

ELASTIC BUCKLING OF CYLINDRICALLY CURVED LAMINATED FIBRE REINFORCED COMPOSITE PANELS WITH ALL EDGES SIMPLY-SUPPORTED UNDER BIAXIAL LOADING

1. APPLICATION AND DERIVATION OF PROGRAM

1.1 Introduction

The Data Item together with ESDUpac A9407 presents a Fortran computer program for calculating the axial buckling loads of curved laminated fibre reinforced composite panels having all edges simply-supported subjected to axial in-plane loading with optional circumferential in-plane loading due to pressure on the panel surface.

This Data Item is arranged in three parts. Section 1 gives information on the application of the program, examples of its application and a summary of the theory from which the program is derived. Section 2 gives the instructions necessary for operating the program, guidance in interpreting the output and an example of input and output of the program. Section 3 gives notes on the program organisation and testing.

The program is applicable to specially orthotropic panels classified as AsBoDs Reference 0 in Table 7.1 of Data Item No. 94003.* If the program is used for laminates with any other forms of lay-ups it will treat them as specially orthotropic. A full list of symmetric and anti-symmetric specially orthotropic lay-ups is given in Data Item No. 82013†.

The panels considered are of rectangular form and uniform thickness having cylindrical curvature across the width. All edges of the panel are assumed to be simply-supported. The panel is subjected to direct loading applied uniformly to the ends of the panel in the direction of the length, which may be combined with similar loading in the circumferential direction along the sides of the panel in the circumferential direction associated with pressure on the panel surface. Sketch 1.1 shows the general configuration of the panel and its loading.

The program provides the minimum elastic buckling load per unit width, N_{xB} , applied to the ends of the panel and the number of half wavelengths into which the panel will buckle, along its length and across its width, for a given circumferential load per unit length, N_y . The panel stiffness matrices **A**, **B**, and **D** are also given together with a check on the panel construction.

The analysis is based upon elastic, thin plate, small deflection theory in which the panel is assumed to behave as a homogeneous orthotropic material whose axes of orthotropy are aligned with the edges of the plate. The theory only takes account of direct in-plane, flexural and twisting deformations as governed by the **A** and **D** stiffness matrices. Through-the-thickness shear deformations are disregarded. Details of the theoretical basis of the program are given in Section 1.3.

Examples on the application of the program are given in Section 1.4.

1.2 Notation

Attention is drawn to the sign convention used in this Item. The convention is that now in common use and differs from that used in Data Item No. 87025 which this Data Item supersedes. The sign convention for in-plane loads has been changed. The convention used in this Item is consistent with

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

† Data Item No 82013 "Laminate stacking sequences for special orthotropy". (Application to fibre reinforced composites.)

that used in Data Items associated with ESDUview and/or ESDUpac programs for the analysis of composites.

Both SI and British Units are quoted but any coherent system of units may be used.

1.2.1 Coordinate System (see Sketches 1.1 and 1.2)

Note: It is implicit in the program that $z = 0$ is midway between the upper and lower surfaces of the laminate.

$0(x, y, z)$ laminate reference axes

$0(\alpha, \beta)$ principal axes of orthotropy of layer

1.2.2 Panel Overall Stiffnesses

A	in-plane stiffness matrix for laminate	N/m	lbf/in
B	coupled in-plane and flexural stiffness matrix for laminate	N	lbf
D	flexural stiffness matrix for laminate	N m	lbf in
S	inverse of A matrix	m/N	in/lbf

1.2.3 Panel Displacements, Curvatures and Strains

K	column matrix of curvature components of plate	m^{-1}	in^{-1}
u	displacement in x -direction	m	in
v	displacement in y -direction	m	in
w	displacement in z -direction	m	in
ϵ	column matrix of mid-panel direct strains and shear strain referred (x, y) coordinates		

1.2.4 Panel Loading and Geometry

a	panel length (in x -direction)	m	in
b	curved panel width (in y -direction along circumference)	m	in
IC	program indicator for selection of b or θ in input		

M	column matrix of bending moment intensities acting in xz - and yz -planes and twisting moment intensity acting about x - and y -axes	N m/m	lbf in/in
m	number of buckle half waves along panel length, a		
N	column matrix of direct load intensities and shear load intensity referred to (x, y) coordinates	N/m	lbf/in
N_{xB}	buckling load per unit width of panel in axial, x -direction	N/m	lbf/in
n	number of buckle half waves across panel width, b		
p	pressure on panel surface (implicit)	N/m ²	lbf/in ²
R	radius of curvature of panel	m	in
λ	parameter associating number of buckle waves with panel length, $\lambda = m\pi/a$	m ⁻¹	in ⁻¹
η	parameter associating number of buckle waves with panel width, $\eta = n\pi/b$	m ⁻¹	in ⁻¹
θ	angle subtended by ends of curved panel	degree	degree

