

VORTEX GENERATORS FOR CONTROL OF SHOCK-INDUCED SEPARATION PART 1: INTRODUCTION AND AERODYNAMICS

1. INTRODUCTION

This Data Item is the first of a sequence of three Items. The objectives here are to describe the situations where control of shock-induced separation may be necessary or desirable, to describe the types of vortex generator that have been used or proposed, and to give an account of the aerodynamic principles and effects of vortex generators, so far as these are relevant to the control of shock-induced separation of a turbulent boundary layer. Results relating to the influence of the type of vortex generator array, to different types of vortex generator, and to the influence of various vortex generator parameters are also presented. Consideration of these results is important in the initial selection of any array of vortex generators, and during any necessary subsequent changes in the array parameters.

Most commonly, where vortex generators have been applied to control shock-induced separation, the vane type of vortex generator has been adopted. In Part 2 (Data Item 93025), recommendations are presented relating to the application of vane-type vortex generators for control of shock-induced separation.

In Part 3 (Data Item 93026), some example applications of vortex generators are described in such a way as to demonstrate the aerodynamic principles involved and their practical consequences.

In many situations the flight conditions at which aircraft operate are constrained by the effects of boundary layer separation. For subsonic jet- and fan-engined aircraft, which cruise typically at a Mach number around 0.8, the margins imposed by airworthiness requirements require the absence of unacceptable buffeting at various flight conditions, the most demanding being those corresponding to higher C_L and/or M_∞ values than apply to the cruise. The degree of buffeting at these higher C_L or M_∞ values is related mainly to the extent of shock-induced separation of the wing boundary layer. Military aircraft may be required to operate and manoeuvre over a wide range of flight conditions, including supersonic speed. Excessive buffeting, and other consequences of shock-induced (and other) separation, limit the manoeuvres that may be carried out.

Many means have been proposed in order to control boundary layer separation and alleviate its adverse effects on flight. Of those entailing only minor changes to the aircraft configuration, the most effective is the application of vortex generators.

Since the principle of boundary layer control by vortex generators was formulated^{1,3}, numerous aircraft have incorporated vortex generators. Generally they have been in the form of small vanes, and not only have they been applied to the wings[†], but in some cases they have been used to improve the flow about the tailplane, fin, fuselage, cowl, and wing-fuselage (or other) junction regions. Although these vortex generating devices are small, their effects can be considerable, and benefits resulting from their adoption have often been significant. For example, in one wind tunnel test, the value of shock-upstream Mach number corresponding to significant separation was increased by vortex generators from 1.3 to slightly more than 1.5. These benefits may be partly offset by adverse effects. For example, under cruise conditions, where the vortex generators would not be needed for separation control, their presence may increase the aircraft drag, usually by a small amount however. Much can be done in choosing configurations to give just the benefits required but with minimum penalties. In principle, the possibilities provided by utilising vortex generators can be taken into account during the various stages of the design of an aircraft and its introduction into service, and also when an existing aircraft is modified, or 'stretched' to yield enhanced performance.

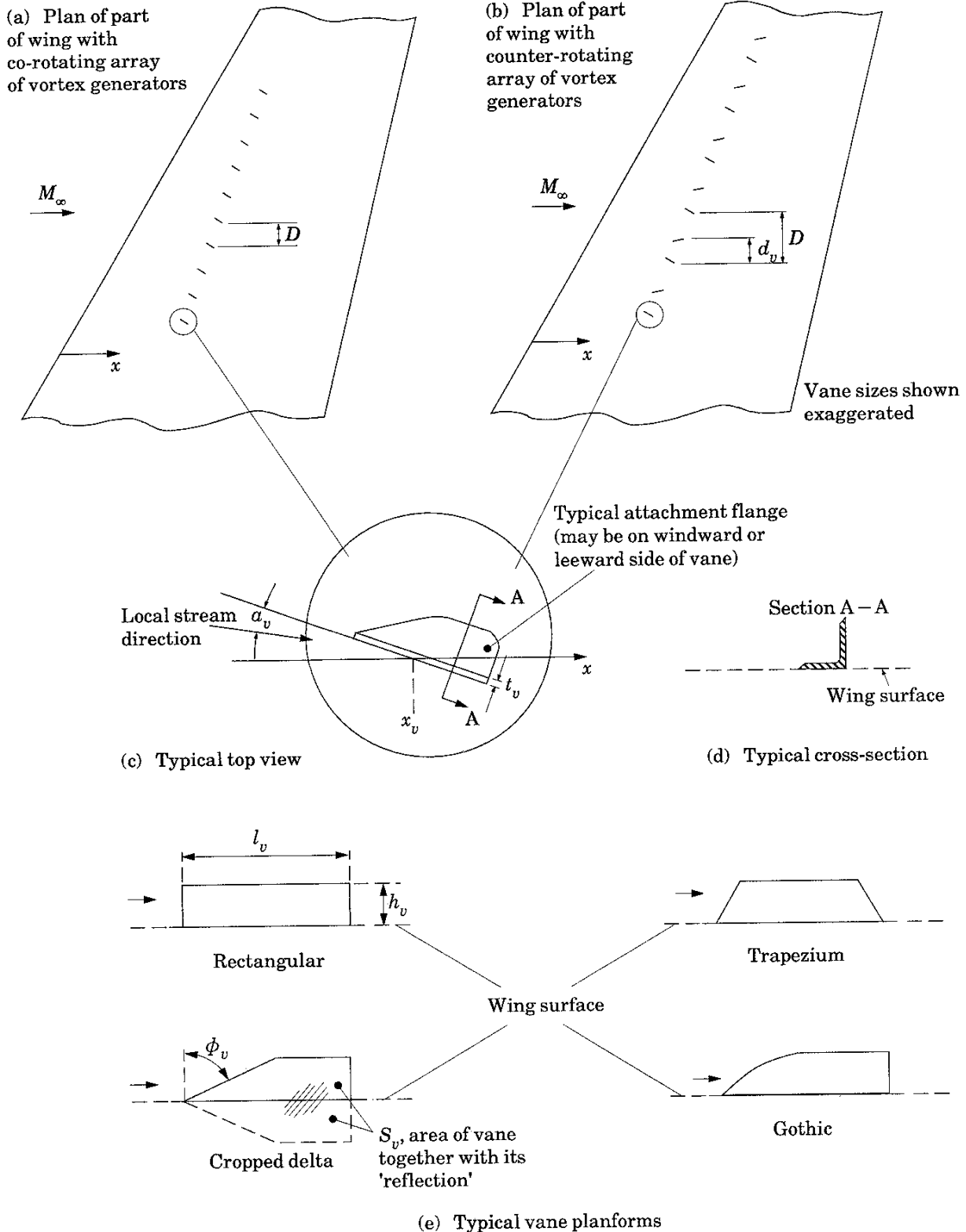
[†] Reference to a wing should be taken to include other aircraft lifting surfaces.

2. NOTATION

A_v	aspect ratio of vane vortex generator (<i>together with its reflection</i>), i.e. $4h_v^2/S_v$, see Sketch 2.1
b	wing span
C	constant
C_{D_v}	drag coefficient of vane, i.e. $2D_v/(\frac{1}{2}\rho_v U_v^2 S_v)$
C_L	lift coefficient based on wing reference area
C_{L_v}	lift coefficient of vane, i.e. $2L_v/(\frac{1}{2}\rho_v U_v^2 S_v)$
C_p	pressure coefficient
c	local chord
$\bar{c}$	geometric mean chord
c_f	local skin friction coefficient
D	lateral distance between neighbouring vortices (for co-rotating vortex generators), or least distance between vortices inducing rotational flow of same sense (for counter-rotating vortex generators)
D_v	drag of vane, i.e., component of aerodynamic force acting on vane in direction of local stream
d	lateral distance between pair of counter-rotating vortices, selected so that induced flow between them is towards surface, see Sketch 6.2
f	fraction of span over which vortex generators extend
f'	number related to f , see Section 8.1.1
H	boundary layer shape parameter, also local pitot pressure
H_∞	pitot pressure in undisturbed flow
h	local height of vortex
h_v	height of vane or other vortex generator device
K	local vortex strength
k_D	$D_v/(\frac{1}{2}\rho_v U_v^2 h_v^2) = 2C_{D_v}/A_v$
k_v	$K/h_v U_v$
L_v	lift of vane, i.e. component of aerodynamic force acting on vane normal to local stream
l_b	streamwise length of separated flow bubble

