights and widths are taken at the root quarter-chord station of the wing or tailplane; spans are true spans



Sketch 1.2 Twin fins



Sketch 1.3 Wing or tailplane parameters

2. INTRODUCTION

This Item gives a method for estimating the contribution of single and twin body-mounted fins and tailplanes to the sideforce, yawing moment and rolling moment derivatives due to sideslip of an aircraft. The method applies at subsonic speeds for low angles of attack ($\alpha < 5^\circ$) and sideslip ($|\beta| \leq 4^\circ$). It complements Item No. 82010 (Derivation 2) that deals with the prediction of single body-mounted fin characteristics for geometries typical of transport aircraft with body height to width ratios close to unity. The current method covers a wider range of body height to width ratios, making it applicable to geometries more typical of some combat aircraft.

The method is based on the techniques proposed in Derivation 22, with additional empirical modification. The fin contribution is calculated via the application of interference factors that modify a basic fin lift-curve slope to allow for the presence of body, wing and tailplane. Those factors are mainly derived from the theoretical calculations made in References 33, 34 and 35, with the introduction of an additional wing interference factor to allow for wing-body sidewash at the fin. The basic lift-curve slope is calculated for the exposed fin geometry mounted on an infinite reflection plane.

A similar approach was followed in the development of Item No. 91031 (Derivation 5) that deals with fin sideslip derivatives at supersonic speeds and Item No. 92029 (Derivation 6) that covers the effects of ventral fins at both subsonic and supersonic speeds.

The method is described in Section 3, its accuracy and applicability are addressed in Section 4, the Derivation and References are listed in Section 5 and worked examples are given in Section 6.

The stability derivatives are given in aero-normalised form for a system of aerodynamic-body axes, see Item No. 86021 (Derivation 4).

3. METHOD

3.1 Sideforce Derivative

3.1.1 Single fin in plane of symmetry

The fin sideforce derivative is calculated by applying an interference factor K_F to a basic fin lift-curve slope a_{1F} that is associated with the geometry of the fin reflected about its root chord. Thus

$$(\Delta Y_v)_F = -a_{1F} K_F S_F / S_W \quad (3.1)$$

where a_{1F} is obtained from Item No. 70011 (Derivation 1) for a straight-tapered planform of aspect ratio $A_F = 2h_F^2 / S_F$, taper ratio λ_F and half-chord sweep $\Lambda_{1/2F}$. The fin root chord is defined as the chord passing through the point of intersection of the fin quarter-chord sweep line and the body surface, see Sketch 1.1.

The position of the wing on the body and with respect to the fin, and the relative magnitudes of the spans of the empennage panels and body diameters are used in the calculation of K_F . Three main cases are considered, as illustrated in Sketch 3.1.

In case (i) there is interference from a wing that is well forward of the fin and from the body, and $K_F = J_W K_{F(B)}$.

In case (ii) there is interference from a wing that is well forward of the fin and from the body and tailplane, and $K_F = J_W K_{F(BT)}$.

In case (iii) there is interference from a wing with a root chord that extends below the fin and from the body, and $K_F = K_{F(BW)}$.

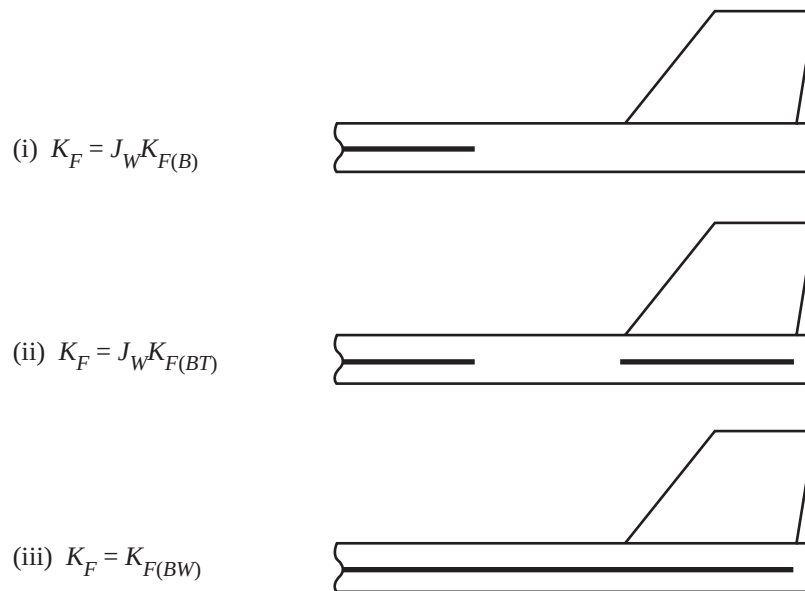
The factor J_W , that is a function of wing height, allows for the influence of wing-body sidewash at the fin. This factor is shown in Figure 1 and is taken directly from Item No. 82010 where it was developed empirically.

