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ESDU 93030

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Three-dimensional elastic stress concentration factors
Plain or countersunk hole in a wide plate subjected to tension, bending or pin loading

Associated software: ESDUpac A9330

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THREE-DIMENSIONAL ELASTIC STRESS CONCENTRATION FACTORS. PLAIN OR COUNTERSUNK HOLE IN A WIDE PLATE SUBJECTED TO TENSION, BENDING OR PIN LOADING

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1. THEORY AND APPLICATION

This Data Item is accompanied by a computer program, ESDUpac A9330, for use in evaluating stress concentration factors along the bore of a plain or countersunk hole in a wide plate subjected to tension, out-of-plane bending or pin loading. The data are derived from a three dimensional analysis of the stress distribution around the hole.

The Data Item consists of two parts. Section 1 describes how the equations were developed, gives examples of stress concentration factor variation along the hole bore for selected geometries and loadings, and describes how the pin loading case was simulated. Section 2 describes the program, its input and its output.

1.1 Introduction

This section explains the derivation of the equations used in ESDUpac A9330 for the various loadings and geometries covered. It also gives examples of the variations in the maximum stress and the stress variation along the hole bore as an aid to the design of members containing holes.

For plain holes, stresses due to remote tension, remote out-of-plane bending and pin loading are treated. For countersunk holes, stresses due to remote tension and remote out-of-plane bending only are considered.

1.2 Notation

Both SI and lbf/in units are quoted but any coherent system of units may be used.

b	straight shank length of countersunk hole	m	in
d	diameter of plain portion of hole	m	in
K_b	stress concentration factor at $\phi = \pm 90°$ along bore of hole in plate under bending where $\sigma_{max} = K_b(6M/t^2)$		
K_p	stress concentration factor at $\phi = \pm 90°$ along bore of hole in plate loaded by pin where $\sigma_{max} = K_p(P/dt)$ (see Sketch 1.2b)		
K_t	stress concentration factor at $\phi = \pm 90°$ along bore of hole in plate under tension where $\sigma_{max} = K_t\sigma = K_t\langle P/Wt\rangle$		
K_w	stress concentration factor at $\phi = \pm 90°$ along bore of hole in plate under wedge loading where $\sigma_{max} = K_w(P/dt)$ (for use in analysis of pin loading, see Sketch 1.2aii)		
M	bending moment per unit width applied to plate acting about y-axis (see Sketch 1.1)	N m/m	lbf in/in
P	applied pin loading in x-direction	N	lbf
t	plate thickness	m	in

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W	width of plate	m	in
x, y, z	cartesian coordinate system used for plate and hole geometry (see Sketch 1.1)		
θ_c	included angle of countersink (see Sketch 1.1)	degree	degree
σ	applied remote tension stress in x-direction	Nm^2	lbf/in^2
σ_{max}	maximum circumferential direct stress in plate at hole perimeter	N/m^2	lbf/in^2
σ_r	radial direct stress applied to hole circumference at location ϕ	N/m^2	lbf/in^2
ϕ	angle measured from x-axis around circumference of hole (see Sketch 1.1)	degree	degree

Sign convention: Stresses - tensile are positive, compressive are negative.

1.3 Development of Data From Finite Element Analyses

The program ESDUpac A9330 (see Section 2) is based on Derivation 1, which employed a comprehensive series of three dimensional finite element calculations for square, elastic isotropic plates. These are dependent on Poisson's ratio and here a Poisson's ratio of 0.3 has been used. The width, W, of the plate was taken to be $10d$ for plain holes and $15d$ for countersunk holes. In practice, extending these dimensions to higher multiples of d may result in significant changes in the stress concentration. The stress concentration factor may decrease by up to 5 per cent for the remote tension case and by up to 10 per cent for the remote out-of-plane bending case. For pin loading a percentage change cannot be established from the available data.

For the case of a plain hole in a wide plate subjected to remote out-of-plane bending, the maximum stress concentration factor at the hole edge is obtainable from both ESDUpac A9330 and Figure 2 of ESDU 09014*, which is based on 'frozen stress' photoelasticity, for which Poisson's ratio is 0.5. The results from ESDU 09014 for a plate width, $W = 10d$, are between 12 and 6 per cent higher than those obtained from ESDUpac A9330 for d/t values between 0.5 and 5.0, respectively. No conclusion has been drawn as to the reason for the discrepancy between the results.