1.2.5 Layer Properties

C	reference number of layer configuration		
C_{max}	maximum number of layer configurations $C \leq 50$		
$E_{\alpha k}, E_{\beta k}$	moduli of elasticity of k th layer in α - and β -directions, respectively	N/m ²	lbf/in ²
$G_{\alpha\beta k}$	shear modulus of k th layer in $\alpha\beta$ -directions	N/m ²	lbf/in ²
nl	number of layers in laminate $nl \leq 200$		
t_k	thickness of k th layer	m	in
Ψ_k	angle of principal axis of orthotropy, α , of k th layer measured from panel reference axis, $0x$, as shown in Sketch 1.2	degree	degree
$\nu_{\alpha\beta k}$	Poisson's ratio for k th layer resulting from load applied in α -direction		

1.2.6 Suffixes

i, j	number signifying position in row or column respectively of element in matrix
k	layer identification number
x, y, z	related to laminate reference axes
α, β	related to layer alignment

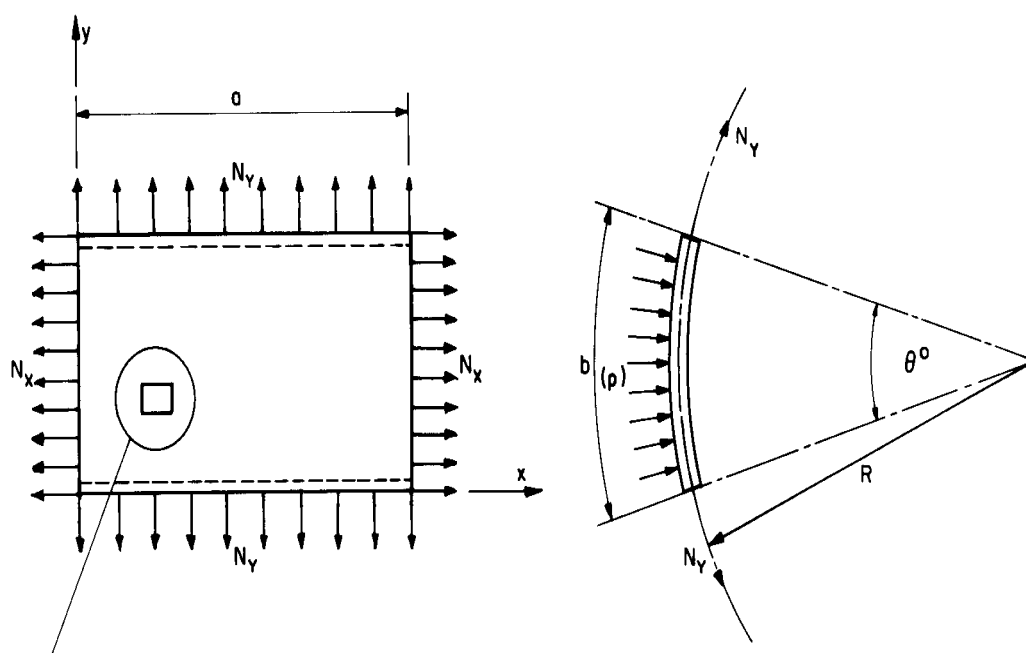
1.2.7 Sign convention

Tensile loads positive, compressive loads negative.

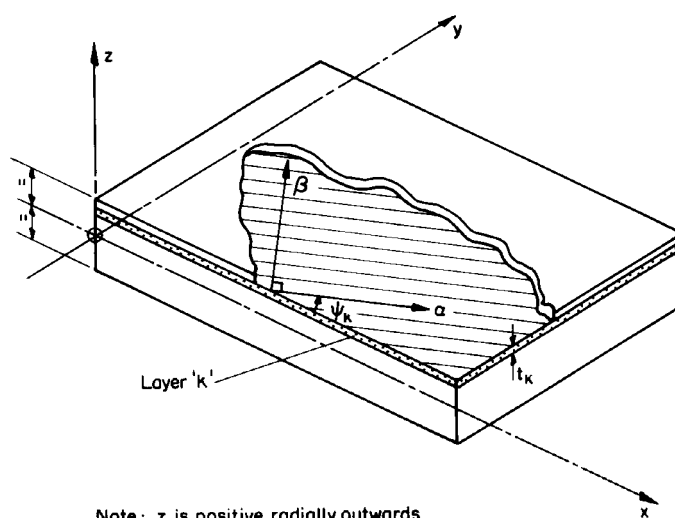
1.2.8 Plate Stiffness Type Code

This is as defined in Table 7.1 of Data Item No. 94003. The code consists of the following letters, as appropriate, after each of the symbols for the symmetric stiffness matrices **A**, **B** and **D**, signifying.

o	all elements zero
s	special orthotropic form



Sketch 1.1 General configuration (note that N_y is shown positive but normally N_y takes a negative value to provide equilibrium).