The interference factors $K_{F(B)}$, $K_{F(BT)}$ and $K_{F(BW)}$ are modified values of the theoretical factors proposed in Derivation 22. The factor $K_{F(B)}$ is given in Figure 2. The factors $K_{F(BT)}$ and $K_{F(BW)}$ are given in Figures 3a to 3g for a body of circular or elliptical cross-section with wing or tailplane at mid-height on the body. Figures 3h and 3i give $K_{F(BT)}$ and $K_{F(BW)}$ for a body of circular cross-section with wing or tailplane in low or high position on the body, and offer guidance for intermediate positions whatever the width to height ratio. For a body of general body width to height ratio, with wing or tailplane at a general height, cross-plots must be made between the cases summarised in Table 3.1 (see Examples, Section 6). Although the data in the Figures are for circular or elliptical cross-sections, the width to height ratio of the body is considered to be the important parameter. The Figures can therefore be employed for any practical cross-sectional shape.

Strictly, theory requires that the trailing edges of all the panels of the empennage be in the base plane of the body. However, the comparisons with experimental data made in Derivation 22 demonstrate that it is possible to relax this restriction and ignore small differences in the longitudinal locations of the tips of each panel. Body diameters can be sensibly defined at the longitudinal station of the root quarter-chord of the panel of interest.

As given, the interference factors $K_{F(BT)}$ and $K_{F(BW)}$ apply when the tailplane or wing root chord extends under the whole of the fin root chord. For the case where the horizontal surface extends under only part of the fin root chord or is just clear of it calculations should be made with and without tailplane or wing interference to establish a tolerance on the predicted sideslip derivatives. Thus for a fin partially affected by the tailplane, calculations would be made with $K_F = J_W K_{F(B)}$ and with $K_F = J_W K_{F(BT)}$. For a fin partially affected by a wing, calculations would be made with $K_F = K_{F(BW)}$ and with $K_F = J_W K_{F(B)} = K_{F(B)}$, as it is recommended that J_W be taken as unity when the wing trailing edge is close to or under the fin, regardless of wing height. For a ratio of body width to tailplane or wing span of about 0.3 the influence of $K_{F(BT)}$ or $K_{F(BW)}$ compared with $K_{F(B)}$ increases the sideforce due to the fin by about 10% for surfaces mounted low or at mid-position on the body. The influence increases with wing or tailplane span, and lessens for surfaces mounted higher on the body.

Figures 2 and 3 are identical to those in Item No. 91031, comparisons with experimental data having confirmed that the empirical modifications to the factors that were derived for supersonic speeds were equally necessary and effective for subsonic speeds.



Sketch 3.1 Interference factors K_F

TABLE 3.1 INTERFERENCE FACTORS OF BODY-WING OR BODY-TAILPLANE ON FIN

Body cross-section width to height ratio	Height on body of wing or tailplane	Figure for
w_{BW}/h_{BW} or w_{BT}/h_{BT}	$2z_W/h_{BW}$ or $2z_T/h_{BT}$	$K_{F(BW)}$ or $K_{F(BT)}$
4	0	3a
3	0	3b
2	0	3c
1.5	0	3d
1	0	3e
0.75	0	3f
0.5	0	3g
1	-1 (low)	3h
1	1 (high)	3i

3.1.2 Twin fins

For a pair of body-mounted fins Equation (3.1) is modified to become

$$(\Delta Y_v)_{2F} = -2k_{2F}(\cos^2 \Gamma)a_{1F}K_F S_F/S_W. \quad (3.2)$$

Here the factor K_F is determined in the same way as K_F for single fins, with h_F measured in the plane of the fin, see Sketch 1.2; k_{2F} is an empirically determined correction factor for twin fins and is given in Figure 4 as a function of w_{crF}/h_{BF} . The variation with $\cos^2 \Gamma$ is due to the decrease in the incidence of the fins if canted and the resolution of the forces on them.

3.2 Yawing Moment and Rolling Moment Derivatives

3.2.1 Single fin in plane of symmetry

To calculate the yawing moment and rolling moment derivatives a point of action has to be established for the sideforce due to the fin. In Item No. 82010 that point was obtained by taking a position dictated by simple assumptions about the loading distribution on the exposed fin and then applying separate numerical correction factors to the longitudinal and vertical positions in order to improve the overall agreement between predicted and experimental values of $(\Delta N_v)_F$ and $(\Delta L_v)_F$. In this way an empirical allowance is made for the fact that some of the fin-induced sideforce is carried on the body.

In detail, for a body-mounted tailplane, the original point of action of the sideforce due to the fin was based on the assumption of nearly-elliptical loading along the fin quarter-chord line. This gave a longitudinal position $0.25c_{rF} + 0.4h_F \tan \Lambda_{1/4F}$ aft of the leading edge of the fin root chord and a vertical position $0.4h_F$ perpendicularly above the fin root chord. These distances were modified through the introduction of the empirical constants 0.7 and 0.85 to become $0.25c_{rF} + 0.7 \times 0.4h_F \tan \Lambda_{1/4F}$ and $0.85 \times 0.4h_F$.

