

COMPUTATION OF STATIC PRESSURE DOWNSTREAM OF A NORMAL SHOCK FOR HYPERSONIC FLIGHT (AMBIENT TEMPERATURE KNOWN)

1. NOTATION, UNITS, CONSTANTS AND DEFINITIONS

1.1 Notation and Units

		SI	British
a	values of p_2/p_{t_2} at $H_p = 32, 47$ or 51 km and at minimum and maximum H_p		
b	values of $\log_{10}[10^4 p_{t_2}/p_{ref}]$ at $H_p = 32, 47$ or 51 km and at minimum and maximum H_p		
$f_1, f_2, f_3 \dots$	denote functional relationships of type f [.....]		
H_p	pressure height	m, km	ft
h	specific enthalpy of air (or gas)	J/kg	lbf ft/slug
M	Mach number		
p	static pressure of air (or gas)*	N/m ² , mb	lbf/ft ²
p_a	ambient static pressure*	N/m ² , mb	lbf/ft ²
p_t	total pressure of air (or gas)*	N/m ² , mb	lbf/ft ²
$[p_2/p_{t_2}]$	ratio of pressures behind a normal shock (equilibrium conditions, see Section 1.3)		
R	gas constant for air	J/kg K	lbf ft/slug K
T	static temperature of air (or gas)	K	K
T_a	ambient temperature	K	K
T_{std}	ambient temperature at height H_p in Standard Atmosphere	K	K
T_t	total temperature of air (or gas)	K	K
V	true airspeed of vehicle	m/s	ft/s
Z	compressibility factor, ($= [p/\rho R_{ref} T]$), see Section 1.3		
γ	ratio of specific heat capacities of air		

For footnotes see end of Notation.

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Δ	denotes increment in affixed parameter		
ρ	air density	kg/m ³	slug/ft ³

Subscripts

a	denotes ambient conditions
max	denotes maximum value
min	denotes minimum value
ref	denotes reference [†] conditions at sea level
std	denotes conditions in Standard Atmosphere [‡]
t_2	denotes parameter(s) for equilibrium gas state at stagnation downstream of normal shock
1	denotes conditions ahead of normal shock
2	denotes conditions behind normal shock
32, 47, 51	denotes parameter applicable to $H_p = 32, 47$ or 51 km

* Note that these pressures are absolute.

[†] "Reference" values are as given in the source data; they do not always correspond with "Standard" values. See Reference 6.

[‡] The "Standard Atmosphere" used here is presented in References 2, 3 and 4. The term embraces what the International Organization for Standardization (ISO) designates as follows:

For $H_p \leq 50$ km (164 042 ft): International Standard Atmosphere,

For $50 \leq H_p \leq 80$ km (262 467 ft): Interim Standard Atmosphere.

(Reference 3 also includes data for greater heights, based on the U.S. Standard Atmosphere, 1976.)

1.2 Constants

Reference pressure ^{**} , p_{ref}	101 324.6 N/m ²	2 116.224 lbf/ft ²
Gas constant for air ^{**} , R_{ref}	287.227 77 J/kg K	3 091.694 lbf ft/slug K

^{**} "Reference" values are as given in the source data; they do not always correspond with "Standard" values. See Reference 6.

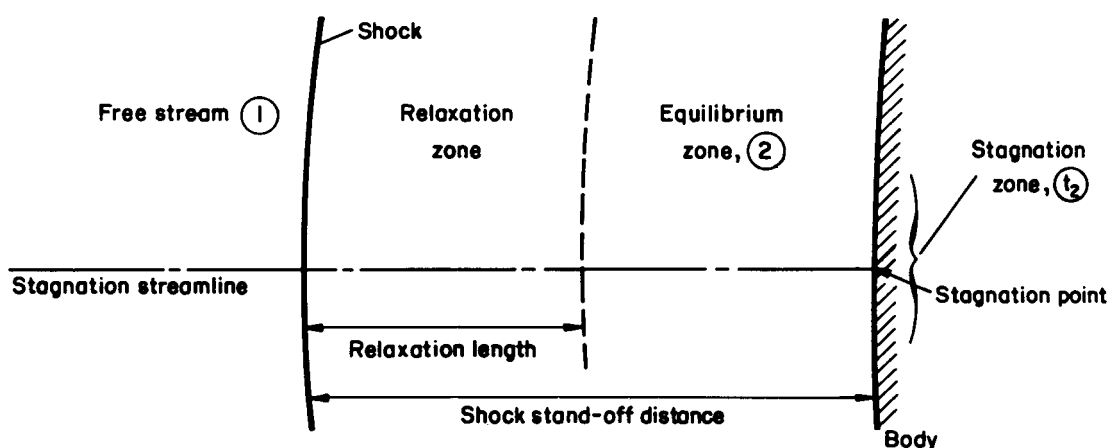
1.3 Definitions (Repeated from Reference 6)

