

FLAT RECTANGULAR ORTHOTROPIC PLATES UNDER UNIFORMLY DISTRIBUTED NORMAL PRESSURE

ELASTIC STRESSES AND DEFLECTIONS FOR VARIOUS FORMS OF EDGE RESTRAINT

1. INTRODUCTION, APPLICATION AND THEORY

1.1 Introduction

This Data Item presents a Fortran computer program to determine the elastic deflections, moments, in-plane forces, stresses and strains of flat rectangular orthotropic plates with various forms of edge restraint under uniformly distributed normal pressure. The program will give, as required:

- (i) the detailed distribution of the deflections and (a) the moments and in-plane forces, (b) the stresses or (c) the strains, throughout the plate for a given single pressure load,
- or (ii) the maximum deflection and the peak values of the above choices (a), (b) or (c) for a range of pressure loads.

The Item consists of two parts. The first part describes the application of the program and its theoretical basis. The second part describes the use of the program, its input and output, and gives example calculations using the program. The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable program for PCs. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

The program is based on the elastic, large deflection, moderate rotation, orthotropic thin plate theory presented in Derivation 1 and is applicable to plates that can be represented as specially orthotropic, for example composite plates (see Data Item No. 94003^{*}). The governing differential equations, formed in terms of stress and displacement functions, are solved for the boundary conditions of elastic rotational and translational restraint using the finite difference method. The moments and in-plane forces in the plate, and subsequently the stresses and strains, are obtained from differential expressions involving the stress and displacement functions (see Section 1.4). The plate is specified by defining its length, width and thickness together with the in-plane and flexural stiffness matrices. The plate loading condition is specified by defining the uniformly distributed normal pressure on the plate and the boundary conditions at the plate edges in terms of values of the elastic rotational and in-plane direct restraints. The edges of the plate are rigidly supported normal to the plane of the plate.

Output option (i) gives the values of the chosen quantities for a single pressure load. These can be given, as required, at each point in the finite difference grid, at alternate points or only at points at which they are likely to peak. Output option (ii) gives the values of the chosen quantities, for a range of pressure loading, but only at the points where they are likely to peak. In either case the computation time, amount of output and the resolution of the analysis may be varied by choosing from three different sizes of finite difference grid (see Section 2.3).

^{*} Data Item No. 94003 "Stiffness of laminated flat plates".

1.2 Notation

Both SI and lbf in units are quoted but any coherent system of units may be used.