l_v	length of vane vortex generator, see Sketch 2.1
l_p	streamwise distance over which full pressure rise downstream of shock occurs with vortex generators present, see Section 7.2
M_L	local Mach number on wing or aerofoil surface
$M_{L_{nsh}}$	local Mach number of external flow normal to isobars immediately upstream of shock (quasi two-dimensional flow)
$M_{L_{sh}}$	value of M_L immediately upstream of shock (two-dimensional flow)
M_∞	freestream Mach number
m_d	drag magnification factor
p	local surface pressure
p_∞	freestream static pressure
$R_{\theta_{sh}}$	Reynolds number based on local boundary layer streamwise momentum thickness at 'shock upstream' conditions
r	distance measured radially from centre of vortex
r_c	vortex core radius
S	wing reference area, = $b\bar{c}$
S_v	planform area of vane vortex generator <i>together with its reflection</i>
s	wing semi-span
t_v	thickness of vane vortex generator, see Sketch 2.1
U_e	local stream speed at edge of boundary layer
U_v	local flow speed at edge of boundary layer at vane location
v	velocity component in y-direction
v_t	tangential or circumferential velocity component
w	velocity component in z-direction
x	streamwise coordinate, zero at local leading edge
x_R	value of x corresponding to point on upper surface where adverse pressure gradient becomes stronger, relatively suddenly, see Section 7.4.1
x_{sh}	value of x corresponding to shock location
x_v	value of x corresponding to vane mid root chord point, see Sketch 2.1
x_0	value of x corresponding to effective origin of vortex

y	co-ordinate in crossflow plane, tangential to surface
z	co-ordinate in crossflow plane, normal to surface
α	angle of incidence of wing
α_v	angle of incidence of vane vortex generator relative to local stream direction
Δ	angle defined by vortex locations in relation to corner
ΔC_D	$\Delta D / (\frac{1}{2} \rho_\infty U_\infty^2 S)$
ΔD	contribution to aircraft drag from array of vortex generators, see Section 8.1.1
$\Delta x_{1/2}$	streamwise distance over which vortex strength reduces by half, see Section 6.4.1
δ	boundary layer thickness (99 per cent thickness) at vortex generator location
ζ	$2\pi z/D$
θ	local streamwise momentum thickness, also angular polar coordinate in Section 6.4 and Appendix A
θ_{sh}	streamwise momentum thickness just upstream of shock location – corresponding to (or evaluated for) a boundary layer flow without vortex generators
ν	kinematic viscosity
ν_E	effective kinematic viscosity
ξ	$2\pi y/D$
ρ_e	local density at edge of boundary layer
ρ_v	density at edge of boundary layer at vane location
ρ_∞	freestream density
σ	angle between vortex paths and local stream direction, also angle between local stream direction and freestream direction
τ	swirl parameter, ratio of circumferential velocity component to axial velocity component in vortex flow, v_t/U_e
ϕ_v	leading edge sweep angle of vane vortex generator, see Sketch 2.1
ω	axial vorticity component in vortex flow, <i>i.e.</i> component parallel to main stream direction
Suffices	
i	denotes initial value, <i>i.e.</i> immediately downstream of vortex generator(s)
max	denotes maximum, in relation to v_t , ω and τ
te	denotes trailing edge



Sketch 2.1 Vane vortex generator features and notation

3. USAGE OF VORTEX GENERATORS TO CONTROL SEPARATION

Any situation where unwanted boundary layer separation occurs (or is predicted to occur), in the flow over an actual or project aircraft, is potentially a candidate for treatment by the application of vortex generators. Specifically, vortex generators have been applied for the following purposes.

- (a) Extending the boundary (in the C_L , M_∞ plane) imposed by unacceptable buffeting resulting from boundary layer separation of the wing flow. See, for example, Reference 7.
- (b) Removing, or reducing, a 'wing dropping' tendency resulting from boundary layer separation occurring asymmetrically over relatively outboard parts of the wing.
- (c) Alleviating a tendency of the aircraft to pitch up at relatively high lift as a result of either
 - (i) wing pitching moment characteristics being affected by boundary layer separation, or
 - (ii) flow field disturbances, due to wing boundary layer separation, influencing the tailplane. (Such disturbances may result from changes in the wing spanwise loading producing locally large downwash changes, which in turn may affect the tailplane loads.)
- (d) Alleviating a reduction in control effectiveness resulting from flow separation occurring either upstream of, or over, the control. Controls on wings, tailplanes and fins have been subject to such treatment (see Reference 8, for example).
- (e) Controlling, or regulating, the development of wing boundary layer separation so as to produce a steady progression of the effects of wing stall. The Harrier aircraft (and its developments) provide the major example of this application of vortex generators¹⁶.
- (f) Reducing local flow separations which produce nuisance effects; for example, local vibrations or (generally small) drag increments. Examples (of the last of these) are separations in junctions and over the rear fuselage³⁰.
- (g) Drag reduction; examples are upswept rear fuselages³⁰ and proposals for high-lift aerofoils⁵⁴.
- (h) On wind tunnel models, in order to remove or control a boundary layer separation which occurs at the test Reynolds number, but not at flight Reynolds numbers. The object here is to improve the wind tunnel representation of the full scale flows, see also Reference 41.

The above list of applications is not exhaustive. It ignores, for example, the important uses of vortex generators in ducts and diffusers.

It should be emphasised here that detailed consideration is given in this Item only to the control of shock-induced separation. However, the action of vortex generators in rendering a boundary layer less prone to separation, or in reducing the extent of a separated flow region, is a general one, and many of the considerations presented (relating, for example, to the effectiveness of mixing) are applicable independently of whether the separation to be controlled is shock-induced or is induced by a more gradual pressure rise.

3.1 Use of Vortex Generators in Design

Vortex generators have generally been applied at a relatively late stage in the design and development of aircraft in order to tackle a problem that was either unexpected or not amenable to reliable prediction; the vortex generators have then been used as a (post-design) 'fix'. However, in some cases (involving both

civil and military aircraft), vortex generators have been considered as a possible means of separation control at an early design stage. As a general principle, it is desirable that the possibilities for separation control by vortex generators should be considered throughout the various design stages of an aircraft, although this does not imply that it would necessarily be desirable for vortex generators actually to be utilised in any particular design. Much would depend on the assessment of the advantages and disadvantages of vortex generators as a means of separation control for the particular aircraft under consideration, as well as an assessment of the overall balance between the costs and the benefits of introducing vortex generators. For a transport aircraft, designing the wing to utilise vortex generators would give the possibility of reducing wing size, and thus aircraft weight. The wing designed with vortex generators might then have a smaller cruise drag (but not drag coefficient) than the wing designed without vortex generators.

A wing might be deliberately designed to have separation margins (in C_L and/or M_∞) which were unnecessarily large for the operating weights of the initial design, so as to allow for possible future developments of the aircraft with higher operating weights. Such a design philosophy would not result in vortex generators being used for the initial design, but nevertheless they could be applicable to the later developments.

As improvements are made in the accuracy of prediction of shock-induced (and other) forms of separation, and in predicting the interactions of vortices and boundary layers (including the effects on drag), so it becomes more practical to consider various candidate means of separation control and boundary layer control at the design stage.

