



ESDU 93030

Issued December 1993

With Amendment A

November 1995

Re-issued February 2012

Three-dimensional elastic stress concentration factors Plain or countersunk hole in a wide plate subjected to tension, bending or pin loading

Associated software: ESDUpac A9330

Institution of
**MECHANICAL
ENGINEERS**



Endorsed by
The Institution of Mechanical Engineers
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Stress Analysis and Strength of Components Committee. This Committee first met in 1964 and now has the following membership:

Chairman	
Prof. C.E. Turner	— Imperial College of Science and Technology
Vice Chairman	
Prof. T.G.F. Gray	— University of Strathclyde
Members	
Mr A.J. Batchelor	— Independent
Dr I.J. Bickley	— Mirrlees Blackstone (Stockport) Ltd
Dr M.S.G. Cullimore	— Independent
Dr L.C. Laming	— Imperial College of Science and Technology
Mr A.B. Smith	— Lloyd's Register of Shipping
Mr J.V. Vint	— Independent.

The Item was accepted for inclusion in the Structures Series by the Aerospace Structures Committee which first met in 1940 and now has the following membership:

Mr J.H. van der Sloot	— Fokker B.V., Schiphol, The Netherlands
Vice-Chairman	
Mr J.K. Bennett	— British Aerospace plc, Space and Communications Division
Members	
Dr P. Bartholomew	— Royal Aerospace Establishment, Farnborough
Mr K. Fitzsimons	— Westland Helicopters Ltd
Mr P.J. Mitchelmore	— British Aerospace plc, Civil Aircraft Division
Mr K.R. Obee	— Independent
Mr B. Popham	— British Aerospace Space Systems Ltd
Mr M.S. Pressnell	— University of Hertfordshire
Mr M. Ranson	— British Aerospace Defence Ltd
Prof. A. Rothwell	— Technische Universiteit Delft
Mr P. Stocking	— Cranfield University
Mr K. van Katwijk*	— European Space Agency, Noordwijk, The Netherlands.

(Continued on inside back cover)

THREE-DIMENSIONAL ELASTIC STRESS CONCENTRATION FACTORS. PLAIN OR COUNTERSUNK HOLE IN A WIDE PLATE SUBJECTED TO TENSION, BENDING OR PIN LOADING

CONTENTS

	Page
1. THEORY AND APPLICATION	1
1.1 Introduction	1
1.2 Notation	1
1.3 Development of Data From Finite Element Analyses	2
1.4 Loading	3
1.4.1 Remote tension and remote out-of-plane bending	3
1.4.2 Pin loading	4
1.5 Typical Variations of Maximum Stress	4
1.5.1 Plain holes	4
1.5.2 Countersunk holes	5
1.5.3 Variation in countersink angle	5
1.5.4 Finite width effect	6
1.5.5 Multiple hole effect	6
1.6 Derivation and References	6
1.6.1 Derivation	6
1.6.2 References	6
2. ESDUpac A9330	14
2.1 Introduction	14
2.2 Program Limitations	14
2.3 Data Input and Output	14
2.3.1 Input file structure	14
2.3.2 Program output and interpretation	15

THREE-DIMENSIONAL ELASTIC STRESS CONCENTRATION FACTORS. PLAIN OR COUNTERSUNK HOLE IN A WIDE PLATE SUBJECTED TO TENSION, BENDING OR PIN LOADING

1. THEORY AND APPLICATION

This Data Item is accompanied by a computer program, ESDUpac A9330, for use in evaluating stress concentration factors along the bore of a plain or countersunk hole in a wide plate subjected to tension, out-of-plane bending or pin loading. The data are derived from a three dimensional analysis of the stress distribution around the hole.

The Data Item consists of two parts. Section 1 describes how the equations were developed, gives examples of stress concentration factor variation along the hole bore for selected geometries and loadings, and describes how the pin loading case was simulated. Section 2 describes the program, its input and its output.

