

BUCKLING OF FLAT RECTANGULAR ORTHOTROPIC PLATES

1. INTRODUCTION AND SCOPE

1.1 Introduction

The Item together with ESDUpac A9405 provides a Fortran computer program for calculating the buckling loads of flat rectangular orthotropic plates under combinations of uniform in-plane biaxial direct and shear loads. The program deals with symmetrical combinations of clamped and simply-supported edge conditions and takes account of out-of-plane shear deformations. Some of the applied loads may be fixed and the remaining load that will cause buckling will be calculated. Alternatively, the ratio of the applied loads one to another, can be specified and the loads in the same ratio that cause buckling calculated.

This Data Item is provided in two parts. Section 1 describes the notation, theory and applications and Section 2 describes the input and output of the program of ESDUpac A9405 and includes example calculations.

The program is available in two formats: (a) a Fortran source code with example input and output files and (b) an executable program for PC's. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

1.2 Notation

Attention is drawn to the sign convention used in this Item. The convention is that now in common use and differs from that used in Data Item No. 73003 which this Data Item supersedes. The sign convention for in-plane loads has been changed. The convention used in this Item is consistent with that used in Data Items associated with ESDUview and/or ESDUpac programs for the analysis of composites.

Both SI and British units are quoted but any coherent system of units may be used.

A_{ij}	amplitude of wave component in buckled plate (see Table 2.3)	m	in
a, b	dimensions of plate in x - and y -directions, respectively	m	in
D_x, D_y	stiffnesses per unit width for bending in xz - and yz -planes, respectively (see Data Item No. 68029*)	N m ² /m	lbf in ² /in
D_{qx}, D_{qy}	stiffnesses per unit width for shear in xz - and yz -planes, respectively	N/m	lbf/in
D_{xy}	stiffness of plate for twisting about x - and y -axes	N m	lbf in
m, n	number of half wave lengths in x - and y -directions respectively		
P_x, P_y, Q	direct and shear loads, per unit width, in x - and y -directions (an additional suffix, b , indicates the load at which buckling first occurs)	N/m	lbf/in

For footnotes see end of Notation.

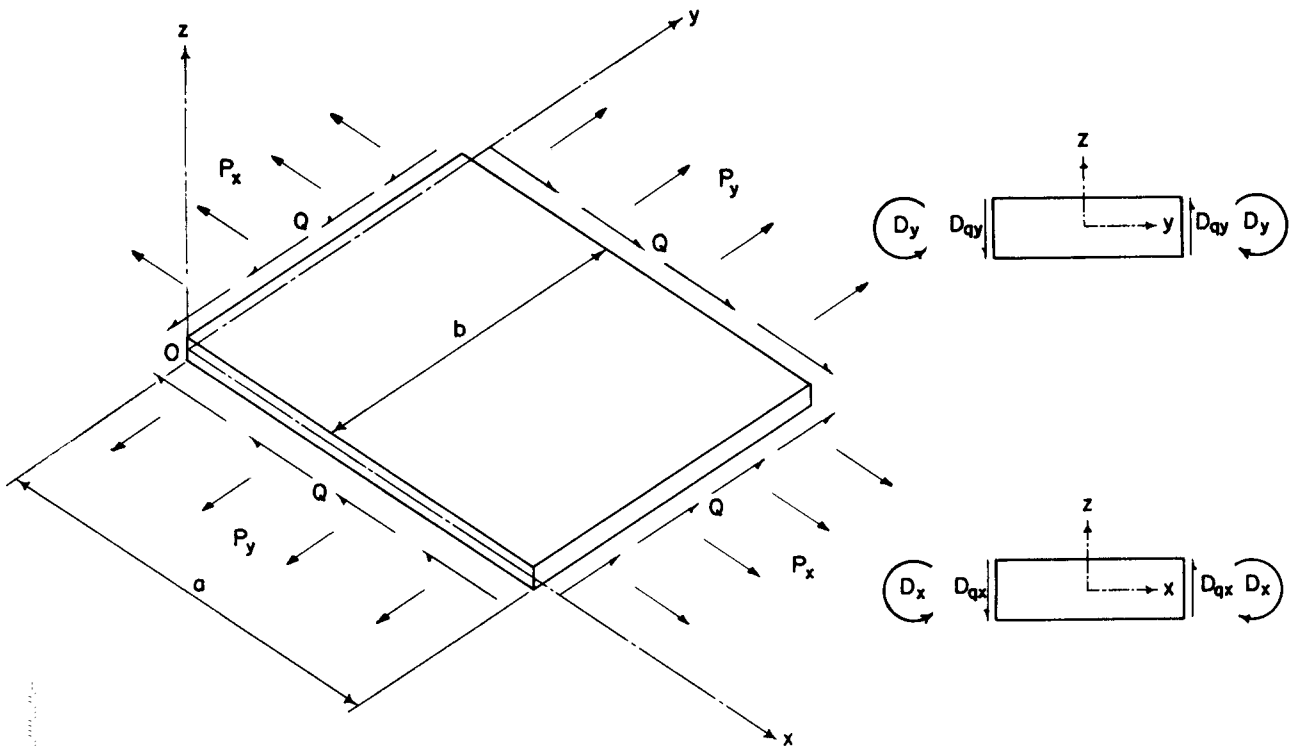
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P_E	$4(\pi/b)^2 D_y / (1 - \sigma_x \sigma_y)$	N/m	lbf/in
σ_x, σ_y	apparent values of Poisson's ratio for bending in xz - and yz -planes respectively such that $\sigma_x/D_x = \sigma_y/D_y$ (see Item No. 68029)		