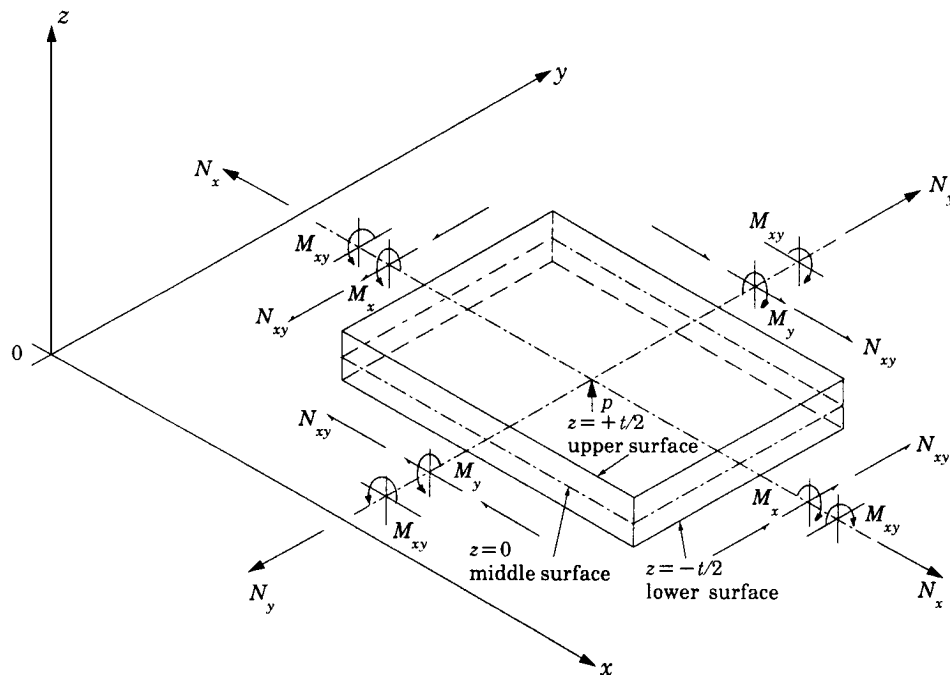
1.2.1 General notation

A	in-plane stiffness matrix for plate (see Data Item No. 94003) with A_{11} for force N_x	N/m	lbf/in
a, b	dimensions of plate in x - and y -directions respectively	m	in
C	number of values of pressure loading for which analyses are required		
D	flexural stiffness matrix for plate (see Data Item No. 94003) with D_{11} for bending under M_x	N m	lbf in
E_x, E_y	elastic moduli in the x - and y -directions respectively for a homogeneous orthotropic plate	N/m ²	lbf/in ²
e	strain in plate (see Section 1.4)		
f	direct stress in plate (see Section 1.4)	N/m ²	lbf/in ²
G_{xy}	elastic shear modulus for a homogeneous orthotropic plate	N/m ²	lbf/in ²
I	program selector for moments and loads, stresses or strains (see Table 2.1)		
J	program selector for data points output (see Table 2.1)		
K_x, K_y	stiffnesses of elastic direct (in-plane) edge restraints per unit length in x - and y -directions respectively	N/m ²	lbf/in ²
M_x, M_y	bending moments per unit length, about y -axis resulting in stresses in x -direction and about x -axis resulting in stresses in y -direction, respectively (see Sketch 1.1)	N m/m	lbf in/in
M_{xy}	twisting moment per unit length about x - and y -axes resulting in shear stress (see Sketch 1.1)	N m/m	lbf in/in
m, n	grid point numbers in quarter plate in x - and y -directions respectively for finite difference analysis ($0 \leq m \leq 8, 0 \leq n \leq 8$, see Sketch 1.2)		
m_{max}	maximum value of m chosen for analysis, $m_{max} = 4, 6$ or 8		
n_{max}	maximum value of n chosen for analysis, $n_{max} = m_{max}$		
N_x, N_y	direct in-plane forces per unit length in x - and y -directions respectively (see Sketch 1.1)	N/m	lbf/in

N_{xy}	shear force per unit length in xy -direction (see Sketch 1.1)	N/m	lbf/in
p	uniform normal resultant pressure, positive applied to plate in z -direction (see Sketch 1.1)	N/m ²	lbf/in ²
q	in-plane shear stress in plate (see Section 1.4)	N/m ²	lbf/in ²
R_x, R_y	stiffnesses of elastic edge rotational restraints per unit length about y -axis (reacting stresses in x -direction) and about x -axis (reacting stresses in y -direction), respectively	N m/(m rad)	lbf in/(in rad)
t	thickness of plate	m	in
w	deflection (normal displacement) of plate middle surface in z -direction	m	in
δ	deflection (normal displacement) of plate middle surface at centre of plate in z -direction	m	in
$0(x, y, z)$	plate reference axes where x and y are parallel to sides a and b respectively and are principal axis of orthotropy		

1.2.2 Suffixes

l	refers to lower surface at $z = -t/2$ in undeflected plate (see Sketch 1.1)
m	refers to middle surface at $z = 0$ in undeflected plate (see Sketch 1.1)
u	refers to upper surface at $z = +t/2$ in undeflected plate (see Sketch 1.1)
x, y, xy	refer to directions of stresses, moments or in-plane forces (see Sketch 1.1)



Sketch 1.1 Sign convention for pressure, moments and in-plane forces on a plate element

