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Parameter estimation of linear systems from frequency response measurements (Computer program)

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PARAMETER ESTIMATION OF LINEAR SYSTEMS FROM FREQUENCY RESPONSE MEASUREMENTS (COMPUTER PROGRAM)

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PARAMETER ESTIMATION OF LINEAR SYSTEMS FROM FREQUENCY RESPONSE MEASUREMENTS (COMPUTER PROGRAM)

1. INTRODUCTION

This Data Item implements a Fortran computer program for the parameter estimation of linear systems from frequency response measurements. The method is primarily based on the procedure described in ESDU 90006^[2] but has been modified to account for the effects of residual modes of response, *i.e.* gain and phase variations of modes whose effects lie predominantly outside the range of frequencies of the analysis data but which nevertheless have an influence within the range of analysis.

The system is assumed modelled as a set of lightly-damped coupled second-order differential equations with constant coefficients and the response determined from solution of these equations is equivalent to the sum of the responses of a related set of single degree-of-freedom systems. It is also assumed that a set of frequency response measurements is available from forced harmonic excitations of the system and that these measurements encompass the major response activity of the modes to be identified. The method has been implemented as an automatic computer-based procedure, but relies on a visual inspection of a Nyquist plot of the measurement data to provide initial estimates of the resonant frequency of each mode of response together with the frequency bands of data associated with each mode. The computer program has been written to deal with a single-input single-output system where displacement measurements obtained at different positions in the system are treated separately.

The program evaluates initial estimates of the natural frequency and damping ratio of each mode together with the coefficients of the complex forcing functions and residual terms. These values provide a starting point for a nonlinear parameter estimation procedure using a Newton-Gauss algorithm. Examples illustrate the use of the computer program to estimate the parameters of systems containing both two and four degrees-of-freedom.

2. NOTATION

Any coherent system of units may be used.

		<i>Symbols used in program input and output</i>
$A_s, B_s, s = 1, n$	coefficients in denominator of quadratic factors	$A(I), B(I), I = 1, n$
$\text{Adj}\{\dots\}$	adjoint of $\{\dots\}$	—
$\underline{B}$	coefficient matrix of $\dot{\underline{z}}(t)$	—
$\text{Det}\{\dots\}$	determinant of $\{\dots\}$	—
$E_s, F_s, s = 1, n$	coefficients in numerator of quadratic factors	$E(I), F(I), I = 1, n$
E_l, F_l	coefficients of low frequency residual terms	Elow, Flow
E_u, F_u	coefficients of high frequency residual terms	Ehigh, Fhigh
i	complex operator	—
J	Objective Function	J
$\underline{K}$	coefficient matrix of $\underline{z}(t)$	—
$\underline{M}$	coefficient matrix of $\ddot{\underline{z}}(t)$	—
N_t	total number of measurements within input data file	—
N_l	lowest data point number used for analysis	—
N_u	highest data point number used for the analysis	—
n	number of degrees-of-freedom (or response modes) used in analysis range of data	n
n_l	lowest resonant frequency mode in analysis range of data	—
n_t	total number of resonant modes of system	—
n_u	highest resonant frequency mode in analysis range of data	—
$R(\omega)$	receptance as function of frequency	—
R_0	constant (complex) error associated with measurement vectors (see Equation (3.16))	—

R_{x0}, R_{y0}	real and imaginary parts of error associated with measurement vectors	—
t	time	—
$\underline{\mathbf{U}}(\omega)$	column matrix of input excitations as functions of frequency	—
$\underline{\mathbf{u}}(t)$	column matrix of input excitations as functions of time	—
$\underline{\mathbf{Z}}(\omega)$	column matrix of displacements as functions of frequency	—
$\underline{\mathbf{z}}(t)$	column matrix of displacements as functions of time	—
γ	unknown parameter	—
Δ	denotes increment	d
δ	phase angle of measurement origin to force reference direction (see Equation (3.15))	—
ζ	damping ratio	—
ω	frequency	—
ω_n	natural undamped frequency	—
ω_r	resonant frequency	—
Superscript		
*	denotes complex conjugate	—
Subscripts		
e	denotes estimate	—
j, k	arbitrary integers	—
l, u	denotes lower and upper, respectively	—
t	denotes total	—
m	denotes measurement	—
r	denotes resonance	—
s	integer associated with mode number	—

A bar below a quantity denotes a matrix or vector quantity.