Separate finite element calculations were made for plain and countersunk holes over a range of hole and countersink sizes for each of the different loading conditions. These results were analysed in Derivation 1, to obtain double-series polynomial equations across the range of geometries examined. The equations, which allow interpolation across the range of geometries shown in Table 1.1, are used in ESDUpac A9330. The large number of finite element results gives confidence in the polynomial fits, which reproduce the calculations across the range of geometries to within 2 per cent.

* ESDU 09014 "Elastic stress concentration factors. Geometric discontinuities in flat bars or strips of isotropic material".

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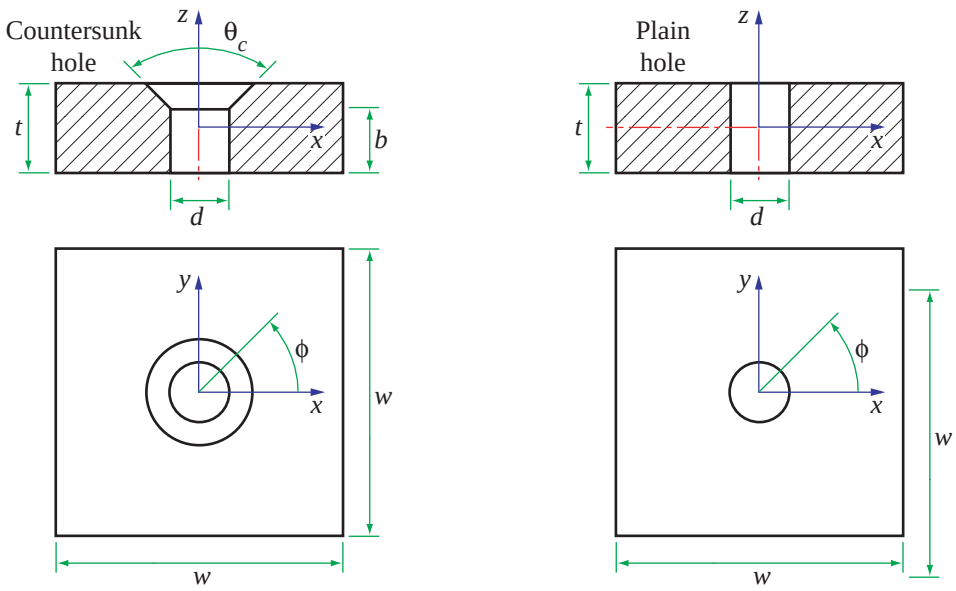
TABLE 1.1 LIMITS APPLYING TO POLYNOMIAL EQUATIONS FOR STRESS CONCENTRATION FACTORS

Limit on	*Range of values*
b/t	0 to 0.1 for countersunk holes
d/t	0.5 to 5 for both plain and countersunk holes

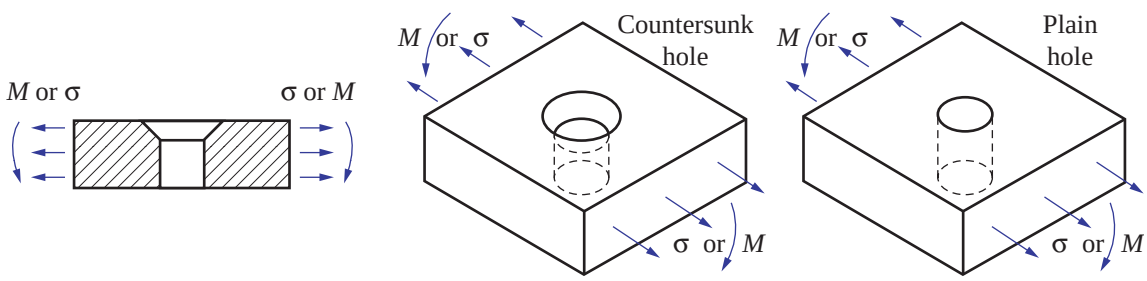
1.4 Loading

1.4.1 Remote tension and remote out-of-plane bending

For plain holes, stresses due to remote tension, remote out-of-plane bending and pin loading are treated. For countersunk holes, stresses due to remote tension and remote out-of-plane bending only are considered. The applied remote tension stress acts in the x-direction ($\phi = 0°$) and remote out-of-plane bending acts about the y-axis.



(i) Hole in a plate with or without a countersink, plan and cross-section views



(ii) Application of load to plate containing a hole with or without a countersink

Sketch 1.1 Plain and Countersunk Hole Geometries and Loading

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