1.2.3 Sign Convention

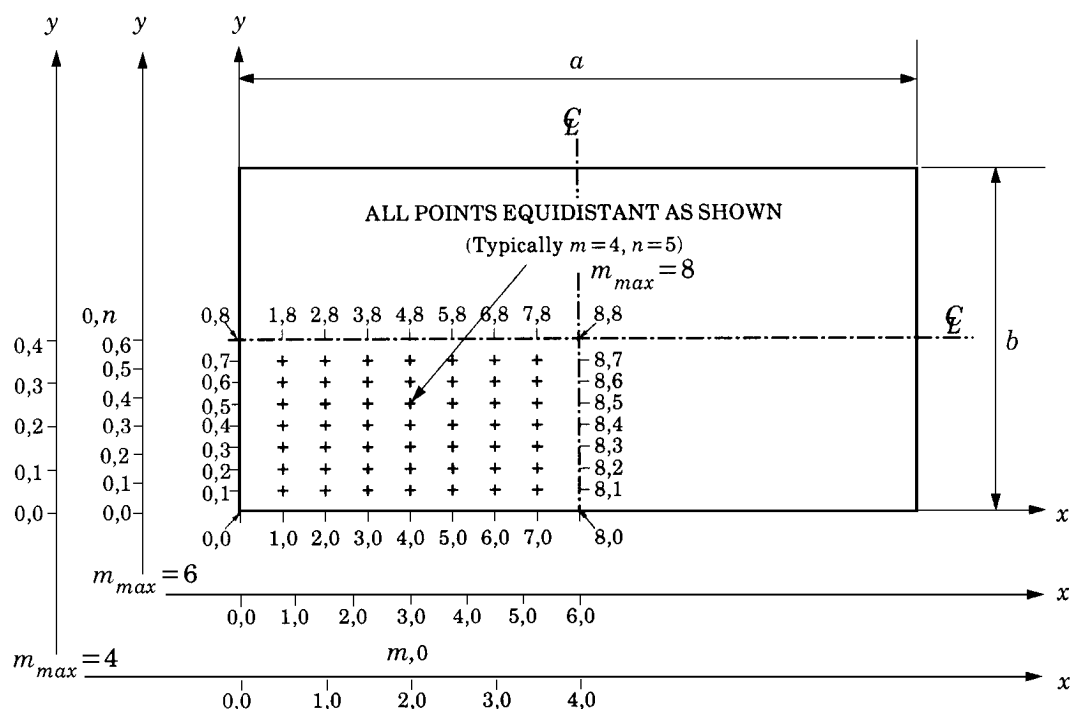
The positive convention for the direct forces, bending moments, shear forces and twisting moments as applied to a plate element is shown in Sketch 1.1.

Pressure (resultant) acts in the positive z -direction.

Deflection is positive in the positive z -direction.

Tensile stresses are positive.

Compressive stresses are negative.



Sketch 1.2 Quarter plate grid system for analysis and definition of m_{max}

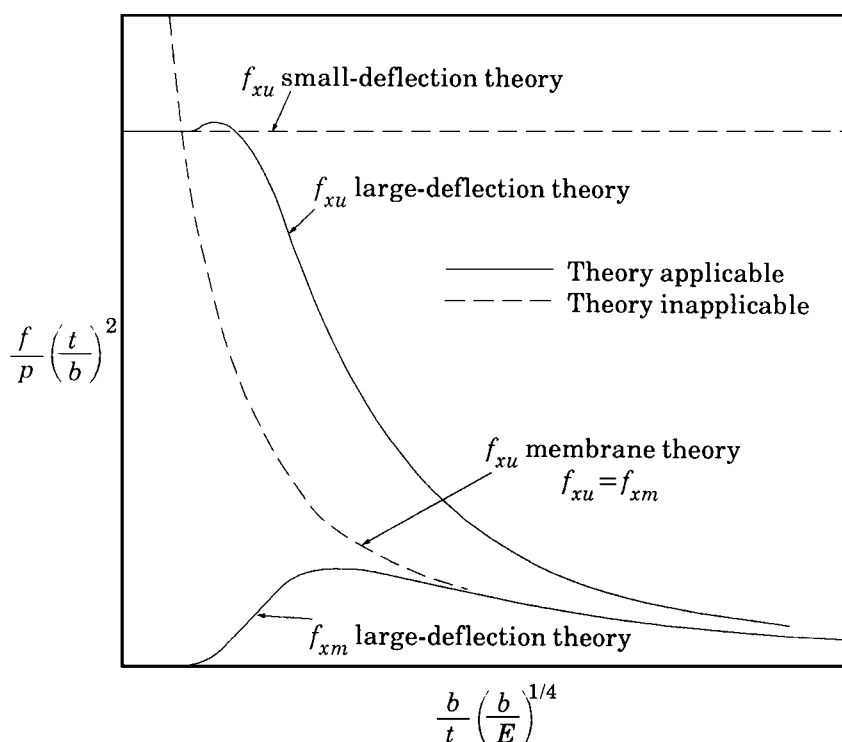
1.3 Application of the Program and General Theoretical Considerations