- (i) **An Ideal Gas** is thermally and calorically perfect. The thermal equation of state is obeyed, *i.e.* $p = \rho RT$, and the specific heat capacities are constant.
- (ii) **A Calorically Imperfect Gas** does not have constant specific heat capacities and their ratio, γ , is a function of temperature. It may be thermally perfect however.

- (iii) **A Real Gas** is one where $p \neq \rho RT$, where γ is not constant, and where gaseous imperfections, such as molecular dissociations, occur. In such a state the gas may also be described as imperfect.
- (iv) **A Relaxation Zone** contains air having the properties of a real gas. The gas is vibrationally excited and, if the temperature is high enough, is in a state of partial dissociation. These states are not stable so that the relaxation zone is followed by an **equilibrium zone** in which the gas has regained thermal equilibrium, see Sketch 1.1.
- (v) **A Frozen Gas State** is one in which the gas may be assumed to retain the initial internal excitations and dissociations that are brought about by some upstream flow process, for example a shock wave. This implies that the length of the relaxation zone is very much greater than the largest flow field dimension of interest.
- (vi) **Compressibility factor, $Z = p/\rho R_{ref} T$** , for the gas conditions relevant to this Data Item is to be seen as a measure of the change in the number density of air due to dissociation, compared to the thermally perfect gas state. It is a function of gas temperature and pressure and, for a diatomic gas, has the following values:

$Z = 1.0$ for all freestream ambient conditions,
 $Z > 1.1$ for dissociation effects to be significant,
 $Z \approx 2.0$ for a completely dissociated gas.

- (vii) **Sensed and Registered (Indicated) Quantities.** “Registered” (“indicated”) values are those provided by transducers (instruments) in an air data system, which have been corrected for any calibration or other inherent errors. Each **transducer** responds to the sensed quantity, which is output by a sensor and conducted by an appropriate part of the air-data system.



Sketch 1.1 Flow conditions in region of air-data sensors

2. INTRODUCTION

This Data Item is ancillary to and is for use in conjunction with Data Item No. 88025, Reference 6, which provides a definitive method of deriving primary air-data parameters for hypersonic flight with an excellent precision, *provided that an accurate value of static pressure, p_2 , can be obtained*. The present Data Item provides an accurate means of deriving this static pressure without having to rely on the use of a static

pressure sensing system which is not practicable for hypersonic flight conditions. The value of static pressure, so derived, is used in the calculation methods laid out in Reference 6 to obtain primary air-data parameter values, such as Mach number, pressure height, true airspeed, equivalent airspeed, *etc.*

This Item uses tables of the ratio, p_2/p_{t_2} , of pressures behind a normal shock, which is given in terms of the values of input parameters which are available from tables given in Reference 6 and *from the sensing and registration of total temperature and total pressure alone.*

For the method of Reference 6 and its use in conjunction with this Data Item, the vehicle forebody on which the sensors are mounted is assumed to be bluff. As a result the leading shock wave is situated ahead of the forebody and is normal to the velocity vector.

Table 8.1 presents values of the ratio of pressures, p_2/p_{t_2} , for the Standard Atmosphere (References 2, 3 and 4) and **Table 8.2** enables corrections to be made for non-standard conditions.