One or two dots above a quantity denotes the first or second derivative, respectively, of that quantity with respect to time.

3. BACKGROUND DESCRIPTION

A previous Data Item, ESDU 90006^[2], developed a method for the estimation of the parameters of a linear system from frequency response measurements. The model of the system was a coupled set of second-order differential equations with constant coefficients expressed as an n_t degree-of-freedom matrix equation of the form

$$\underline{M} \ddot{\underline{z}}(t) + \underline{B} \dot{\underline{z}} + \underline{K} \underline{z}(t) = \underline{u}(t) , \quad (3.1)$$

where the variables $\underline{z}(t)$ and $\underline{u}(t)$ are column vectors of size $n_t \times 1$ comprising the response variables and input variables, respectively. The coefficients $\underline{M}$, $\underline{B}$ and $\underline{K}$ are square matrices in which the off-diagonal elements represent coupling between the various degrees-of-freedom.

It was shown that Equation (3.1) could be re-expressed in the frequency domain as the following equation

$$\underline{z}(\omega) = \frac{\text{Adj}\{\underline{M}(\text{i}\omega)^2 + \underline{B}(\text{i}\omega) + \underline{K}\} \underline{u}(\omega)}{\text{Det}\{\underline{M}(\text{i}\omega)^2 + \underline{B}(\text{i}\omega) + \underline{K}\}} . \quad (3.2)$$

The present Item and associated computer program implement a single-input single-output system and so the output and input vector quantities, $\underline{z}(\omega)$ and $\underline{u}(\omega)$, respectively, in Equation (3.2) are reduced to scalar quantities, $z_i(\omega)$ and $u_j(\omega)$, representing excitation at a single point, j , and response at a single location, i , at which the set of measurements are taken. In terms of system receptance, $R(\omega)$, Equation (3.2) may then be written as

$$R(\omega) = \frac{\text{Adj}\{\underline{M}(\text{i}\omega)^2 + \underline{B}(\text{i}\omega) + \underline{K}\}}{\text{Det}\{\underline{M}(\text{i}\omega)^2 + \underline{B}(\text{i}\omega) + \underline{K}\}} , \quad (3.3)$$

where $R(\omega) = z_i(\omega)/u_j(\omega)$.

The denominator of Equation (3.3) may be reformulated into the product of n_t quadratic factors in $(\text{i}\omega)$ and then separated into n_t terms given by

$$R(\omega) = \sum_{s=1}^{n_t} \frac{E_s + \text{i}\omega F_s}{(\text{i}\omega)^2 + \text{i}\omega A_s + B_s} . \quad (3.4)$$

The coefficients A_s and B_s are approximated by

$$B_s \approx (\omega_r)_s^2 , \quad s = 1, n_t \quad (3.5a)$$

$$\text{and} \quad A_s \approx 2\zeta_s(\omega_r)_s , \quad s = 1, n_t , \quad (3.5b)$$

where $(\omega_r)_s$ and ζ_s , $s = 1, n_t$ are the resonant frequencies and damping ratios, respectively, of the n_t modes of the system. Equations (3.5a) and (3.5b) are approximate since the resonant frequencies are approximations to the natural frequencies of the system.

The transformed model representation, Equation (3.4), representing the original coupled Equation (3.1), is now an equation consisting of n_t independent frequency response functions, each driven by a complex forcing function quantified by E_s and F_s , and characterised as a second-order response by the variables A_s and B_s .

The new set of unknown parameters A_s , B_s , E_s and F_s are not identifiable directly with the physical qualities of the system but are related to them. They are found by equating the coefficients, having the same power of $i\omega$, Equations (3.3) and (3.4). (The relationships for a two degree-of-freedom system are given in ESDU 90006^[2] as Equations (5.10).)