1.3.1 General consideration of plate behaviour

The program is based on the elastic, large deflection, moderate rotation ($\partial w/\partial x$ and $\partial w/\partial y \leq 0.2$) orthotropic thin plate theory presented in Derivation 1. Elastic thin plate behaviour (a/t and $b/t \geq 20$) can be considered in three phases: (1) plates with small deflections ($\delta/t \leq 0.5$) in which bending effects dominate and linear bending theory may be applied, (2) plates with large deflections and moderate rotation in which membrane stresses interact with the bending effects and non-linear theory should be used, as in this Item, and (3) very thin plates with very large deflections in which membrane stresses dominate and membrane theory may be applied. The plate edge conditions affect the relative importance of these effects for the stresses at various points in the plate.

Sketch 1.3 shows the relationship of the phases and the various theories for the stresses at the centre of a simply-supported square, isotropic plate under uniform normal pressure, presented in the form of convenient non-dimensional parameters. The behaviour of other stresses, for other edge conditions and for plates with significant orthotropy, is more complex but the trends are generally similar.

The stresses at the outer surfaces of the plate, the upper and lower surfaces, arise from a combination of bending and membrane effects while the stresses on the middle surface (of a specially orthotropic plate) arise from the membrane effect only. Either of these effects may dominate at a particular point in the plate. Where bending is not present the stresses will be the same at both the outer and the middle surfaces. Where bending only is present the middle surface stress will be zero. The stresses vary in the x - and y -directions and are most likely to peak at the centre of the plate, the mid-point of each edge or near the diagonals, depending on the edge restraints and the plate orthotropy.



Sketch 1.3 Elastic thin plate behaviour for the stresses at the centre of a plate according to various theories

This Data Item provides a program based on a mathematical model that can be used to describe the behaviour of a laminated composite plate. The model, which has been constructed using the assumptions described in Section 1.4, contains a very large number of independent variables. Test data that can be used to verify its accuracy are only available for particular sets of the independent variables and these are reported on in Derivation 2. Sufficient data with which to verify the model over its complete range are highly unlikely to become available in the foreseeable future. The model is based upon well known principles of elastic behaviour which have been employed successfully in many models of structural elements and components. Within the validity of the assumptions made it is therefore reasonable to assume that the agreement demonstrated in comparisons with the available sets of test data gives an indication of the accuracy of the model when applied to properly designed, structural, laminated composites employing current materials. However, because of the lack of comprehensive test data consideration should be given to testing the structure.

1.3.2 Use of the two forms of analysis

A choice of two forms of output is provided:

- (i) the detailed distribution of the deflections and (a) the moments and in-plane forces, (b) the stresses or (c) the strains, throughout the plate for a given single pressure load,
- or
- (ii) the maximum deflection and the peak values of the above choices (a), (b) or (c) for a range of pressure loads.

These two options can be used in a complementary manner. Option (i) can be used to study the detailed distribution of the deflections, moments and in-plane forces, stresses or strains across the plate in any direction. The results of this form of analysis for an isotropic plate are illustrated in Figure 1.

Option (ii), the analysis of the plate over a range of pressure loads, can be used to provide a picture of the changing level of the plate maximum deflection and peak moments, forces, stresses or strains with increasing pressure as shown in Figure 2. This form of analysis can be used to establish a pressure to give a desired moment, force, stress or strain level for subsequent detailed analysis at one pressure level (option (i)). The moments and forces obtained for either option can be used as input to the failure analysis program of Data Item No. 84018*.