* Data Item No. 68029 "Information on the use of Data Items on sandwich panels".

1.2.1 Sign Convention

Tensile loads and shear loads are positive and compressive loads are negative, see Sketch 1.1.



Sketch 1.1

1.3 Theory and Applications

The program of ESDUpac A9405 is based on the sandwich panel theory set out in Data Item No. 68029. It may be applied to any panel that has principal planes of bending parallel to its edges with a common neutral plane. The plate axes are taken to lie in this neutral plane. The program is applicable to plates manufactured from isotropic materials, sandwich panels and reinforced laminated composite plates that are specially orthotropic. Data Item No. 68029 gives guidance on the estimation of plate stiffnesses for sandwich panels including the through-the-thickness shear stiffnesses. Data Item No. 89013*, together with

* Data Item No. 89013 "Transverse (through-the-thickness) shear stiffnesses of fibre reinforced composite laminated plates".

ESDUpac A8913, provides a method of estimating the through-the-thickness shear stiffnesses for composite laminated plates. Data Item No. 94003* deals with estimation of plate in-plane, flexural and twisting stiffnesses of laminated plates.

The buckling loads computed are based on a 6×6 term series for the wave form of the buckled plate (see also Section 2.4.2 and Table 2.3).

The program was used to calculate buckling loads for Data Item Nos 67024 to 26, 67032 and 68030 to 33. These Data Items may be used as a guide to the range of validity of the program. Data Item No. 81047† provides a computer program that may be used to input the layer properties of a laminated composite plate and that calculates the required stiffnesses and calls the principal subroutine of this Data Item to calculate the buckling load.

1.4 DERIVATION

1. BRUCE, J.
ROBINSON, J.R. The buckling and bending of orthotropic sandwich panels in compression and shear. Unpublished work at Handley Page Ltd, 1956.
2. BOWDLER, H.J.
MARTIN, R.S.
PETERS, G.
WILKINSON, J.H. Solution of real and complex systems of linear equations. *Numerische Mathematik*, 8, pp. 217-234, 1966.

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

† Data Item No. 81047 "Buckling of flat rectangular orthotropic plates (isotropic, orthotropic and laminated composite plates and sandwich panels).".

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2. ESDUpac A9405*

2.1 Introduction

ESDUpac A9405 contains a Fortran computer program that accepts the properties of an orthotropic plate in dimensionless form together with the in-plane loading and, for defined edge conditions, calculates the loads at which buckling will occur. Section 2.2 gives the scope of the program, Section 2.3 details the input requirements and Section 2.4 gives an explanation of the output including all messages. Section 2.5 provides a selection of examples that illustrate the use of the program and that may also be used as test cases.

The program is available in two formats: (a) a Fortran source code with example input and output files and (b) an executable program for PC's. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume. Information on how to mount and run the program is contained in the introduction to the Software Volume.

