

Stress analysis of laminated flat plates

ESDU Copyright material. For current status contact ESDU.

Endorsed by
The Royal Aeronautical Society

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes Item No. 74039, was monitored and guided by the Aerospace Structures Committee. This Committee first met in 1940 and now has the following membership:

Chairman

Mr K. Fitzsimons — Westland Helicopters Ltd

Members

Prof. P. Bartholomew	— QinetiQ, Farnborough
Dr R. Biswas	— BAE SYSTEMS Avionics plc (Stanmore)
Mr A. Dickson	— BAE SYSTEMS (Warton)
Mr S. Georgiou	— BAE SYSTEMS Avionics plc (Stanmore)
Mr A. Missenden	— Cranfield Aerospace Ltd
Mr P.J. Mitchelmore	— Independent
Mr M.S. Pressnell	— Independent
Mr B. Popham	— Astrium Space Ltd
Prof. A. Rothwell	— Technische Universiteit Delft, The Netherlands
Mr J.H. van der Sloot	— Independent
Mr K. van Katwijk*	— European Space Agency, Noordwijk, The Netherlands.

* Corresponding Member

The technical work involved in the assessment of the available information and the construction and subsequent development of the Data Item was undertaken by

Mr M.E. Grayley — Former Head of Strength Analysis Group.

The person with overall responsibility for the work in this subject area is Mr A.C. Quilter, Head of Strength Analysis Group.

STRESS ANALYSIS OF LAMINATED FLAT PLATES**CONTENTS**

	Page
1. INTRODUCTION	1
2. NOTATION	1
2.1 System of Reference Axes	1
2.2 Strains	2
2.3 Stresses	2
2.4 Plate Curvatures	2
2.5 Plate Loading Intensities	3
2.6 Plate Stiffnesses	3
2.7 Geometry	3
2.8 Thermal Loading	3
2.9 Suffixes	4
2.10 Sign Convention	4
3. LAYER STRESSES DUE TO EXTERNAL LOAD	5
3.1 Layer Stresses Under External Load for Balanced Laminates	6
4. BUILT-IN LAYER STRESSES	6
4.1 Method of Calculation	6
4.2 Constraint of Individual Layers to Datum Dimensions	7
4.3 Deformations in the Unrestrained Plate	8
4.4 Built-In Layer Stresses in the Unrestrained Plate	9
5. LAYER STRESSES UNDER EXTERNAL LOAD AND BUILT-IN STRESSES	10
5.1 The Plate Deformations	10
5.2 The Layer Stresses	10
5.2.1 The layer stresses in balanced laminates	11
6. UNBALANCED LAMINATES HELD FLAT	11
6.1 Deformations and Stresses When There is No In-Plane Loading Applied	12
6.2 Deformations and Stresses When an External In-Plane Loading is Applied	12

7.	EXAMPLES	12
7.1	Stresses in a Balanced Laminated Plate	12
7.2	Built-in Stresses in a Sandwich Panel with Laminated Face Plates	14
7.2.1	Plate stiffness and transformation matrices	15
7.2.2	Stresses in a face plate prior to assembly to core	16
7.2.3	Stresses in the assembled sandwich	18
	7.2.3.1 Face plate stresses in the assembled sandwich	19
	7.2.3.2 Core stresses in the assembled sandwich	20
8.	SUMMARY TABLE	21

STRESS ANALYSIS OF LAMINATED FLAT PLATES

1. INTRODUCTION

This Item presents formulae for the elastic analysis of the stresses in flat laminated plates built up from thin orthotropic layers that may have different material elastic and thermal properties, thicknesses and orientations of their principal axes of orthotropy. Stresses in laminated plates arise either as a result of externally applied loading or as a result of the method of manufacture.

Current methods of production of laminated plates usually require the assembly to be cured at a temperature above the working temperature range of the plate. Upon cooling to working temperatures the different orientations of the layers together with their often different elastic and thermal properties lead to the development of a system of self-equilibrating stresses. Similarly, internal stresses may be generated in plates cured at room temperature but which are subjected to a wide range of working temperatures.

The formulae presented relate to uniform stress systems away from the plate boundaries or other geometric discontinuities. No allowance is made for the deformations associated with the effects of through-the-thickness shear.

Methods of calculation of the stiffnesses and transformation matrices referred to extensively in this Item are provided in Item No. 94003*.