1.1 Introduction

This section explains the derivation of the equations used in ESDUpac A9330 for the various loadings and geometries covered. It also gives examples of the variations in the maximum stress and the stress variation along the hole bore as an aid to the design of members containing holes.

For plain holes, stresses due to remote tension, remote out-of-plane bending and pin loading are treated. For countersunk holes, stresses due to remote tension and remote out-of-plane bending only are considered.

1.2 Notation

Both SI and lbf/in units are quoted but any coherent system of units may be used.

b	straight shank length of countersunk hole	m	in
d	diameter of plain portion of hole	m	in
K_b	stress concentration factor at $\phi = \pm 90^\circ$ along bore of hole in plate under bending where $\sigma_{max} = K_b(6M/t^2)$		
K_p	stress concentration factor at $\phi = \pm 90^\circ$ along bore of hole in plate loaded by pin where $\sigma_{max} = K_p(P/dt)$ (see Sketch 1.2b)		
K_t	stress concentration factor at $\phi = \pm 90^\circ$ along bore of hole in plate under tension where $\sigma_{max} = K_t \sigma = K_t \langle P/Wt \rangle$		
K_w	stress concentration factor at $\phi = \pm 90^\circ$ along bore of hole in plate under wedge loading where $\sigma_{max} = K_w(P/dt)$ (for use in analysis of pin loading, see Sketch 1.2aii)		
M	bending moment per unit width applied to plate acting about y-axis (see Sketch 1.1)	N m/m	lbf in/in
P	applied pin loading in x-direction	N	lbf
t	plate thickness	m	in

Issued December 1993
With Amendment A, November 1995
Re-issued February 2012

W	width of plate	m	in
x, y, z	cartesian coordinate system used for plate and hole geometry (see Sketch 1.1)		
θ_c	included angle of countersink (see Sketch 1.1)	degree	degree
σ	applied remote tension stress in x-direction	N/m ²	lbf/in ²
σ_{max}	maximum circumferential direct stress in plate at hole perimeter	N/m ²	lbf/in ²
σ_r	radial direct stress applied to hole circumference at location ϕ	N/m ²	lbf/in ²
ϕ	angle measured from x-axis around circumference of hole (see Sketch 1.1)	degree	degree

Sign convention: Stresses - tensile are positive, compressive are negative.

1.3 Development of Data From Finite Element Analyses

The program ESDUpac A9330 (see Section 2) is based on Derivation 1, which employed a comprehensive series of three dimensional finite element calculations for square, elastic isotropic plates. These are dependent on Poisson's ratio and here a Poisson's ratio of 0.3 has been used. The width, W , of the plate was taken to be $10d$ for plain holes and $15d$ for countersunk holes. In practice, extending these dimensions to higher multiples of d may result in significant changes in the stress concentration. The stress concentration factor may decrease by up to 5 per cent for the remote tension case and by up to 10 per cent for the remote out-of-plane bending case. For pin loading a percentage change cannot be established from the available data.

For the case of a plain hole in a wide plate subjected to remote out-of-plane bending, the maximum stress concentration factor at the hole edge is obtainable from both ESDUpac A9330 and Figure 2 of ESDU 09014*, which is based on 'frozen stress' photoelasticity, for which Poisson's ratio is 0.5. The results from ESDU 09014 for a plate width, $W = 10d$, are between 12 and 6 per cent higher than those obtained from ESDUpac A9330 for d/t values between 0.5 and 5.0, respectively. No conclusion has been drawn as to the reason for the discrepancy between the results.

Separate finite element calculations were made for plain and countersunk holes over a range of hole and countersink sizes for each of the different loading conditions. These results were analysed in Derivation 1, to obtain double-series polynomial equations across the range of geometries examined. The equations, which allow interpolation across the range of geometries shown in Table 1.1, are used in ESDUpac A9330. The large number of finite element results gives confidence in the polynomial fits, which reproduce the calculations across the range of geometries to within 2 per cent.