Section 3 discusses the problem of deriving static pressure, p_2 , using sensors mounted on the flight vehicle as the initial pressure source and considers solutions. It also discusses the precision with which Mach number, as a typical primary parameter, may be derived.

Section 4 is concerned with those characteristics of non-ideal air which are relevant to the computation of static pressure behind a normal shock for speeds not greater than $M \approx 10$.

Section 5 describes the computation of static pressure starting from the use of tables of the ratio of pressures for the Standard Atmosphere (Tables 8.1 and 8.2), and, subsequently, offering corrections for non-standard conditions using Table 8.3. The residual uncertainties for the computation are discussed.

Section 7 gives a worked example, which is for the same flight condition as that given in Reference 6.

3. DERIVATION OF STATIC PRESSURE

3.1 General Considerations

The evaluation of such primary air-data parameters as Mach number or airspeed presents novel problems in hypersonic flight compared to the situation for airspeeds up to $M \approx 2$. For lower airspeeds, calculation methods use ideal gas laws to obtain Mach number from values of total and static air pressures. In general, these calculations are made by an electronic analysis sub-system within the air-data system installed on the flight vehicle. The pressures are registered by transducers responding to sensed data output by total and static pressure sensors and, in general, the whole procedure is one of the functions of an air-data system. Whilst it is still possible to make use of the vehicle total pressure and total temperature sensors at higher speeds, the static pressure sensor mounted on the surface of the flight vehicle is likely to become unusable as a means of deriving ambient air pressure. The usual relationships between sensed static pressure and ambient pressure break down when the air behind a leading shock wave departs so far from being an ideal gas that it is both calorically and thermally imperfect. It is then not possible to evaluate, by any known calibration methods, that difference between sensed surface static and ambient pressures known as “static error” or “position error”, which is required for calculation of the primary air-data parameters*. Changes in the methods of analysis are obligatory for such gas states, so that the air behind the leading shock wave is treated as a real gas. Ambient static pressure cannot be derived directly and it is necessary, initially, to

* For supersonic speeds up to, at least, $M \approx 2$, calibration tests may be performed in flight at cruise heights. These make use of high accuracy radar height measurements in conjunction with data from research aircraft and from meteorological balloons. Such calibrations enable ambient pressure to be derived from the aircraft static pressure measurements to a precision which is satisfactory for the evaluation of such primary air-data parameters as height, airspeed and Mach number.

derive the static pressure behind the leading shock wave. Pressure and temperature are interdependent, however, and it is possible to make use of Gas Tables to interpret the pressure and temperature information output by the vehicle sensors.

Reference 6 pays some attention to the instrumentation errors and the residue uncertainties associated with calibration deficiencies, but the effects on the static pressure instrumentation due to the precision constraints imposed by the calculation method are not fully addressed. However, these aspects have been considered carefully in Reference 8, which makes the following conclusions.

- (a) For the measurements of total temperature and total (*i.e.* pitot) pressure, the requirements imposed on the sensing and registering apparatus are thought to be reasonable and capable of being met. Reference 7, which considers the systems available for air-data measurement at these speeds, makes similar conclusions.
- (b) For the measurement of static pressure the constraints for conventional pressure sensing apparatus are too severe. Part of the problem was acknowledged in Paragraph 6.3 of Reference 6, which states:
 - (i) “The sign of the residual uncertainty in the registered value of the static pressure must be the same as that for the total (*i.e.* pitot) pressure in every case.
 - (ii) The magnitude of the residual uncertainty in the registration of static pressure must be very close to that for total pressure in every case.”

3.2 Methods Available

3.2.1 Sensing of surface static pressure

In the method of Reference 6 it is assumed that surface static pressure sensing will be used – for which the transducer registers differential pressure relative to the absolute value of total pressure. Section 6.3 of Reference 6 lays down a minimum value for the residual uncertainty in gauge pressure of ± 0.050 mb, or ± 0.10 lbf/ft². However, not only is the severity of this requirement unrealistic for such instrumentation but, for some regimes of flight, where the value of total pressure is very low, it is not severe enough, see Section 3.2.3.