The principal formulae derived in this Item are summarised in Table 8.1.

2. NOTATION

Attention is drawn to the sign convention used in this Item. The convention is that now in common use and differs from that used in Data Item No. 74039 which this Data Item supersedes. The sign convention for moments and curvatures has been changed and, as a consequence, the signs of many of the equations are also changed. It also employs a different sign for the coupling stiffnesses for unbalanced plates (see Data Item No. 94003). The convention used in this Item is consistent with that used in Data Items associated with ESDUview and/or ESDUpac programs for the analysis of composites.

Both SI and British units are quoted but any coherent system of units may be used.

2.1 System of Reference Axes

$O(x, y, z)$ plate reference axes

$O(\alpha, \beta)$ principal axes of orthotropy of a layer (used as additional set of reference axes)

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

2.2 Strains

$\mathbf{e}$	$\begin{bmatrix} e_x \\ e_y \\ e_{xy} \end{bmatrix}$	column matrix of direct strains and shear strain referred to (x, y) coordinates
$\bar{\mathbf{e}}_k$	$\begin{bmatrix} e_\alpha \\ e_\beta \\ e_{\alpha\beta} \end{bmatrix}_k$	column matrix of direct strains and shear strain of k th layer referred to (α, β) coordinates of that layer
$\mathbf{T}_k$		matrix relating strain components in (x, y) coordinates to strain components in (α, β) coordinates of k th layer (see Item No. 94003)
$\boldsymbol{\epsilon}$	$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix}$	column matrix of mid-plate direct strains and shear strain referred to (x, y) coordinates

2.3 Stresses

$\mathbf{f}_k$	$\begin{bmatrix} f_x \\ f_y \\ f_{xy} \end{bmatrix}_k$	column matrix of direct stresses and shear stress in k th layer referred to (x, y) coordinates	N/m ²	lbf/in ²
$\bar{\mathbf{f}}_k$	$\begin{bmatrix} f_\alpha \\ f_\beta \\ f_{\alpha\beta} \end{bmatrix}_k$	column matrix of direct stresses and shear stress in k th layer referred to (α, β) coordinates of that layer	N/m ²	lbf/in ²

2.4 Plate Curvatures

w		deflection of plate in z -direction	m	in
$\mathbf{K}$	$\begin{bmatrix} -\partial^2 w / \partial x^2 \\ -\partial^2 w / \partial y^2 \\ -2\partial^2 w / \partial x \partial y \end{bmatrix}$	vector of plate middle plane curvatures and twist components	m ⁻¹	in ⁻¹

2.5 Plate Loading Intensities

M	$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$	column matrix of bending moment intensities acting in xz - and yz -planes and twisting moment intensity acting about x - and y -axes respectively	N m/m	lbf in/in
N	$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}$	column matrix of direct load intensities and shear load intensity referred to (x, y) coordinates	N/m	lbf/in

2.6 Plate Stiffnesses^{*}

A	in-plane stiffness matrix for plate (see Item No. 94003)	N/m	lbf/in
B	coupled in-plane and flexural stiffness matrix for plate (see Item No. 94003)	N	lbf
b_k	matrix relating stress and strain in (x, y) coordinates for k th layer (see Item No. 94003)	N/m ²	lbf/in ²
C_k	matrix of k th layer relating stress and strain in (α, β) coordinates (see Item No. 94003)	N/m ²	lbf/in ²
D	flexural stiffness matrix for plate (see Item No. 94003)	N m	lbf in

2.7 Geometry

n	number of layers in plate		
t_k	thickness of k th layer	m	in
z	distance from reference plane along z -axis	m	in

2.8 Thermal Loading

k_α, k_β	coefficient of linear thermal expansion for layer material in α - and β -directions respectively	°C ⁻¹	°C ⁻¹
T_u	assembly temperature	°C	°C

^{*} Units quoted against each matrix defined in this section are the units of all the elements of that matrix.