In either case the computation time, amount of output and resolution of the analysis can be determined by the choice of the finite difference grid size and in option (i) the amount of output can be further varied by choosing to output every point or only alternate points (defined by J , see Table 2.1).

1.3.3 Example results

Figure 1 shows the variation of the stresses across the width of an isotropic square plate under uniformly distributed normal pressure. In this example the edges of the plate are elastically restrained in rotation and free in translation. It can be noted that while the total stress in the x -direction has a value even at the edge, the middle surface stress in the x -direction is zero at the edge. This is because the bending part of the total stress in the x -direction is present at the edge due to the elastic rotational restraint; the absence of in-plane restraint allows no direct membrane effect at the edge but the membrane effect develops as deflection increases away from the edge. There is, however, a compressive middle surface stress, f_{ym} , in the y -direction at the edge reacting the tensile membrane stress in the middle of the plate.

Figure 2 shows the variation of the deflection at the centre of an isotropic plate with increasing pressure when the plate has different forms of rotational and in-plane translational restraint. Each of the curves shows the distinct non-linear behaviour of the analysis. As is to be expected the deflection is highest when in-plane translational and rotational restraints are zero (edges free) and lowest when these are infinite (edges fixed).

Sketch 1.3 shows the behaviour of the outer surface stress at the centre of a flat isotropic square plate under uniform normal pressure. The data obtained from the program can be used to provide such presentations but only for variation in load and geometry with the plate orthotropy fixed throughout.

1.4 Theoretical Basis of the Program Analysis

The program is based on the elastic, non-linear, large deflection, moderate rotation, orthotropic thin plate theory presented in Derivation 1 which assumes that the plate material is homogeneous. The method of solution of the governing differential equations for the various boundary conditions by finite difference analysis outlined in Derivation 1 is detailed in Derivation 2. The analysis is applied to one quarter of the rectangular plate on the assumption that the plate is uniformly loaded and that its boundary conditions are symmetrical about the two orthogonal centre-lines.

* Data Item No. 84018 "Failure analysis of fibre reinforced composite laminates".

The stresses given by the program, assuming that the plate is homogeneous, take the forms:

$$\left. \begin{aligned} f_x &= \frac{12M_x z}{t^3} + \frac{N_x}{t}, \\ f_y &= \frac{12M_y z}{t^3} + \frac{N_y}{t}, \\ \text{and} \quad q &= \frac{6M_{xy} z}{t^3} + \frac{N_{xy}}{t}. \end{aligned} \right\} \quad (1.1)$$

If the plate cannot be considered homogeneous through the thickness, such as for a laminated composite plate, these stress equations are not applicable but the moments, forces and strains can be used.

The strains given by the program take the forms:

$$\left. \begin{aligned} e_x &= \frac{A_{22}N_x - A_{12}N_y}{A_{11}A_{22} - A_{12}^2} + z \left(\frac{D_{12}M_y - D_{22}M_x}{D_{11}D_{22} - D_{12}^2} \right), \\ e_y &= \frac{A_{11}N_y - A_{12}N_x}{A_{11}A_{22} - A_{12}^2} + z \left(\frac{D_{12}M_x - D_{11}M_y}{D_{11}D_{22} - D_{12}^2} \right), \\ \text{and} \quad e_{xy} &= \frac{N_{xy}}{A_{33}} + \frac{z}{2} \frac{N_{xy}}{D_{33}}. \end{aligned} \right\} \quad (1.2)$$

In these expressions z is taken as $+t/2$, 0 and $-t/2$ for the upper, middle and lower surfaces respectively, see Sketch 1.1. The moments and forces are determined from second order differential expressions for the displacement functions and stress functions respectively, which incorporate orthotropic plate moduli equivalents of the laminate **D** and **A** stiffness matrices.

1.5 DERIVATION

This section gives the sources which have assisted in the preparation of this Data Item.

1. BASU, A.K.
CHAPMAN, J.C. Large deflection behaviour of transversely loaded rectangular orthotropic plates. *Proceedings of the Institution of Civil Engineers*, Vol. 35, pp. 79-110, 1966.
2. PENNING, R.L. Rectangular orthotropic plates under pressure - finite difference solution. ESDU Memorandum No. 100, April 1995.