3.1 Residual Effects

The foregoing analysis and the method outlined in ESDU 90006^[2] have assumed that the frequency range covered by the set of measurements and the analysis includes the resonant characteristics of all the modes of interest of the system and that other low or high frequency modes that may have also been excited have negligible influence within the frequency range of the analysis data. Although it is a necessary condition in the analysis that the resonant frequencies of the modes of interest are well separated from the resonant frequencies of the unwanted modes these latter modes may well have an influence within the frequency range of the analysis due to residual response effects.

For example in a vibration flutter test of a wing structure low frequency rigid body modes and other high frequency modes may well have been excited by the input excitation but could lie outside the range of flutter frequencies of practical interest.

An approximation to these residual effects can be made in the following way.

Consider the system receptance, Equation (3.4), rewritten as three identical terms each covering a different range of frequency response modes, as follows:

$$R(\omega) = \sum_{s=1}^{n_l-1} \left\{ \frac{E_s + i\omega F_s}{(i\omega)^2 + i\omega A_s + B_s} \right\} + \sum_{s=n_l}^{n_u} \left\{ \right\} + \sum_{s=n_u+1}^{n_t} \left\{ \right\} . \quad (3.6)$$

The first and third terms in Equation (3.6) represent the low frequency modes ($s = 1$ to $n_l - 1$) and high frequency modes ($s = n_u + 1$ to n_t), respectively. The second term represents the n modes of interest ($n = n_u - n_l + 1$) which are those modes whose response is covered by the range of frequencies of the measurement and analysis data and so are to be identified in the analysis.

The response effects of the low frequency terms can be approximately accounted for by assuming that frequencies within the range of the analysis data are much greater than the resonant frequencies of the low frequency modes, i.e. $\omega \gg (\omega_r)_s$ for $s = 1, (n_l - 1)$. The system receptance of these modes is then approximated by

$$R(\omega)_{s < n_l} \approx \sum_{s=1}^{n_l-1} \frac{E_s + i\omega F_s}{(i\omega)^2} \equiv \frac{E_l + i\omega F_l}{-\omega^2} \quad (3.7)$$

and represents a mass residual effect.

Similarly for the high frequency modes that are not to be specifically identified, an approximation given by $\omega \ll (\omega_r)_s$ for $s = (n_u + 1)$, n_t is suitable, which results in a system receptance given by

$$R(\omega)_{s > n_u} \approx \sum_{s = n_u + 1}^{n_t} \frac{E_s + i\omega F_s}{B_s} \equiv E_u + i\omega F_u \quad (3.8)$$

and represents a stiffness residual effect.

With the use of the latter two approximations, Equation (3.6) reduces to

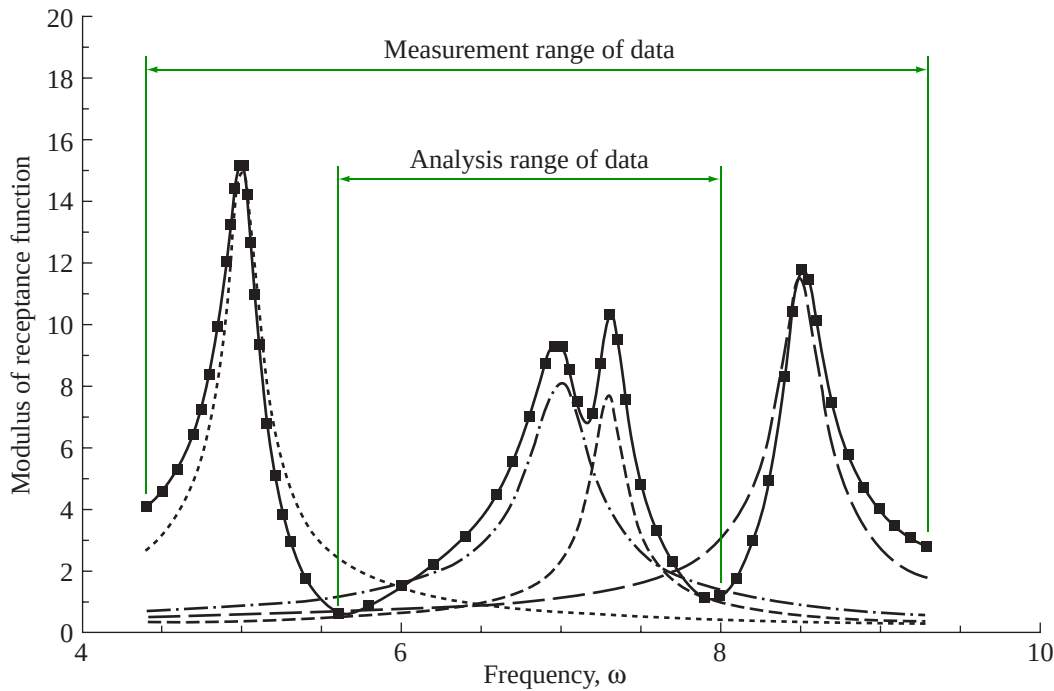
$$R(\omega) = \frac{E_l + i\omega F_l}{-\omega^2} + \sum_{s=1}^n \left\{ \frac{E_s + i\omega F_s}{(i\omega)^2 + i\omega A_s + B_s} \right\} + E_u + i\omega F_u, \quad (3.9)$$