4. VORTEX GENERATORS AND THEIR APPLICATION TO AIRCRAFT

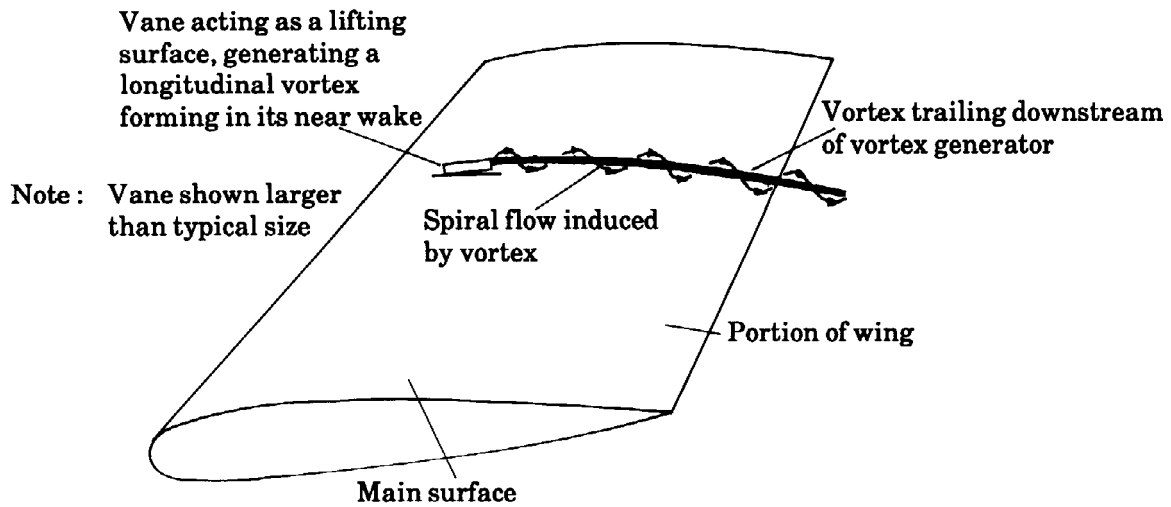
4.1 Vortex Generator Devices and Arrays

Most commonly, vortex generators consist of small vanes fixed at right angles to the airframe surface and inclined to the local stream direction at an angle less than about 25 deg. Usually these vanes are simple flat plates (often having rounded or chamfered edges). They exhibit a variety of planforms and generally have a low aspect ratio ($A_v < 2.0$), see Sketch 2.1. Each vane acts as a lifting surface and trails a longitudinal vortex over the main surface, see Sketch 4.1. An array of vortex generators consists of a number (usually between 4 and 50) of individual devices, arranged in one of the following ways:

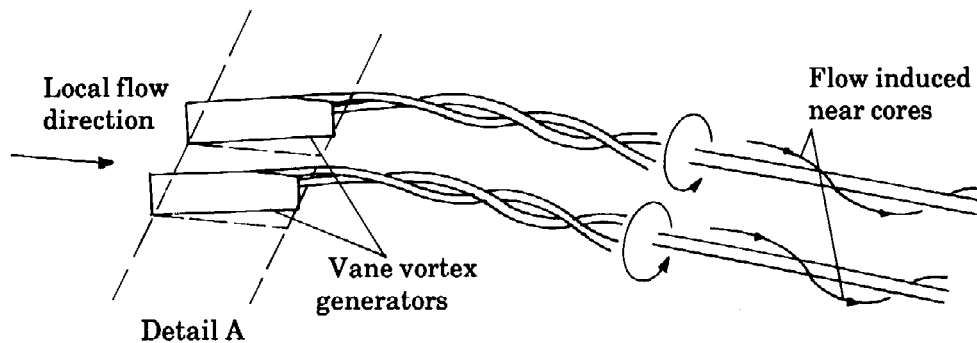
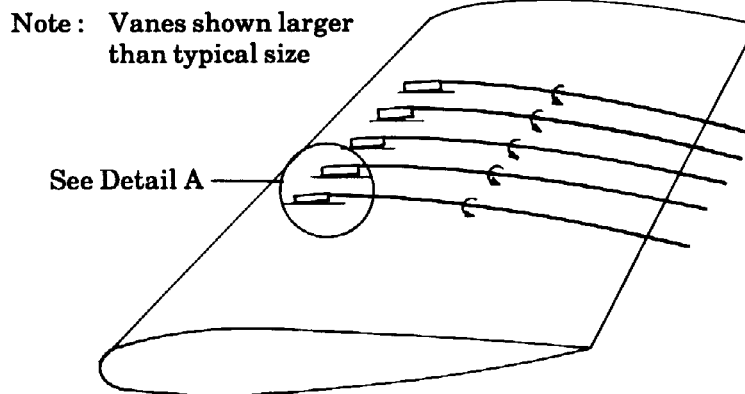
- (a) the individual vortices generated are co-rotating, *i.e.* the spiral flows produced by the vortices have the same rotational sense, see Sketch 4.2,
- (b) the vortices produced are in counter-rotating pairs, see Sketch 4.3,
- (c) the vortices are arranged in a biplane layout, which consists essentially of two counter-rotating layouts, see Sketch 4.4.

Of these types, (a) and (b) are the more common.

The aerodynamic effects produced by vortex generators are much influenced by the type of array adopted, as well as by the size, shape, orientation and location of the individual devices. Each type of array has its particular merits, and these are described and compared in Section 6.

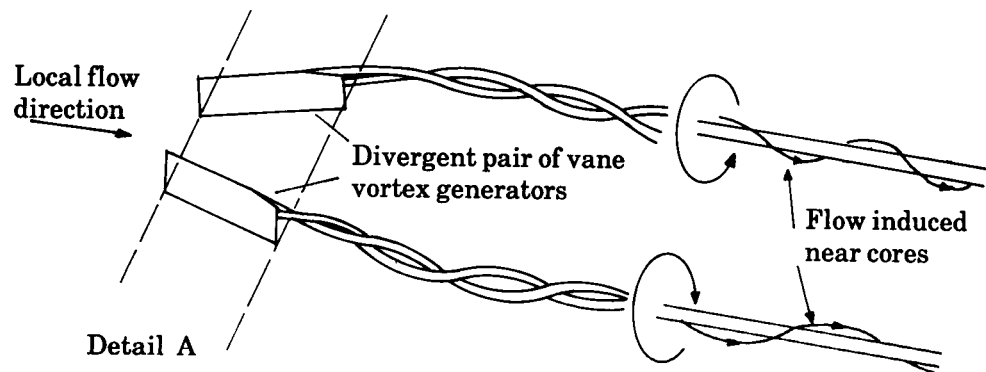
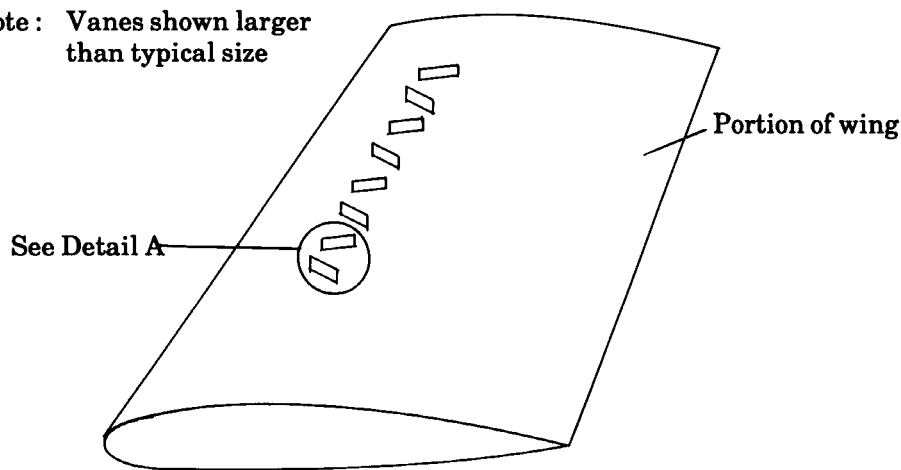