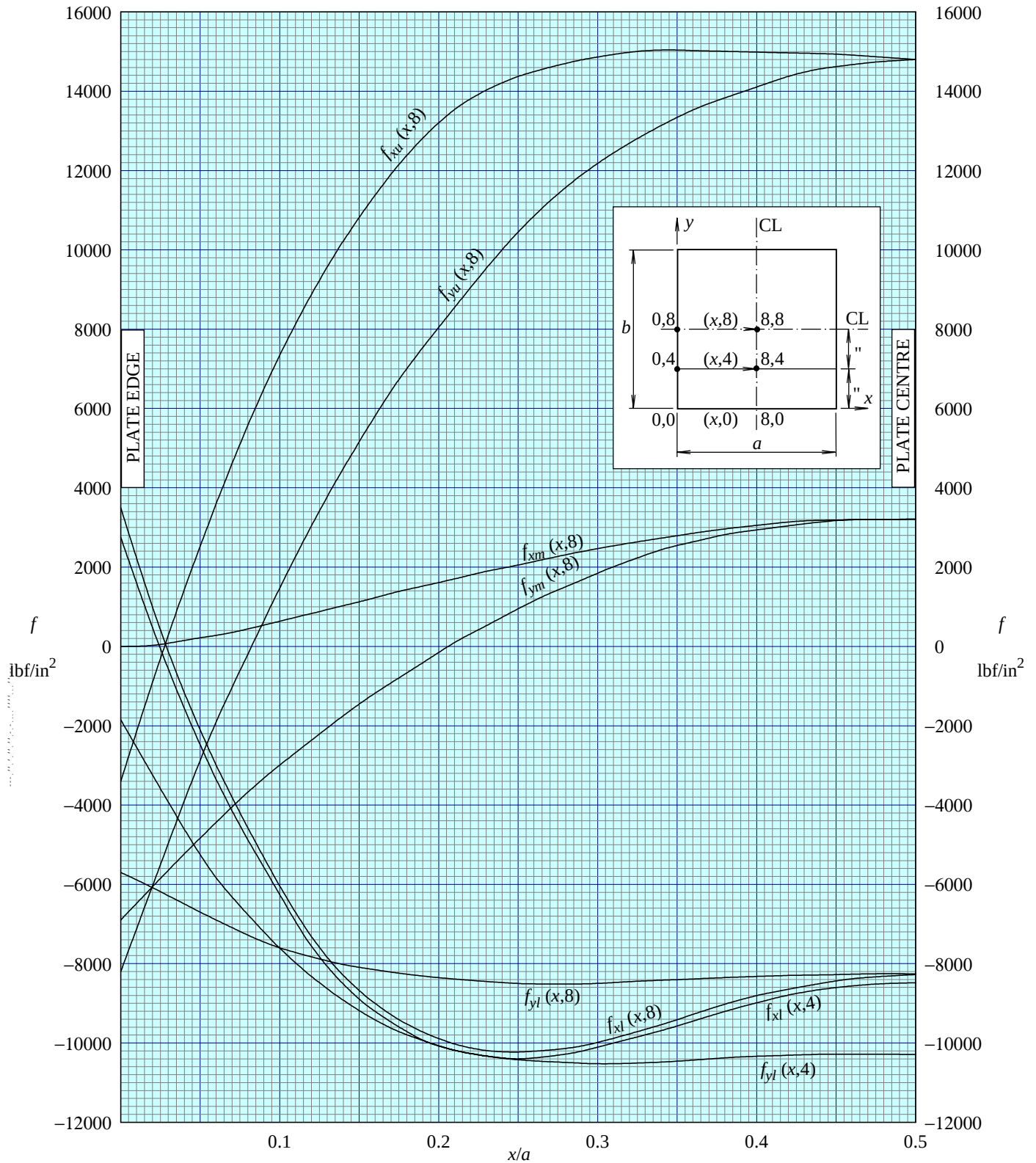


FIGURE 1 STRESS DISTRIBUTION ACROSS A FLAT SQUARE ISOTROPIC PLATE UNDER UNIFORM PRESSURE