where l and u refer to the lower and upper residuals, respectively.

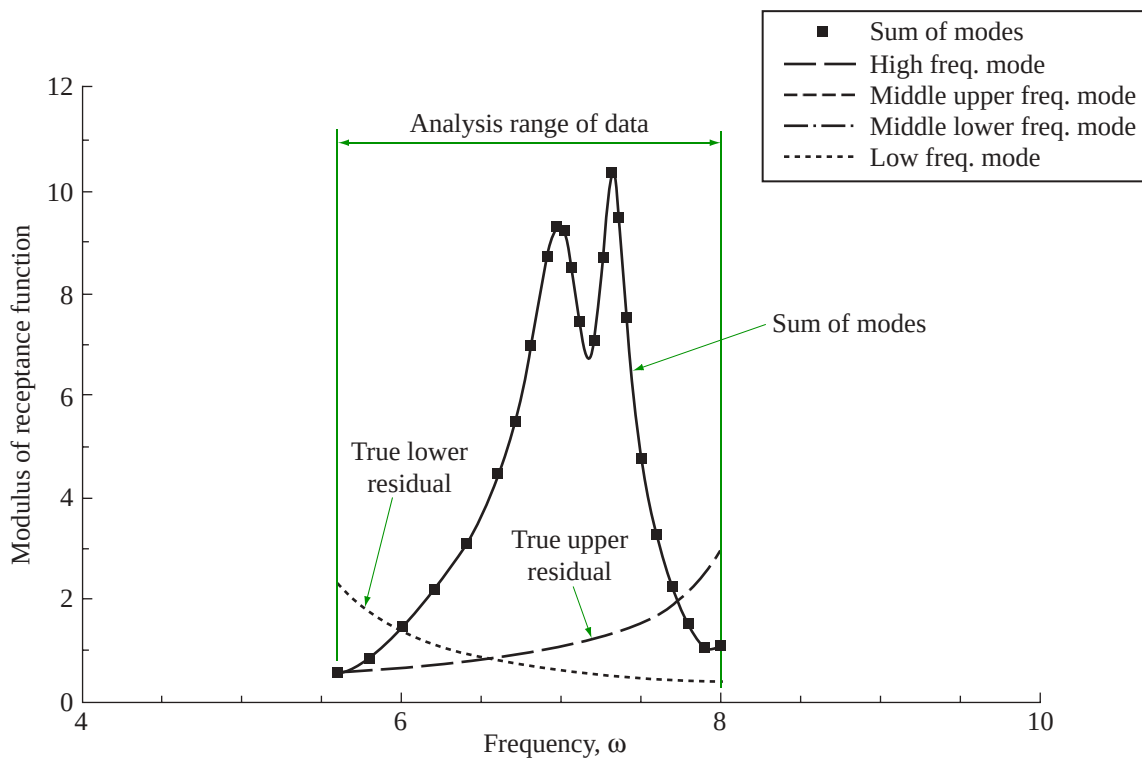
A graphical illustration of residual effects is given in Sketch 3.1 where the modulus of a receptance function is plotted versus frequency. The system is known to consist of only four degrees-of-freedom and the measurement range of the data as indicated in Sketch 3.1(a) is seen to contain the majority of the response activity of all four response modes. Sketch 3.1(a) shows the sum of the modal responses together with the four individual responses, plotted as various types of dotted line, that summed together equal the total response. It may be noted that the sum of the responses is a vector sum and so can be smaller than the arithmetic sum of the individual responses.