* ESDU 09014 "Elastic stress concentration factors. Geometric discontinuities in flat bars or strips of isotropic material".

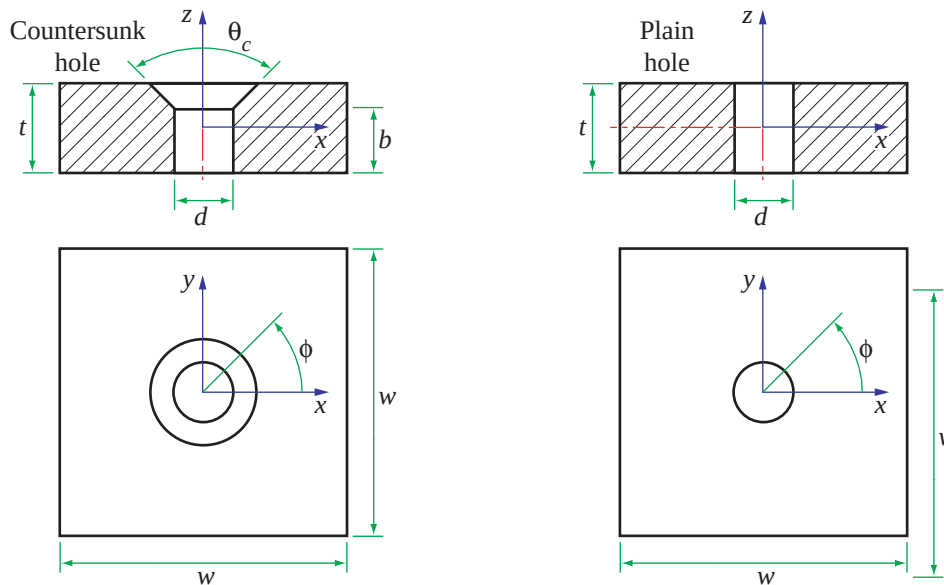
TABLE 1.1 LIMITS APPLYING TO POLYNOMIAL EQUATIONS FOR STRESS CONCENTRATION FACTORS

Limit on	Range of values
b/t	0 to 0.1 for countersunk holes
d/t	0.5 to 5 for both plain and countersunk holes

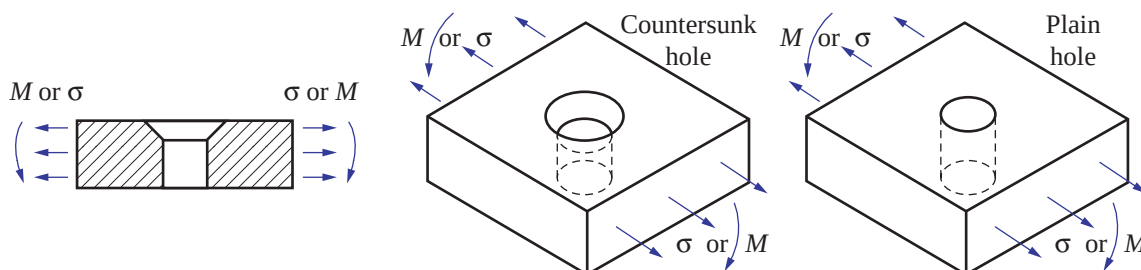
1.4 Loading

1.4.1 Remote tension and remote out-of-plane bending

For plain holes, stresses due to remote tension, remote out-of-plane bending and pin loading are treated. For countersunk holes, stresses due to remote tension and remote out-of-plane bending only are considered. The applied remote tension stress acts in the x -direction ($\phi = 0^\circ$) and remote out-of-plane bending acts about the y -axis.



(i) Hole in a plate with or without a countersink, plan and cross-section views



(ii) Application of load to plate containing a hole with or without a countersink

Sketch 1.1 Plain and Countersunk Hole Geometries and Loading