

EXAMPLE OF STATISTICAL TECHNIQUES APPLIED TO ANALYSIS OF EFFECTS OF SMALL CHANGES

1. NOTATION AND UNITS

F	F Variance Ratio test statistic, see Section 5.2 of Reference 1
$s_{\Delta\gamma}$	sample standard deviation of $\Delta\gamma$
$\hat{s}_{\Delta\gamma}$	best estimate of population standard deviation of $\Delta\gamma$ based on small sample
$\hat{s}_{\Delta\gamma}^2$	best estimate of population variance of $\Delta\gamma$ based on small sample
z	standardised Normal variable, see Reference 1
γ	per cent climb gradient
$\Delta\gamma$	measured value minus scheduled value of per cent climb gradient
$\overline{\Delta\gamma}$	sample mean value of $\Delta\gamma$
$\mu_{\Delta\gamma}$	population mean value of $\Delta\gamma$
$\sigma_{\Delta\gamma}$	population standard deviation of $\Delta\gamma$
$\sigma_{\Delta\gamma}^2$	population variance of $\Delta\gamma$
$\sigma_{\overline{\Delta\gamma}}$	standard error of the mean for $\Delta\gamma$

Subscripts

b	denotes basic model of aircraft
m	denotes fully modified version of aircraft
s	denotes scheduled value
v	denotes “special variant” of aircraft

2. INTRODUCTION

This Data Item uses the results of two sets of second segment climb gradient* measurements to provide, in Sections 3 and 4, simple examples of the use of statistical significance testing and confidence intervals. An introduction to statistical techniques for use in analysis of aircraft performance data is given Data Item No. 91017 (Reference 1).

* For definition of “second segment climb gradient” and its “associated conditions”, see the relevant Airworthiness Requirements. For the purposes of this Data Item, see Section 3 of Data Item No. 76011.

3. EXAMPLE 1

3.1 The Data

- (a) During the certification trials of the “basic” model of a new twin turbo-prop transport aircraft the second segment climb gradient was established from a large number of tests as exceeding the scheduled gradient by a mean margin of 0.09% gradient, giving,

$$\overline{\Delta\gamma_b} = 0.09\% \text{ gradient.} \quad (3.1)$$

In addition, the results of these tests, supported by theoretical analysis, indicated a value of 0.35% gradient for the standard deviation of the difference between measured and scheduled values of second segment climb gradient, including the variability due to measurement errors (see Reference 3). Therefore,

$$s_{\Delta\gamma_b} = 0.35\% \text{ gradient.} \quad (3.2)$$

However, since these values are derived from a large number of tests and the value of standard deviation is further supported by theoretical analysis, they can be taken as being population values. Consequently,

$$\mu_{\Delta\gamma_b} = 0.09\% \text{ gradient} \quad (3.1(a))$$

$$\sigma_{\Delta\gamma_b} = 0.35\% \text{ gradient.} \quad (3.2(b))$$

- (b) Following certification of the basic model it was decided to offer a “Special Variant” embodying a number of minor aerodynamic changes. The effect of these changes on second segment climb gradient was estimated to be negligible but it was decided to carry out a small number of check climbs to verify this conclusion, giving rise to the following data.

<i>Test number</i>	<i>Measured climb gradient %</i>	<i>Scheduled climb gradient %</i>	$\Delta\gamma_v$ %
1	2.38	2.20	0.18
2	2.42	2.25	0.17
3	1.95	2.29	−0.34
4	2.11	2.31	−0.20
5	2.26	2.35	−0.09
6	2.43	2.38	0.05
7	2.83	2.42	0.41
8	2.74	2.44	0.30
9	2.46	2.47	−0.01
10	2.40	2.50	−0.10

Following the procedures of Sections 2.4 and 2.5 of Reference 1, these values led to the following results for arithmetic mean and standard deviation.

$$\overline{\Delta\gamma_v} = 0.04 \% \text{ gradient} , \quad (3.3)$$

$$\hat{s}_{\Delta\gamma_v} = 0.23 \% \text{ gradient} . \quad (3.4)$$

3.2 The Analysis

- (a) The differences apparent between the mean and standard deviation values from the two sets of data cast some doubt on the accuracy of the pre-test forecast of “negligible effect”. For this forecast to be verified, the mean and standard deviation values had to be shown to agree within acceptable limits, taking account of the levels of variability inherent in such measurements (see Sections 5.2 and 3.2 of Reference 1).
- (b) When carrying out a comparison between two sets of results of the within-set variability, it is usual to compare the variance values, rather than compare standard deviation values directly. This is done by using the Fisher's Variance Ratio “ F ” test (see Section 5.2 of Reference 1) which may be applied as follows.

Since $\sigma_{\Delta\gamma_b} > \hat{s}_{\Delta\gamma_v}$, the value of F may be calculated using

$$F = \frac{\sigma_{\Delta\gamma_b}^2}{\hat{s}_{\Delta\gamma_v}^2} = \frac{0.35^2}{0.23^2} = 2.32 . \quad (3.5)$$

The corresponding numbers of degrees of freedom are, respectively, ∞ and 9. Consequently, the value of F read from Table 11.5 of Reference 1, at the 5% level, is 2.71. Since the calculated value of F is less than 2.71, the difference between the variance values can be taken to be “not significant” and the population variance can be taken to apply equally to the “Special Variant” results.

- (c) Since the population variance and standard deviation are known (see (b) above) the procedure described in paragraph 3.2.2 of Reference 1 can be used to test the significance of the difference observed between the mean of the “Special Variant” sample of check climbs and the (population) mean of the results achieved by the basic aircraft.

The difference between these two means is

$$\begin{aligned} \overline{\Delta\gamma_v} - \mu_{\Delta\gamma_b} &= 0.04 - 0.09 \\ &= -0.05 \% , \end{aligned} \quad (3.6)$$

and from Equation (3.1) of Reference 1 the standard error of the mean may be calculated as

$$\begin{aligned} \sigma_{\overline{\Delta\gamma}} &= \frac{0.35}{(10)^{1/2}} = \frac{0.35}{3.16} \\ &= 0.11 \% \text{ gradient} . \end{aligned} \quad (3.7)$$