Sketch 4.1 Vortex trailing over main surface



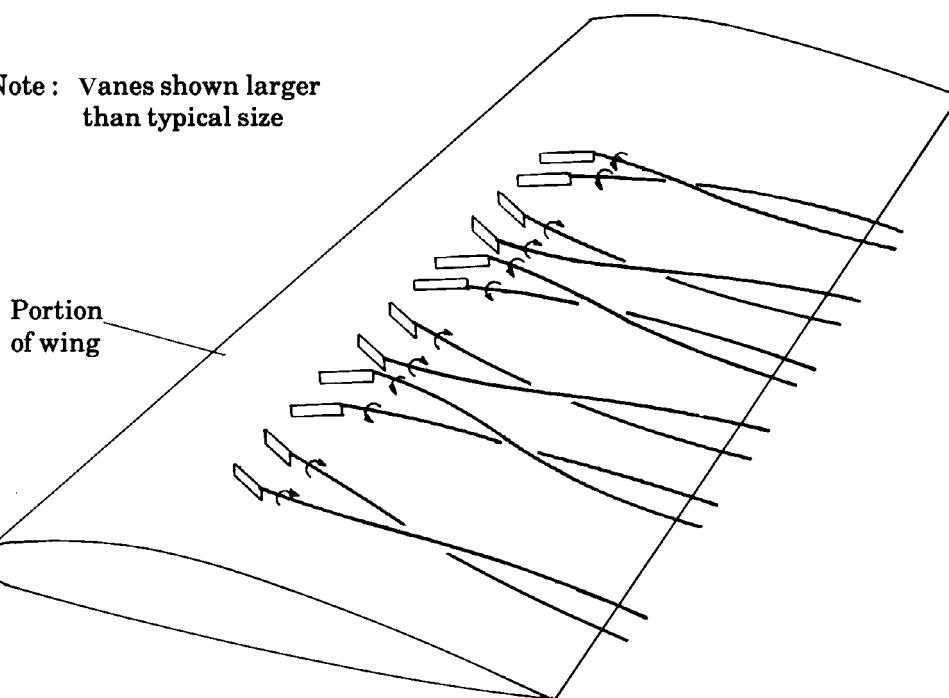
Sketch 4.2 Co-rotating vortex generators

Note: Vanes shown larger than typical size



Sketch 4.3 Counter-rotating vortex generators

Note: Vanes shown larger than typical size

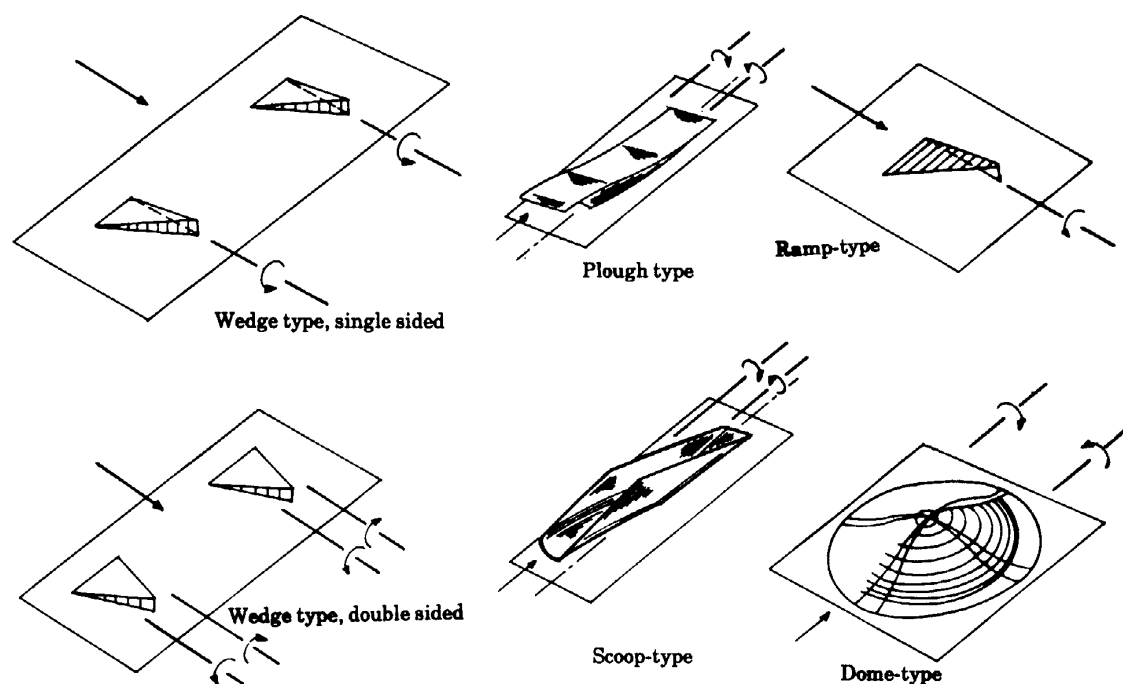


Sketch 4.4 Biplane array of vortex generators

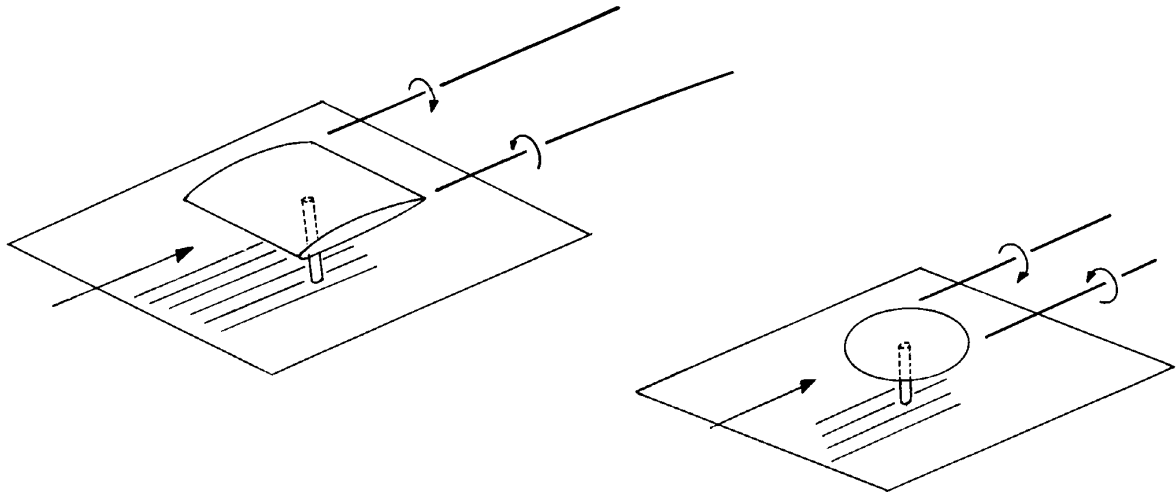
Although vanes fixed at right angles to the main surface are the most commonly used type of vortex generator device on aircraft, a wide variety of forms has been proposed, see Sketch 4.5. Virtually any obstacle or depression which disturbs the flow over a surface produces a wake which is dominated by one or more streamwise vortices. For aircraft application, it is important that the generation of the vortex wake is efficient, *i.e.* without unnecessary drag, and, because of this, the wedge and ramp types of McCullough *et al*⁴ and Collins and Stephens⁵, soon ceased to attract attention, as did certain other shapes (see Schubauer and Spangenberg¹²), such as the plough, scoop and dome (see Sketch 4.5).

Efficient production of a vortex wake is a feature of the wing-type of vortex generator, see Sketch 4.6, as well as the vane-type. The wing-type relies, like the vane-type, on a lifting surface to produce a rolled-up trailing vortex system but, in contrast to the vane-type, the small lifting surfaces are mounted a little distance 'above' the main surface or even slightly ahead of it. Each wing-type generator is inclined at an angle of incidence to the local stream and produces a pair of counter-rotating vortices trailing downstream. This contrasts with a vane generator which is mounted directly on to the main surface, and produces a single vortex. Wing-type vortex generators have been used as a means of generating pairs of counter-rotating vortices where the individual vortices forming a pair remain nearly equal in strength, even when the direction of the local stream changes. This property is of particular significance when the application is to a swept wing, where the local flow direction may change significantly with changing flight conditions. If the wing-type generators have a circular planform, then the relative strengths of the individual trailing vortices forming a counter-rotating pair are virtually unaffected by flow direction changes.