T_w working temperature °C °C

2.9 Suffixes

a relates to stresses arising when an external loading is applied

d relates to prescribed deformations from datum

k layer identification number

p relates to loads and moments required to hold plate to datum at working temperature

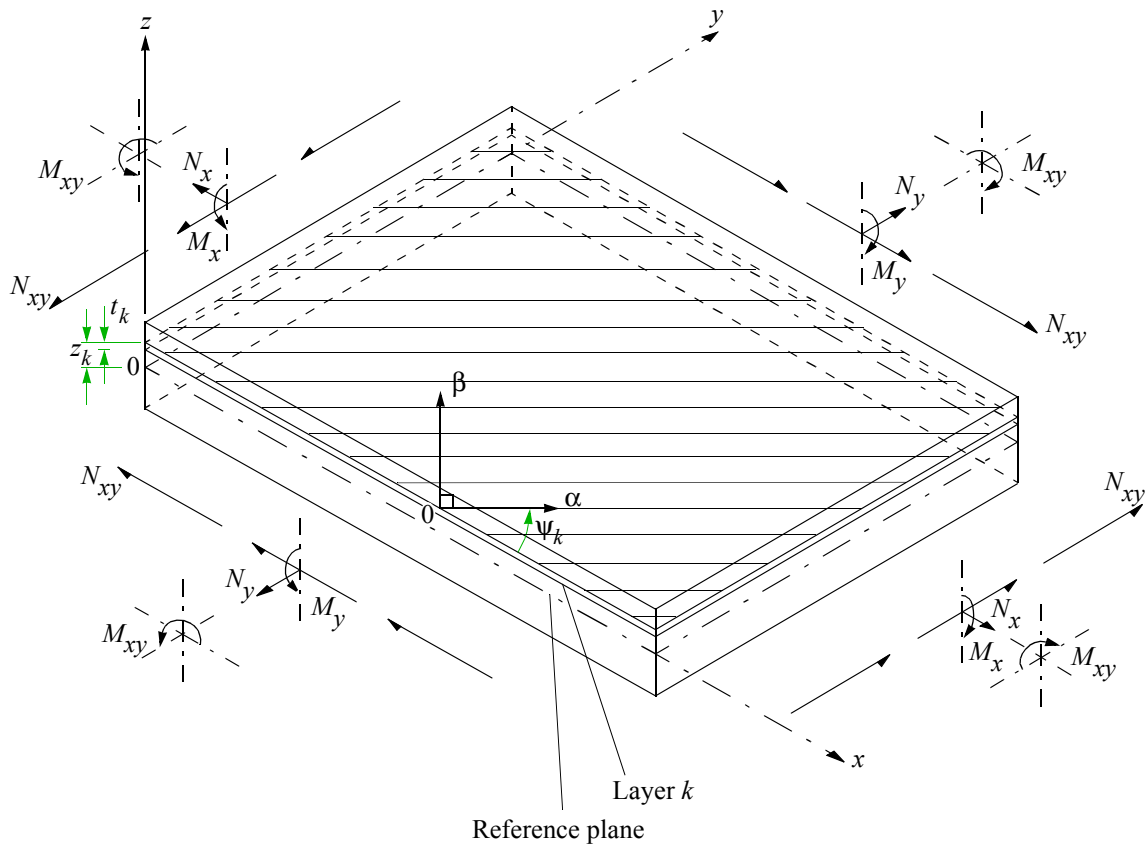
u relates to deformation of unrestrained plate from datum

θ relates to constraint due to thermal effects

2.10 Sign Convention

The positive convention for direct and shear forces and bending and twisting moments is shown in Sketch 2.1.

Tensile stresses are positive and compressive stresses are negative.



Sketch 2.1 Sign convention for forces and moments

NOTE: This Data Item supersedes Data Item No. 74039. The sign convention for moments has been changed to accord with current practice.

3. LAYER STRESSES DUE TO EXTERNAL LOAD

If a laminated plate is subjected to known force and moment intensities, $\mathbf{N}$ and $\mathbf{M}$, then, from Equation (3.4) of Item No. 94003 the associated plate deformations are given by

$$\begin{bmatrix} \boldsymbol{\varepsilon} \\ \mathbf{K} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix}. \quad (3.1)$$

Under the action of $\mathbf{N}$ and $\mathbf{M}$ the strain at distance z from the central, or reference, plane of the plate about which the plate stiffnesses were calculated (see Section 3 of Item No. 94003) is

$$\mathbf{e} = \boldsymbol{\varepsilon} - \mathbf{K}z \quad (3.2)$$

so that, using Equation (4.3) of Item No. 94003, the associated stress is

$$\mathbf{f}_{ak} = \mathbf{b}_k(\boldsymbol{\varepsilon} + \mathbf{K}z). \quad (3.3)$$