The basis of the Fortran computer program of ESDUpac A9405 is described in Section 1 of this Data Item. Every reasonable effort has been made to ensure that the program performs the intended calculations satisfactorily. No representation as to the suitability or fitness of the program for any particular purpose is made and no liability for any loss occasioned by any persons as a direct or indirect result of use of the program whether arising from negligence or otherwise can be accepted. In no event shall ESDU or any individuals associated with the development of the program be liable for any damages, including loss of profit or consequential loss, arising out of or in connection with the program.

2.2 Scope of the Program

The program may be used for the following calculations:

- (a) calculation of the unknown load in any combination P_{bx}/P_E , P_{by}/P_E and Q_b/P_E in which two loads are prescribed,
- (b) calculation of P_{bx}/P_E , P_{by}/P_E and Q_b/P_E when they are in prescribed proportions to one another.

The program deals with the following edge conditions:

- (i) all edges simply-supported,
- (ii) edges parallel to x-axis clamped and edges parallel to y-axis simply-supported,
- (iii) edges parallel to x-axis simply-supported and edges parallel to y-axis clamped,
- (iv) all edges clamped.

2.3 Data Input

Prior to data input 3 lines are provided for run identification information. These lines may be empty or can carry up to 77 alpha numeric characters each. The contents of these lines appear in the output.

Following run identification information five distinct sets of data input are required. They are inputs that

* The current version of this ESDUpac is shown in the Volume contents and also in the examples of program output in Section 2.5.

control the number of calculations, define the edge conditions, control the loading case, control the calculation accuracy and finally provide the plate aspect ratio, dimensionless stiffnesses and loads.

2.3.1 Number of calculations

Provision is made for more than one calculation per run. The first number specified is therefore the number of calculations required and therefore the number of separate sets of data to be provided.

TABLE 2.1 Input parameters

Entry order	Interpretation	Section number for notes
(1)	Run identification information. 3 lines that may be empty or carry any message composed of up to 77 alpha numeric characters per line.	2.3
(2)	Integer specifying the number of separate calculations to be carried out. The data as defined in steps (3) to (7) inclusive must follow for each separate calculation required.	2.3.1
(3)	Single integer specifying the edge conditions for the current calculation. 1 All edges simply-supported. 2 Edges parallel to x-axis clamped and edges parallel to y-axis simply-supported. 3 Edges parallel to x-axis simply-supported and edges parallel to y-axis clamped. 4 All edges clamped.	2.3.2
(4)	Single integer specifying loading type for the current calculation. 1 Given P_x/P_E and P_y/P_E, find Q_b/P_E 2 Given Q/P_E and P_y/P_E, find P_{bx}/P_E 3 Given Q/P_E and P_x/P_E, find P_{by}/P_E 4 Given any convenient set of numbers that are in proportion with P_x, P_y and Q and have the correct signs, find P_{bx}/P_E, P_{by}/P_E and Q_b/P_E	
(5)	Two integers must be entered. The first specifies the number of iterations permitted in the search for the root in the current calculation. Any number in the range 5 to 1000 is permitted. The recommended value is 200. The second integer specifies the reciprocal of the maximum permitted error in the root in the current calculation. Any number in the range 100 to 1 000 000 may be specified. The recommended value is 50 000.	2.3.3

TABLE 2.1 Input parameters

Entry order	Interpretation	Section number for notes
(6)	Plate aspect ratio followed by five non-dimensional stiffness ratios, all real numbers all positive. $a/b, D_x/D_y, D_{qy}/D_{qx}, D_{xy}(1 - \sigma_x \sigma_y)/D_y, P_E/4D_{qy}, \sigma_x$	2.3.4
(7)	Input loads. For loading types 1, 2 or 3 of step (4) enter $P_x/P_E, P_y/P_E, Q/P_E$ using zero for the unknown load ratio and where the applied loads are zero. Known load ratios may be positive (tension) or negative (compression) and are real numbers. For loading type 4 in step (4) enter any convenient set of real numbers that are in proportion with P_x, P_y and Q observing the sign convention.	
(8)	Repeat Steps (3) to (7) inclusive for each separate calculation specified in Step (2).	

2.3.2 Specification of edge conditions and loading combinations

These are controlled by integers that take a value of 1, 2, 3 or 4 and are fully specified in Table 2.1.

2.3.3 Accuracy and limits on the calculation

The solution involves an iterative procedure in the search for the dominant root of the characteristic equation. The number of iterations allowed and the accuracy of the test for convergence can be varied. Values of 200 iterations and one part in 50 000 for the accuracy are recommended. These can be altered if the program fails to find the required root (see Section 2.4.3).