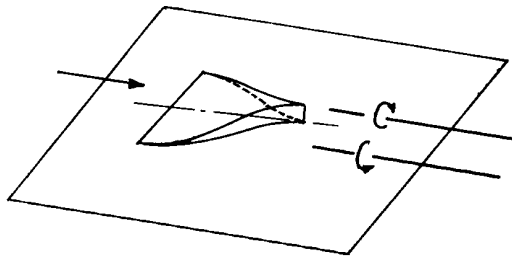
Other vortex generator device forms have been proposed by Wheeler²⁸ and Kuethe¹⁸, see Sketch 4.7. The latter forms were termed 'wave elements' or described as having a 'wave type'. The Wheeler and Kuethe forms are of particular interest as 'low-profile' or 'sub-boundary layer' vortex generators and then have heights of the order of 1/10 to 1/4 of the local thickness of the undisturbed boundary layer. The flow mechanisms, whereby streamwise vorticity is produced in the wake of these forms, differ from that of the wing-like flow over a vane or similar device and it appears likely that the Wheeler and Kuethe forms are more appropriate for use as sub-boundary layer devices than at a size comparable with that of the boundary layer height, which is more usual in aircraft applications. However, aerodynamic data on these forms of vortex generator are lacking, and it is not clear whether either form is preferable to the vane-type, even when used as sub-boundary layer devices.



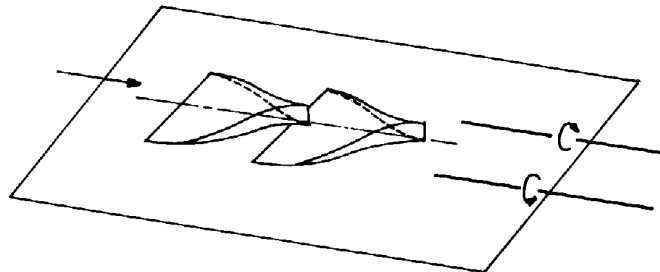
Sketch 4.5 Various early types of vortex generators



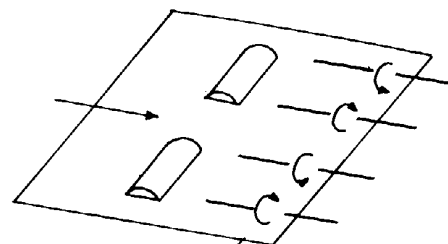
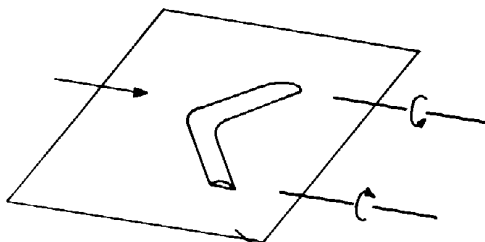
Sketch 4.6 Wing-type vortex generators



Wheeler singlet



Wheeler doublet



Kuethe or wave-element types

Sketch 4.7 Wheeler and Kuethe forms of vortex generator

4.1.1 Vortex generator jets

Using jets issuing from suitably shaped orifices in the main surface to generate vortices and to control separation offers a number of advantages^{9,13,43,46,56}. The avoidance of significant drag penalties under conditions where the control of the boundary layer is unnecessary is one of these. Another advantage stems from the fact that jets do not necessarily become ineffective, as vanes do, when submerged by local regions of separated flow or thick shear layers, since the jets may penetrate into the outer regions of higher speed to form their vortices. Data relating to the application of jet vortex generators for control of shock-induced separation are presented in References 13 and 56.

4.1.2 Active vortex generators

If the vortex generator system considered is 'active' (*i.e.* it involves the action of valves or other moving parts), the implications of its installation and operation (including airworthiness) need to be taken into account.

4.2 Factors Influencing Choice of Type of Vortex Generator Device

Some of the factors affecting the choice of device used to generate the vortices have been alluded to in Section 4.1. The ability of the device to generate a vortex wake of adequate strength at an appropriate height above the main surface is essential, and such strengths can certainly be achieved by a variety of vane- and wing-type generators. A dominant parameter linking vortex strength to the effectiveness of vortex-induced mixing is K/hU_e . A value of the order of unity for this parameter is necessary for there to be a vigorous rate of mixing; however, it may not be necessary for it to be as high as possible for each vortex (or vortex pair) generated, even though the local rate of mixing produced by the vortex flows reduces as this parameter reduces. This is because reductions in the parameter could, in certain situations, lead either to a greater range of vortex generator effectiveness (in the case, for example, of a counter-rotating array), or to a reduction in the adverse effects of the array. Also, it may not always be necessary, or desirable, to achieve the maximum degree of control.

The following is a list of reasons for choosing a particular form of vortex generator device. Not all the reasons are necessarily relevant to any particular application.

- (a) The device can produce a vortex (or vortex pair) with the desired value of K/hU_e .
- (b) The device can produce the vortex wake 'efficiently', *i.e.* with small adverse effects and low device drag. A device that employs power may need to be efficient as regards power usage.
- (c) The device produces a stable vortex wake with a vortex (or vortices) having an initially small core size. The significance of vortex core size is referred to in Section 6.4.
- (d) The device produces a vortex wake which is insensitive to changes in flow direction at the device. This feature is particularly important for vortex generators applied to swept wings where the flow direction may change significantly with changes in C_L and/or M_∞ . For vane vortex generators, vortex strength sensitivity to changes in local flow direction is less when their aspect ratio is low. Particularly low sensitivity to flow direction is provided by wing-type vortex generators.
- (e) The device is simple to make and attach to (or incorporate into) the surface.
- (f) The device is not prone to damage or loss.
- (g) The device is not hazardous to personnel (due to sharp edges, for example).
- (h) The devices do not detract from the aesthetic appearance of the aircraft.

5. FUNDAMENTAL AERODYNAMIC EFFECTS OF VORTEX GENERATORS

The beneficial effects of vortex generators, in providing a degree of control of boundary layer separation, are ascribed to the influence of the set of vortices which trail over the main surface. These vortices cause 'overturning' motions in the fluid near the main surface, see Figure 1¹³. The induced secondary motions produce an enhanced rate of mixing and consequent greater rate of momentum interchange between the outer inviscid flow and the retarded fluid in the boundary layer¹². Some of the outer flow is drawn 'down' towards the main surface, and some of the boundary layer fluid is swept away from the surface in 'tongues', see Figure 2¹³. Corresponding to this increased rate of mixing, the average surface shear stress at locations downstream of the vortex generators is increased. The degree and extent of these increases in shear stress depend primarily on the vortex strengths and paths, although other features of the vortex flows have some influence^{12,13,15,48}. The vortex flows thus result in the boundary layer having an increased resistance to separation, and also yield improvements in pressure recovery and other flow characteristics in those cases where separation may still occur locally.

In addition to the increases in the relatively large scale mixing produced by the vortex flows, a possible further influence of the vortices is that of suppressing turbulence (to some degree) as a result of flow stabilisation associated with induced streamline curvature, see for example Reference 19. This has been suggested as the reason for the persistence of vortices over a remarkably long distance 'through' turbulent flows, and it could contribute to observed reductions in the rate of growth of the (mean) boundary layer associated with vortex generators.

6. FACTORS INFLUENCING SEPARATION CONTROL BY VORTEX GENERATORS

Early work with vane-type vortex generators, applied at low speeds to diffusers and aerofoils^{1,3} and to transonic flows over aerofoils and wings⁸, revealed trends in their effectiveness for separation control related to various design parameters such as vane angle of incidence and height, and to the location of the vanes relative to the region of adverse pressure gradient. Initially, attention was given principally to the changes produced by the vortex generators in the velocity profiles of the boundary layer at locations upstream of the adverse pressure gradient. Subsequently, it was found that the success of an array of vortex generators in restraining the development of boundary layer separation depended critically on the strengths and locations of the streamwise vortices in the precise region of the adverse gradient; this was found to be far more important than the details of the boundary layer velocity profiles just upstream. This observation justifies the view that effective vortex generators provide a continuous mixing influence from the vortices, *i.e.* one which extends through the region of adverse pressure gradient, see Reference 13.

In a flow with a shock wave, the region of most severe adverse pressure gradient on the surface occurs in the locality of the 'foot' of the shock, and extends aft a short distance from this locality. With changes in the flight conditions, (*i.e.* with changes in the values of C_L and/or M_∞), the position of any shock wave will vary, see for example Reference 44, and the range of relevant shock locations may cover a large proportion of the chord. Thus, the strengths and orientation of the vortices may need to be such as to produce the desired enhanced mixing effects over a considerable streamwise range. The need to achieve a desired range of effectiveness of separation control from an array of vortex generators is a dominant factor in the choice of vortex generator parameters, see Sections 6.1 to 6.3. It is especially important in the consideration of counter-rotating and biplane arrays.