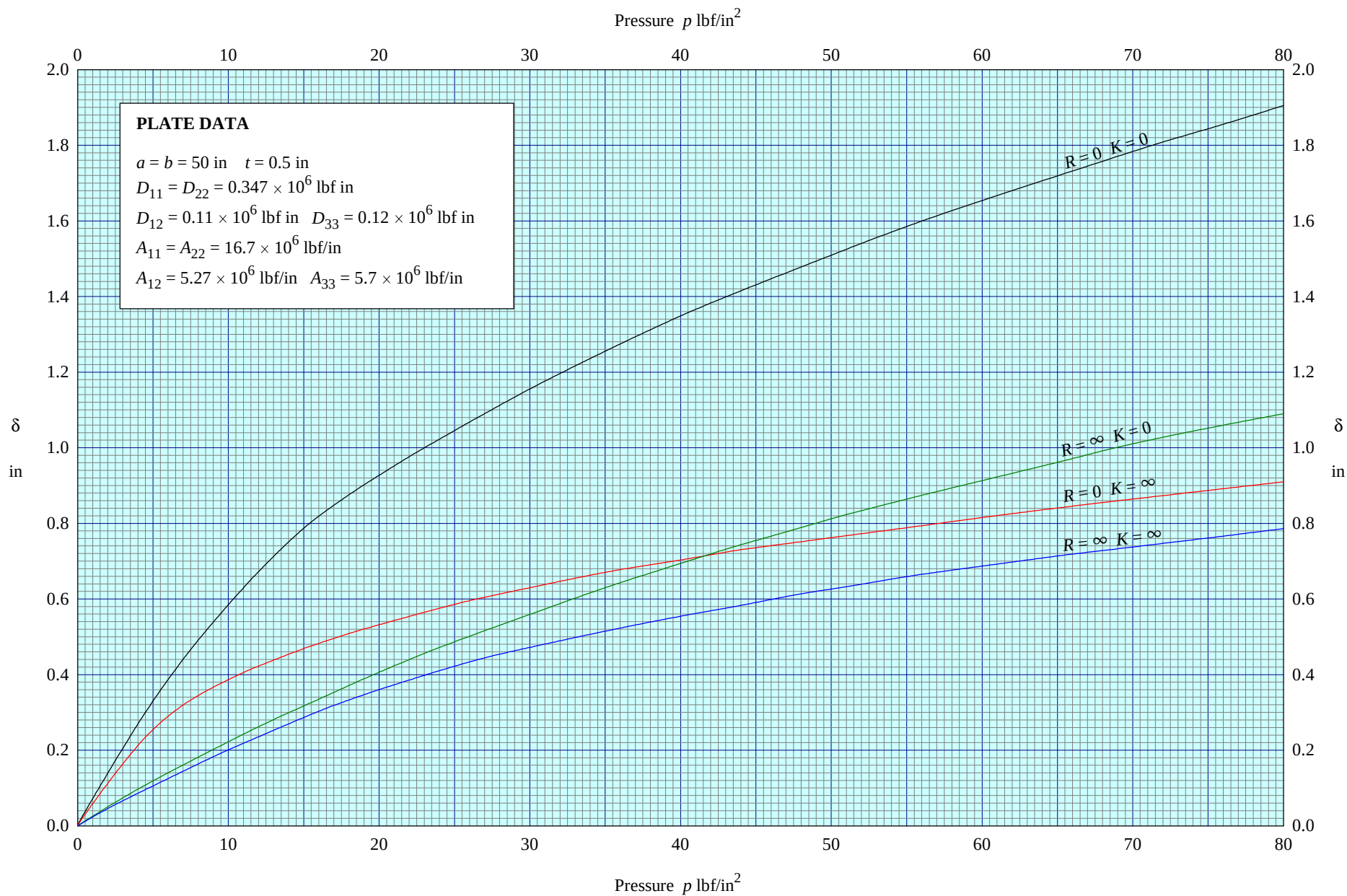


FIGURE 2 PLATE CENTRE DEFLECTIONS FOR FLAT ISOTROPIC PLATE OVER A RANGE OF PRESSURE LOADING. EFFECT OF DIFFERENT EDGE CONDITIONS