2.3.4 Plate aspect ratio, dimensionless stiffnesses and loadings

The plate aspect ratio and the dimensionless forms in which the plate properties should be supplied are set out in Table 2.1. These should be provided in strict order. The dimensionless loadings are then provided. In these, when the loading control is 1, 2, or 3, the unknown load is given zero value. When the loading control is 4, that is loads entered in prescribed ratio, any set of loads in the desired ratio, taking due account of the sign convention, can be entered.

Where two known dimensionless loads are supplied the program automatically checks whether the known loads will cause buckling before the unknown load is applied. (If this is the case then the calculation is terminated, see Section 2.4.3, message number 14.) Because of this the program does not automatically provide the stabilising tensile load that will make the known load(s) the buckling load(s). Such a load combination may be found by trial by inserting values of the stabilising tensile load, and one of the known loads if two are present, and determining that value that gives the known load(s) as buckling load(s). Where a set of numbers proportional to P_x, P_y and Q is specified, the program automatically determines whether or not the loading combination is stable. In these cases actual loads, of the correct sign, may be input.

2.4 Program Output and Interpretation

The output of a successful run is provided in three parts, first the run identification information then the input data check and finally the calculated buckling loads. Section 2.5 gives examples of the output of the program.

2.4.1 Input data check

This first lists the total number of calculations in the run. Thereafter the calculations are each given a sequential case number. The edge conditions and loading controls are stated followed by the root solving controls. The input plate aspect ratio and stiffnesses are then output and Table 2.2 lists the form of symbols used in the output against those of the notation.

TABLE 2.2 Notation

a/b	$\equiv a/b$	D_x/D_y	$\equiv D_x/D_y$
D_{qx}/D_{qy}	$\equiv D_{qx}/D_{qy}$	$D_{xy}(1 - \text{sigx} \cdot \text{sigy})/D_y$	$\equiv D_{xy}(1 - \sigma_x \sigma_y)/D_y$
$PE/4D_{qy}$	$\equiv P_E/(4D_{qy})$	sigx	$\equiv \sigma_x$

2.4.2 Results

The results are provided in the dimensionless form P_x/P_E , P_y/P_E and Q/P_E in the x -, y - and xy -directions, respectively, for calculations based upon 6×6 and 5×5 term series. The solutions for the two sets of results are provided to indicate if the limits of the program have been exceeded. Table 2.3 sets out the buckled wave forms associated with 6×6 and 5×5 term series for the four edge condition cases dealt with. Values for these two different length series are provided to help with assessing the validity of the computed buckling loads. Large differences between the buckling loads computed for the 6×6 and 5×5 term series are a warning that the range of the program has been exceeded but small differences do not guarantee that it has not. The Data Items noted in Section 1.3 are the best guide to the range of validity of the program. Additional help in the assessment of results may be obtained by calculating buckling loads for ratios of a/b on either side of the ratio under consideration.

TABLE 2.3 Buckled waveforms

Buckled waveform	m and n	
	6×6 series	5×5 series
All edges simply-supported $\sum_{i=1}^m \sum_{j=1}^n A_{ij} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$	$m = 6 \quad n = 6$	$m = 5 \quad n = 5$
Edges parallel to x -axis clamped, edges parallel to y -axis simply-supported $\sum_{i=1}^m \sum_{j=2}^n A_{ij} \sin \frac{i\pi x}{a} \left[\cos \frac{(j-2)\pi y}{b} - \cos \frac{j\pi y}{b} \right]$	$m = 6 \quad n = 7$	$m = 5 \quad n = 6$
Edges parallel to x -axis simply-supported, edges parallel to y -axis clamped $\sum_{i=2}^m \sum_{j=1}^n A_{ij} \left[\cos \frac{(i-2)\pi x}{a} - \cos \frac{i\pi x}{a} \right] \sin \frac{j\pi y}{b}$	$m = 7 \quad n = 6$	$m = 6 \quad n = 5$
All edges clamped $\sum_{i=2}^m \sum_{j=2}^n A_{ij} \left[\cos \frac{(i-2)\pi x}{a} - \cos \frac{i\pi x}{a} \right] \left[\cos \frac{(j-2)\pi y}{b} - \cos \frac{j\pi y}{b} \right]$	$m = 7 \quad n = 7$	$m = 6 \quad n = 6$