The paths taken by vortices behind an array of vortex generators are governed by the strength, rotational sense, height and lateral position of each vortex at the longitudinal or streamwise position at which it is generated. These vortex paths are conveniently represented as a series of vortex positions (points) in successive planes normal to the main stream flow[†]. Jones⁶ found that the paths of the vortices could be

[†] Main stream flow refers to the flow on which the vortex effects are superimposed.

predicted fairly accurately, at least in low speed flow, using greatly simplifying assumptions. These assumptions are equivalent to supposing that the flow is ideal (*i.e.* inviscid) and that the gradient of the streamwise velocity disturbance component in the x direction is small (near the surface) compared with $\partial v/\partial y$ and/or $\partial w/\partial z$. They amount to treating the flow in planes normal to the main stream using two-dimensional ideal flow theory. Three-dimensional effects and viscous effects were found to be of minor importance. Wendt *et al*⁴⁸ confirmed that vortex paths could be calculated in this way. In principle, the assumptions should be applicable equally to subsonic and supersonic flows.

By adopting this two-dimensional approach, and considering the induced flows of neighbouring and image vortices, the vortex paths for co-rotating arrays are shown to be markedly different from those of counter-rotating and biplane arrays. For co-rotating arrays, the sum of the induced velocities normal to the surface produced by neighbouring and image vortices tends to be very small, and the lateral induced velocities tend to be equal, so vortex positions in successive crossflow planes remain evenly spaced, and at a constant height above the main surface, except near the ends of an array. For counter-rotating arrays, the induced normal velocities may not cancel, and the lateral induced velocities differ from one vortex to another, so the relative vortex positions change in successive crossflow planes. Far downstream, the vortices in counter-rotating arrays migrate away from the main surface, and their effectiveness in producing any significant enhancement in the mixing processes is much reduced. The vortices in biplane arrays similarly 'migrate' and also become ineffective far downstream. On the other hand, at streamwise locations well upstream of those where vortex migration has occurred, the mixing effects of counter-rotating arrays and biplane arrays may be significantly better than those of otherwise 'similar' co-rotating arrays. Consequently, where the streamwise range of required vortex generator effectiveness is large, co-rotating arrays should be selected; where this range is restricted, the choice of array type is more open, with counter-rotating and biplane arrays probably offering the most control. A wide choice between maximum effectiveness and range of effectiveness is available for counter-rotating and biplane systems by suitable selection of the vortex strength and spacing to height parameters (see Reference 2 and Sections 6.2 to 6.3).

Detailed analysis of the effects of numerous different vortex generator layouts¹³ has shown how these qualitative results can be refined and made more precise, so permitting selection of a suitable vortex generator array to be made with relative confidence. In presenting the aerodynamic principles involved in this analysis, together with their implications for the application of vortex generators to control shock-induced separation on aircraft, it is convenient to deal separately with the following aspects.

The influence of various types of vortex array on the degree and range of mixing effectiveness, Sections 6.1 to 6.3.

Consideration of vortex decay, with implications for vortex generator effectiveness, Section 6.4.

Assessment of the degree of control of shock-induced separation achievable with vortex generators, Section 7.

Factors relating to the application of vortex generators to wings or other lifting surfaces, including adverse effects, Sections 8.1 to 8.4.

Factors relating specifically to flows near junctions, such as wing-fuselage junctions, Section 8.5.

Multiple arrays of vortex generators, Section 8.6.

6.1 Co-Rotating Arrays

6.1.1 Vortex paths

Utilising the assumptions of ideal, two-dimensional crossflow⁶, the induced flows of an infinite array of equal strength, uniformly-spaced co-rotating vortices produce a uniform (lateral) velocity of the vortices, with $w = 0$ and

$$v = \frac{K}{4\pi h} t \coth t, \quad (6.1)$$

where $t = 2\pi h/D$.

If h/D is small,

$$v \sim \frac{K}{4\pi h}. \quad (6.2)$$

The lateral velocity component may be expressed as an angle of inclination, σ , between the vortex paths and the main stream direction

$$\sigma = \sin^{-1}(v/U_e), \quad (6.3)$$

so that, for small values of h/D ,

$$\sigma = \sin^{-1}\left(\frac{K}{4\pi h U_e}\right). \quad (6.4)$$

Typically, for co-rotating arrays, $K/hU_e \sim 1.0$, so that $\sigma \sim 5$ deg for 'widely' spaced vortices. With the same value of K/hU_e and $D/h = 4.0$, $\sigma \sim 8$ deg.

Observations reported in Reference 13 indicate that co-rotating vortices in real flows may remain at roughly the same height above the surface. However, vortex core growth (which occurs more rapidly in adverse pressure gradients) produces a displacement away from the surface, as may a growth in mean boundary layer thickness. Also, vortices near the ends of an array tend to be displaced, towards the surface at one end, but away from it at the other – where there is a tendency for vortices to 'curl up'.

6.1.2 Vortex spacing

The importance of vortices being widely spaced laterally, relative to their height above the surface, in order to achieve a vigorous mixing effect, is illustrated for co-rotating vortices by the measured total pressure contours shown in Figure 3¹³.

Effects of vortex spacing on mixing can be assessed by use of the ideal, two-dimensional crossflow theory used to predict the vortex paths. Consider an initially straight line of fluid particles lying transverse to the flow and parallel to the main surface (conventionally represented as a plane). The distortions produced in this line of fluid particles as it moves downstream can be evaluated. Results of four such calculations (for $D/h_i = 2.0, 4.0, 6.0$ and 8.0) with the initial line of fluid particles taken to be at half the vortex height[†]

[†] The choice of this height is arbitrary. Making it quite different from the height of the vortices obviates the need to model the vortex core flows. For vortices embedded well within a boundary layer, a height greater than the vortex height would be of greater value in revealing the effects of mixing.

above the surface are shown in Figure 4. In Figure 4 the location of the initial straight line of fluid particles is shown as well as the locations of these particles after they have moved downstream through a distance Δx equal to $10h_i$ (taking, in each case, $K/h_i U_e = 1.0$).

Comparing these sketches with the contours of pitot pressure shown in Figure 3 reveals similarities between the computed fluid particle locations and the pitot contours, especially for the cases $D/h_i = 4$ and 6. For $D/h_i = 2$ there is little similarity, but this might be expected considering the form adopted by the line of particles in this case. What is seen is that the calculated mixing is poor at $D/h_i = 2$, and this is also observed in practice.

Mixing advantages may be expected to occur from at least two effects: tongues of (boundary layer) fluid being swept away from the main surface, and fluid outside the boundary layer (or in the outer part of the boundary layer) being swept close to the surface (this is synonymous with boundary layer thinning). Judged on these criteria the effects at $D/h_i = 2.0$ are relatively weak. Only a very thin tongue of fluid is transported away from the surface, and nowhere would the boundary layer be significantly thinned. This is in contrast to the effects at $D/h_i \geq 4$. These show that a significant tongue of fluid is transported from the surface, and that much of the boundary layer would be thinned to a marked degree. Thus the results shown in Figure 4 give some theoretical endorsement to the well-established need (Reference 13), based on experimental results, to utilise a sufficiently high value of D/h_i , and further suggest that a value as high as 8 (and possibly higher) may be almost as effective as values of 4 or 6. A further factor tending to favour higher values of D/h_i is that these correspond to fewer vortex generators (of fixed height) needed to cover a given spanwise extent, and therefore to reduced adverse effects, see Section 8. As pointed out by Pearcey¹³, and given added weight by Wendt *et al*⁴⁸, any arrangement with closely spaced vortices is likely to be ineffective also as a result of the viscous interaction (*i.e.* damping) between vortices, since this leads to a rapid diminution in vortex strengths in the streamwise direction.