3.2.2 Use of tables of the ratio of pressures

The use of Tables 8.1 to 8.3 enables static pressure, p_2 to be derived in terms of the known value of total pressure (see Section 5.3) and it follows that the criteria (i) and (ii) of Section 3.1, item (b) are satisfied. Section 5.3 shows that the only other parameter required for using the tables is total temperature, which can be measured satisfactorily.

Table 8.2 (and Table 8.1) are based on the assumption that ambient temperature is that of the Standard Atmosphere of References 2, 3 and 4. The data are given for heights of 32 km, 47 km and 51 km at which changes occur in ambient temperature lapse-rate. Limiting case data are also give appropriate to the minimum and maximum heights applicable to the Data Item (see Section 5.1 for these heights).

Table 8.3 presents information for adjusting data, evaluated using Tables 8.1 and 8.2, to non-standard conditions. This information is dependent on specific enthalpy and on the ambient temperature lapse-rate.

As an example of the calculation of a primary air-data parameter, Section 3.2.3 considers the precision with which Mach number may be derived using the uncertainties discussed in Section 5.

3.2.3 Precision of computed primary air-data parameters

The methods of Reference 6 have been used to calculate primary air-data parameters with allowances for the uncertainties implicit in the derivation of static pressure. Since Mach number is involved directly in the efficient control of a flight vehicle and of any air-breathing propulsion unit, its precision requirement is likely to be the most stringent. Although this precision requirement cannot be established firmly, the value of 1% proposed in Reference 7 is considered to be a satisfactory target. This value is closely in line with the known characteristics of the intake control system in the "Concorde" aircraft. Reference 8 evaluates the achievable precisions in Mach number, as follows.

For the method discussed in Section 3.2.1, the precision is 4.9% (this value is very close to the 4.3% derived in Reference 6, Case 1, which is for an almost identical condition).

For the method discussed in Section 3.2.2, *i.e.* using tables of the ratio of pressures, a precision of about 1.3% is obtained. This value assumes that ambient air temperature is known to within 5 K – a pessimistic assumption. An assumption of 1 – 3 K would enable the target precision of 1% to be bettered.

4. GENERAL CONSIDERATIONS FOR NON-IDEAL AIR

4.1 Characteristics of Thermally Perfect, Calorically Imperfect Air ($Z = 1$, $\gamma \neq 1.4$)

The general form of the relationships of the ratio of pressures for real gases is derivable from the equations given in Appendix A of Data Item No. 83029 (Reference 5) and in Reference 1. That Appendix is concerned with the treatment of temperature-dependent quantities for total temperatures between 400 and 1000 K and provides a method for deriving primary air-data parameters between $M \approx 2$ and $M \approx 4$. In this temperature range air departs from a calorically perfect state so that the ratio of specific heats no longer equals 1.4. Such a gas state may be described as a ($Z = 1$, $\gamma \neq 1.4$) gas. The variation of the specific heats may be estimated assuming that the vibrational heat capacity is characterised as a simple harmonic vibrator. Reference 1 uses this assumption to derive an integrated form of the adiabatic energy relationship which links total temperature, ambient temperature and Mach number. An isentropic relationship is also derived in the Reference linking the ratio (total/static) temperatures behind a normal shock with the ratio (total/static) pressures. These relationships are given in terms of the vibrational temperature characteristic for air.

The relationships given in Reference 1 are solved either directly or by iteration to give:

$$\text{static temperature, } T_2 = f_1(M, T_a), \quad (4.1)$$

$$\text{total temperature, } T_{t_2} = f_2(M, T_a), \quad (4.2)$$

$$\text{total specific enthalpy, } h_{t_2} = f_3(M, T_a), \quad (4.3)$$

$$\text{ratio (total/static) pressure, } p_{t_2}/p_2 = f_4(T_{t_2}, T_2), \quad (4.4)$$

$$\text{velocity aft of shock, } V_2 = f_5(M, T_a, T_2), \quad (4.5)$$

$$\text{static pressure, } p_2 = f_6(M, T_a, T_2, p_a). \quad (4.6)$$