To illustrate the effect of residuals an analysis range of data that includes the middle two response modes has been chosen, a frequency range between $\omega \approx 5.6$ to 8.0 rad/s being appropriate. The sum of the modal responses in this frequency-range together with the residual of the lowest frequency and the residual of the highest frequency have been replotted in Sketch 3.1(b). These are the true lower residuals as seen from the individual response of Sketch 3.1(a). Equations (3.7) and (3.8) are used as approximations to account for these residual effects. When more than one higher and/or lower frequency mode exists outside the range of analysis, then Equations (3.7) and (3.8), respectively, are used to approximate the combined effect of these modes.

Careful consideration should be given to the range of frequencies chosen for the measurement and the analysis so as not to exclude modes that are closely coupled to those under investigation.



(a) Individual mode responses and response of sum of modes



(b) Analysis range of data showing true upper and lower residuals

Sketch 3.1 Illustration of residual effects

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3.2 Initial Parameter Estimates

Prior to implementation of the full parameter estimation procedure it is necessary to find initial estimates of the unknown parameters. These consist of the modal numerator and denominator coefficients (E_s , F_s , A_s and B_s , $s = 1, n$) and the coefficients of the residual terms (E_l , F_l , E_u and F_u) as identified from Equation (3.9). In a complete and exact description of the system the sum of the imaginary parts of the numerator coefficients for all the modes of a system is zero, that is

$$F_n = - \sum_{s=1}^{n_l-1} F_s, \quad (3.10)$$

as shown in a two degree-of-freedom system in Equation (5.7) of ESDU 90006^[2]. However, for all practical systems, analysis is completed within a range of frequencies that does not include all the resonances of the system and so this constraint is not used in the computer program implementation.

The denominator coefficients can be estimated in the following way as described in Section 4 of ESDU 90006^[2]. From an examination of the Nyquist diagram of the measurement data the number of degrees-of-freedom or modes present in the system will produce a characteristic circular arc plot with a resonant frequency, ω_r , as the point on the curve where the rate of change of arc length with frequency is a maximum. An initial estimate of the natural frequency of each mode may then be taken, for lightly damped systems to which this Item is applicable, as the resonant frequency. Also from each circular arc the damping ratio, ζ , of each mode may be estimated with the use of one of the methods outlined in ESDU 90006^[2]. Initial estimates of the parameters A_s and B_s , $s = 1, n$ are then found from

$$B_s = (\omega_n^2)_s \approx (\omega_r^2)_s, \quad s = 1, n \quad (3.11a)$$

$$\text{and} \quad A_s = (2\zeta\omega_n)_s \approx (2\zeta\omega_r)_s, \quad s = 1, n. \quad (3.11b)$$

The unknown parameter set, γ , consists of the following parameters:

$$[\gamma_k]_{k=1, (2n+4)} = [E_l, F_l, E_u, F_u, (E_s, F_s)_{s=1, n}]. \quad (3.12)$$

The initial estimates, γ_k , can be found in a manner as described in Section 5.4 of ESDU 90006^[2].

The measured responses, $R_m(\omega)$, differ from the mathematical model responses, $R_e(\omega)$, and these differences are denoted by

$$\varepsilon_j = (R_m - R_e)_j, \quad j = N_l, N_u, \quad (3.13)$$

where $(N_u - N_l + 1)$ is the total number of measurements used in the analysis. An Objective or Cost Function, J , is formed as the sum of the squares of these difference quantities as follows.

$$J = \sum_{j=N_l}^{N_u} \varepsilon_j \varepsilon_j^*, \quad (3.14)$$

where * refers to complex conjugate and the summation is taken over all the measurements used in the analysis.