6.1.3 Streamwise extent of control

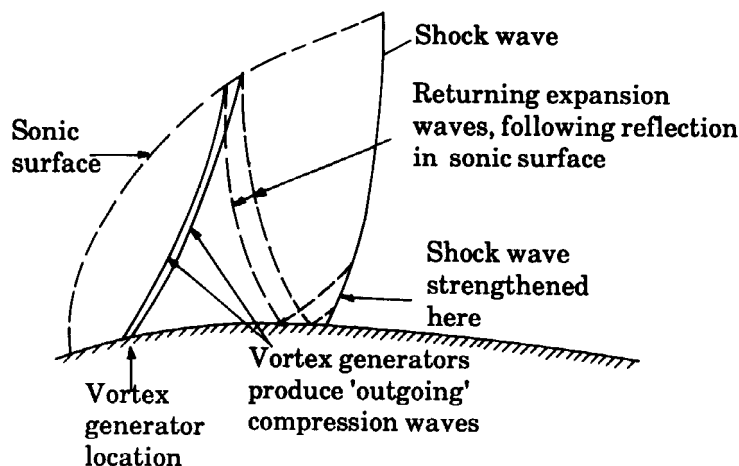
Vortex generators should be selected to provide an appropriate degree of separation control over the full range of flight conditions (C_L , M_∞) for which such control is needed. They might therefore have to allow for a wide range of shock locations, and thus provide a streamwise extent of vortex generator effectiveness which is a significant proportion of the local chord.

As noted in Section 6.1, the mixing influence of vortices occurs as a continuous and progressive effect on the boundary layer. This being so, there has to be a certain distance, extending downstream from the vortex generators, for mixing to 'accumulate' and so provide an appreciable degree of control of shock-induced separation. Thus the vortex generators should be placed a sufficient distance ahead of any shock which is strong enough to produce a separation that requires control. In attempting to determine how great this distance should be in any particular case, it may be noted that Figure 4 indicates that a distance for $(x - x_i)$ of $10h_i$ appears to give a significant mixing when $K/h_i U_e = 1.0$. Where lower values of $K/h_i U_e$ are employed, the distance required would be greater, in inverse proportion.

Another consideration, which might influence the choice of this minimum distance, is the need to avoid placing vortex generators where they could introduce significant adverse interference with the main flow over the surface. Examples of adverse interference effects which could occur are as follows.

A system of shock waves might develop in the 'channels' formed by neighbouring vane (or other) vortex generators, and these could interact with the main shock system, possibly leading to the main shock pausing near the location of the vortex generators instead of moving steadily over the surface with varying C_L or M_∞ . Such a pausing could lead in turn to undesirable features in aerodynamic characteristics.

Compression waves emanating from the vortex generators could return downstream as expansion waves (following reflection in the sonic surface) and lead to an increase in the strength of the main shock, see Sketch 6.1.



Sketch 6.1 Disturbances from vortex generators affecting shock strength

Utilising results and arguments presented in this Section and in Section 6.4, and neglecting differences between x_v and x_i in relation to $(x_{sh} - x_v)$, it is suggested that co-rotating vortex generators should be placed as follows,

$$\text{for good mixing effectiveness} \quad 10h_i \leq (x_{sh} - x_v) \leq 100h_i,$$

$$\text{for full effectiveness} \quad 10h_i \leq (x_{sh} - x_v) \leq 75h_i.$$

The generators may need to be placed significantly more than $10h_i$ upstream of the shock to avoid adverse interference with the main flow. This last consideration is most important when the vortex generating devices are either relatively large or closely spaced, although data from which a general rule could be derived are lacking. In practice it appears that a minimum distance of 10 per cent chord (approximately) usually extends downstream from any effective vortex generators to the location of the shock tending to produce separation. Finally, to generate vigorous mixing, the vortex strength parameter, K/hU_e , evaluated close to the vortex generators, ought be of the order of unity, but data are insufficient to permit a more precise recommendation.

6.1.4 Widely spaced vortices and sub-boundary layer vortex generators

Some applications of vane vortex generators to the wings of jet transport aircraft have adopted array configurations with lateral spacing ratios, D/h_v , as high as 15 to 20. (Since vanes produce vortices with an initial height, h_i , quite close to h_v , the values of D/h_i for these co-rotating arrays are also as high as 15 to 20.) Unfortunately there is a lack of experimental data relating to the effectiveness of vortex generators with such high values of D/h_i . With widely-spaced vortex generators, the flow downstream of any single vortex generator should be largely free of the effects attributable to the induced velocities of neighbouring vortices, and studies of the effects of interactions between a single vortex and a shock wave may then be of use. An example of such a study is given in Reference 40.

The widely spaced configurations, referred to in this Section, have probably been adopted following a process of tuning, *i.e.* the final configurations have been arrived at following tests with a progression of different layouts and geometries and the configuration judged to give the best overall performance selected.

It is likely that such tuned, or optimised, configurations provide significant separation control with either negligible or very small adverse effects, as compared with configurations that provide the greatest possible separation control at the expense of somewhat larger adverse effects.

A situation where values of D/h_i greater than 8 (say) may be chosen, arises when it is important to produce effective mixing involving those fluid particles that are initially further from the surface than (about) twice the vortex height. This would be most applicable in considering vortex generators which produce vortices relatively low down in the boundary layer, *i.e.* sub-boundary layer vortex generators, (SBVGs). Using the approach of Section 6.1.2, Figure 5 indicates mixing effects of co-rotating vortices with $D/h_i = 4, 8$ and 16. For $D/h_i = 4$, three values for the initial height of the line of fluid particles are considered: $h_i/2$ in Figure 5a, h_i in Figure 5b, and $2h_i$ in Figures 5c and 5d. Figures 5c and 5d show ineffective mixing. In Figures 5e and 5f, for $D/h_i = 8$ and 16 respectively, considerably more effective mixing is indicated for particles having initial height $2h_i$.

Although few results are available for SBVGs (see References 30, 49, 57), it is possible that SBVGs provide levels of control significantly greater than would be expected on the basis of extrapolating results from vane vortex generators of a conventional size. There is also the possibility of beneficial developments of the form of SBVGs. Such possibilities should be considered during the process of selecting and refining a vortex generator layout.

6.2 Counter-Rotating Arrays

6.2.1 Vortex paths

In extensive wind tunnel tests of various vortex generator layouts reported by Pearcey¹³, certain counter-rotating arrays were among the most effective. However, the streamwise extent of the effectiveness of counter-rotating arrays can be rather limited, and this restriction in effectiveness is primarily related to the eventual movement of the vortices away from the surface with increasing distance from the vortex generators. Since the vortex paths are relatively well predicted by the method of Jones⁶, it is possible to explain the relative effectiveness of the various counter-rotating systems and feasible to select the initial vortex strengths and spacings so as to produce (within limits) a desired combination of maximum effectiveness and extent of effectiveness.

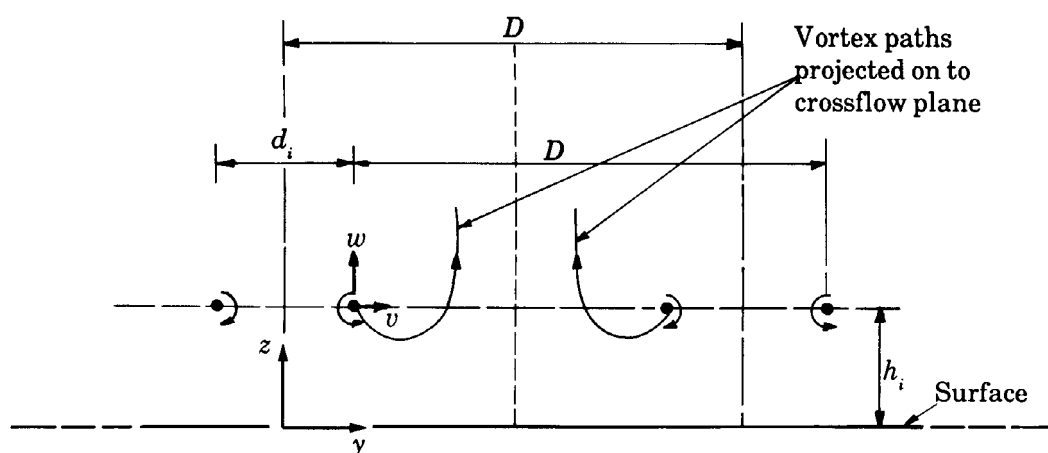
Consider an infinite regular array of counter-rotating vortices, the (constant) cell span of which is D , and denote by d the (variable) distance between two neighbouring vortices chosen so that in the region between them the induced flow is towards the surface, see Sketch 6.2 (these vortices would be produced by vanes that form a divergent pair, see Sketch 4.3). In contrast to arrays of co-rotating vortices, there is no net lateral velocity of the whole array induced by the vortices and their reflections in the main surface. However, within each cell, there are induced motions and as a result the vortex spacings and height above the surface change with distance downstream of the vortex generators. The vortex paths may conveniently be represented as a motion of the vortices in a single crossflow plane onto which the vortex locations are projected. Within the assumptions of ideal two-dimensional crossflow, the paths of the vortices in this plane are given by the equation