Comparisons with experimental data made in the development of the present method have confirmed that these moment arms remain satisfactory when used in conjunction with the values of $(Y_v)_F$ predicted by

Equation (3.1). Relative to the moment reference centre and in geometric-body axes the moment arms for the yawing moment and rolling moment are

$$m_F = m_{0F} + 0.25c_{rF} + 0.28h_F \tan \Lambda_{1/4F} \quad (3.3)$$

and
$$z_F = z_{crF} + 0.34h_F. \quad (3.4)$$

At an angle of attack α , the yawing moment and rolling moment derivatives expressed in aerodynamic-body axes are

$$(\Delta N_v)_F = -(\Delta Y_v)_F [(m_F/b_W) \cos \alpha + (z_F/b_W) \sin \alpha] \quad (3.5)$$

and
$$(\Delta L_v)_F = (\Delta Y_v)_F [(z_F/b_W) \cos \alpha - (m_F/b_W) \sin \alpha]. \quad (3.6)$$

3.2.2 Twin Fins

For a pair of body-mounted fins m_F remains as in Equation (3.3), but as in Sketch 1.2 geometric considerations require z_F to be replaced by

$$z_{2F} = 0.5w_{crF} \sin \Gamma + z_{crF} \cos \Gamma + 0.34h_F. \quad (3.7)$$

Then
$$(\Delta N_v)_{2F} = -(\Delta Y_v)_{2F} [(m_F/b_W) \cos \alpha + (z_{2F}/b_W) \sin \alpha] \quad (3.8)$$

and
$$(\Delta L_v)_{2F} = (\Delta Y_v)_{2F} [(z_{2F}/b_W) \cos \alpha - (m_F/b_W) \sin \alpha] \sec \Gamma. \quad (3.9)$$

Comparisons with experimental data have shown that the moment arms incorporated in Equations (3.8) and (3.9), although based on work for single-fin configurations, give satisfactory predictions of the derivatives for twin-fin configurations.

3.3 Effect of Body-mounted Tailplane

Apart from its influence on the fin sideforce through the factor $K_{F(BT)}$, the addition of the tailplane to a body also produces a contribution to ΔY_v and ΔN_v due to interference with the body. Therefore, in the calculation of the effect of an empennage it is necessary to estimate the small additional effect of adding the tailplane to the body in the absence of any fin. Adoption of the method in Derivation 22 gives this contribution as

$$(\Delta Y_v)_T = K_{B(T)}(Y_v)_B \quad (3.10)$$

where
$$K_{B(T)} = K_{B(T)max} \left\{ 1 - \left[1 - \left(\frac{2z_T}{h_{BT}} \right)^2 \right]^{1/2} \right\} \quad (3.11)$$

and $K_{B(T)max}$ is reproduced in Figure 5 for a tailplane mounted tangentially at the top or bottom of the body. Equation (3.11) gives a variation between the maximum value and the value of zero for a tailplane in mid-position.

The body-alone sideforce derivative

$$(Y_v)_B = -0.5\pi w_{BT} h_{BT} / S_W \quad (3.12)$$

is a modified form of the slender-body theory value, which is regarded as a simple approximation that is

adequate for the calculation of the relatively small tailplane contribution.

The yawing moment contribution is

$$(\Delta N_v)_T = -(\Delta Y_v)_T (m_T / b_W) \cos \alpha, \quad (3.13)$$

where m_T is the moment arm measured to the mid-root-chord of the tailplane.

There is also a small tailplane contribution to ΔL_v due to the antisymmetric loading induced across the tailplane by the presence of the fin. An empirical modification of the work in Derivation 3 suggests that for single fins the body-mounted tailplane contribution is,

$$(\Delta L_v)_T = 0.13 S_F b_T / S_W b_W. \quad (3.14)$$

To provide a first approximation to the corresponding contribution for twin-fin configurations, the antisymmetric loading that underlies Equation (3.14) is assumed still to act on each of the exposed panels of the tailplane, with a point of action at one third of the exposed tailplane span, but any contribution to the rolling moment from a fin-induced loading distribution across the body between the fins is ignored. The predicted contribution is then

$$(\Delta L_v)_T = \frac{0.13 S_F (b_T + 2w_{crF})}{S_W b_W}. \quad (3.15)$$

But there were too few experimental data available for any comprehensive verification of this approximation.

3.4 Empennage Values

The stability derivatives due to the empennage are obtained by adding the single or twin fin contributions to the tailplane contributions, as summarised in Table 3.2.

TABLE 3.2

<i>Derivative</i>	<i>Contribution</i>	<i>Equation</i>
ΔY_v	$(\Delta Y_v)_F$	(3.1)
	or $(\Delta Y_v)_{2F}$	(3.2)
	$(\Delta Y_v)_T$	(3.10)
ΔN_v	$(\Delta N_v)_F$	(3.5)
	or $(\Delta N_v)_{2F}$	(3.8)
	$(\Delta N_v)_T$	(3.13)
ΔL_v	$(\Delta L_v)_F$	(3.6)
	or $(\Delta L_v)_{2F}$	(3.9)
	$(\Delta L_v)_T$	(3.14) or (3.15)