$$\operatorname{co} \operatorname{sech}^2 \zeta + \operatorname{cosec}^2 \xi = C, \quad (6.5)$$

where $\zeta = 2\pi z/D$ and $\xi = 2\pi y/D$. The value of C for a particular path is determined by the initial location of the vortex denoted, for example, by

$$y = d_i/2, \quad z = h_i, \quad (6.6)$$

or, alternatively, by the vortex location at any subsequent time. Figure 6 shows the form of these vortex paths. The speed at which a vortex 'moves along' its path is also relevant, and this can be found from the velocity components induced at the vortex by the other vortices and the reflection vortices. These velocity components are proportional to the vortex strength, as also are the 'mixing' velocities imposed by the array of vortices on the boundary layer fluid. If d_i is less than $D/2$, the vortices initially move towards the surface (as a result of the dominant effect of the nearest neighbouring vortex). However, they also move away from the vortices that were initially their nearest neighbours (thus d increases). When d becomes greater than $D/2$ the vortices move away from the surface. As stated earlier (and shown for co-rotating vortices), the vigour of the mixing effect produced by a vortex array is related primarily to the spacing of the vortices relative to their height above the surface. As the vortices of the counter-rotating array reach the stage where they move away from the surface, the rate at which they produce a mixing effect starts to diminish.



Sketch 6.2 Notation for counter-rotating vortices

6.2.2 Effects on mixing

An indication of the importance of the locations of the vortices in counter-rotating arrays is presented in Figures 7 and 8. In these Figures, which utilise the two-dimensional ideal flow theory, the vortex locations projected onto a crossflow plane are shown at time intervals corresponding to fluid particles moving through a distance in the main stream equal to $10h_i$, where (as previously) the value of $K/h_i U_e$ has been taken as unity. The effect of changing the value of $K/h_i U_e$ may be readily assessed since the intervals between the points indicated on the projected vortex paths are directly proportional to this parameter. From Figures 7 and 8, for a given D/h_i , relatively low values of d_i/h_i result in relatively rapid motion of the vortices, which is first towards the surface, and subsequently away from it. Thus with $D/h_i = 5$ and $d_i/h_i = 1.25$ (Figure 7a), the vortices initially remain close to the surface (*i.e.* within a distance of h_i from it) but only up to the end of the second time interval (corresponding to a streamwise distance of $20h_i$ from the vortex generators), after which the movement away from the surface is rapid. This compares with $D/h_i = 5$ and $d_i/h_i = 2.5$ (Figure 7b) where the vortices immediately start to move away from the surface, but since this motion is relatively slow, the vortices are closer to the surface after the end of the third time interval than those having the configuration parameters $D/h_i = 5$, $d_i/h_i = 1.25$. There is no point in having d_i/h_i greater than 2.5 (with $D/h_i = 5$) because the motion of the vortices will then not only be always away from the surface, but will be more rapid than that with $d_i/h_i = 2.5$. Figure 7c, showing the vortex motion with $D/h_i = 5$ and $d_i/h_i = 4.0$, illustrates this.

For a given of D/h_i , reducing the value of d_i/h_i (below $D/2h_i$) reduces the minimum value of h/h_i . As indicated in Figure 7a, low values of h/h_i (resulting from small values of d/h_i) also produce a more rapid motion of the vortices in the crossflow plane (for a given value of $K/h_i U_e$), so the enhanced mixing rate produced by the vortices moving close to the surface tends to be sustained for a relatively short streamwise extent. However, the choice of a relatively intense but short-lived mixing effect is available if needed.

The effect of increasing the cell width D relative to h_i is also of interest, and Figures 8 illustrates corresponding effects to those of Figure 7 for a value of $D/h_i = 10$ rather than 5. Thus, Figure 8a shows vortex paths for $D/h_i = 10$ and $d_i/h_i = 1.25$, Figure 8b relates to $D/h_i = 10$ and $d_i/h_i = 2.5$, whilst Figure 8c treats $D/h_i = 10$ and $d_i/h_i = 4.0$. The general effect of increasing D/h_i on the vortex motions is to reduce the magnitudes of the velocity component normal to the surface, whereas the motions parallel to the surface are hardly changed. Compared with $D/h_i = 5$ the vortices remain close to the surface over a considerably longer streamwise distance.

An assessment of the mixing propensities of various configurations of counter-rotating arrays of vortices may be made by considering the distortions induced in 'lines' of fluid particles – as was done for co-rotating arrays. For a co-rotating array of equal strength vortices the presentation is quite simple since, as the relative positions of the vortices remain constant[†], so does the degree of distortion produced in any given streamwise distance. For other types of array, the relative positions of the vortices vary, so the degree of distortion produced in a given streamwise distance also varies. Thus, for completeness, the distortion produced, say over an incremental streamwise distance of $10h_i$, should be shown for a number of different portions of the flow, for example for ranges of $(x - x_i)$ of 0 to $10h_i$, $10h_i$ to $20h_i$, $20h_i$ to $30h_i$ and so on. However, since the distortions calculated provide, at best, only an **indication** of the mixing effects of the vortices, and also in order to limit the number of distortions shown, the distortions corresponding to just two portions of each flow are given, namely $0 \leq (x - x_i) \leq 10h_i$ and $50h_i \leq (x - x_i) \leq 60h_i$. These are denoted for convenience as 'early' and 'late' distortions respectively^{††}. These distortions are shown in Figures 9 and 10 for the six different vortex arrays of Figures 7 and 8. It is seen that whereas the early distortions appear to be significant, showing a greatly thinned region of fluid near the surface, and large tongues of fluid moved away from the surface, most of the late distortions are quite negligible. Only for the case with $D/h_i = 10$ and $d_i/h_i = 2.5$ is the late distortion significant.

A more detailed comparison of the effects of changing the parameters D/h_i and d_i/h_i on the distortions could include consideration of 'intermediate' distortions in addition to early and late results. However, the results presented give a fair indication of the influence of these parameters.

A comparison of distorted 'lines' of fluid particles, calculated using the two-dimensional ideal flow theory, with measured boundary layer distortions, is given in Figure 11 for a system of counter-rotating vortices described by Pearcey¹³ in which the vortices were initially equally spaced and where $D/h_i = 5.25$.

The general features of the vortex-induced mixing for this system are fairly well predicted by the two-dimensional approach, although the growth of the boundary layer and vortex dissipation, which have significant effects in the observed flows, particularly for $(x - x_i)/D \geq 6.4$ (i.e. $(x - x_i)/h_i \geq 33.6$), are of course not modelled by the ideal flow calculations. Figure 11 adds confidence that the simple approach yields reliable indications of mixing effects. Further discussion of this comparison is given in Appendix B.

Given the generally desirable feature of having relatively few vortices to cover any given spanwise extent, there are benefits in choosing $D/h_i = 10$, rather than $D/h_i = 5$, assuming that the initial vortex height is unchanged. Pearcey¹³ found this to be the case in practice in wind tunnel tests.

Further consideration of the effectiveness of counter-rotating vortex generator arrays is presented in Section 8.2.

6.2.3 Comparison with co-rotating arrays

In comparison with co-rotating systems it would appear that the mixing effects of counter-rotating systems should be greater, but only locally. The performance of vortex generators producing counter-rotating arrays

[†] Also the strength of the vortices is assumed to be constant, and the array end effects are ignored.

^{††} Early and late refer to the sequence of events for fluid particles convecting with the main stream.