Sketch 1.2 Element of laminate

1.3 Derivation

1.3.1 Sources

The buckling analysis of a curved composite laminated panel in the program of ESDUpac A9407 is based on Derivations 1 and 2. The formulation of the stiffness matrices used in the program is derived from Data Item No. 94003.

1.3.2 Theory

The analysis is based upon elastic, thin shell, small deflection, classical laminate theory, in which the cylindrical panel is assumed to be made of homogeneous orthotropic material whose axes of orthotropy are aligned with the x- and y-axes of the panel. Through-thickness forces and shear deformations are neglected.

The basic load deformation relationships for the laminate are given by Equation (4.4) of Data Item No. 94003 in which:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \mathbf{K} \end{bmatrix}. \quad (1.1)$$

The theory considers only specially orthotropic lay-ups of the form AsBoDs, type 0 in Table 7.1 of Data Item No. 94003, for which the $\mathbf{B}$ matrix equals zero and

$$A_{13} = A_{23} = D_{13} = D_{23} = 0. \quad (1.2)$$

The buckling analysis is based on the assumption that the simply-supported boundary conditions can be

satisfied by distributions of the form

$$\begin{aligned} u &\propto \cos(\lambda x) \sin(\eta y) \\ \text{and} \quad v &\propto \sin(\lambda x) \cos(\eta y) \\ w &\propto \sin(\lambda x) \sin(\eta y) \end{aligned} \quad (1.3)$$

as detailed in Reference 3 Chapter 11.

The buckling load per unit width for the curved panel is obtained from the expression

$$N_x = -[(H_{12}^2/H_{11}) + H_{22} + N_y \eta^2]/\lambda^2, \quad (1.4)$$

where

$$\begin{aligned} H_{11} &= \lambda^4 S_{22} + 2[S_{12} + (S_{33}/2)]\lambda^2 \eta^2 + S_{11} \eta^4, \\ H_{12} &= (\lambda^2/R), \\ H_{22} &= \lambda^4 D_{11} + 2(D_{12} + 2D_{33})\lambda^2 \eta^2 + D_{22} \eta^4, \end{aligned} \quad (1.5)$$

in which values of S_{ij} , are obtained from the relationship

$$\mathbf{S} = \mathbf{A}^{-1}. \quad (1.6)$$

For a given panel Equation (1.4) will give a series of loads depending on values of m and n . The minimum value of N_x is chosen as the panel buckling load per unit width, N_{xB} . Note that for practical composite curved panels as for isotropic panels initial imperfections of form and material may cause buckling at loads other than the values of N_{xB} given.

It is implicit in Equation (1.4) that N_y is balanced by pressure, p , on the surface of the panel (see Sketch 1.1).

1.3.3 Derivation and references

Derivation

This section lists selected sources that have assisted in the preparation of this Item.

1. GEIER, B. Das Beulverhalten versteifter Zylinderschalen, Teil 1: Differentialgleichungen (in German, ESDU translation). *Z. Flugwiss.*, 14, Heft 7, pp 306-323, 1966.
2. GEIER, B. Das Beulverhalten versteifter Zylinderschalen, Teil 2: Beullasten (in German, SEGGELE, P. ESDU translation). *Z. Flugwiss.*, 15, Heft 12, pp 477-490, 1967.

References

The references given are recommended sources of information supplementary to that in this Item.

3. TIMOSHENKO, S.P. *Theory of elastic stability*. McGraw-Hill, New York Chapter 11, second edition, 1961.
- GERE, J.M.
4. LEISSA, A.W. Buckling of laminated composite plates and shell panels. Ohio State University Research Foundation, Columbus. AD-A162723, OSURF 762513/713464, AFWAL-TR-85-3069, 464 pp, January 1985.
5. SOLDATOS, K.P. Buckling and vibration of cross-ply laminated circular cylindrical shell panels. TZIVANIDIS, G.J. *Z. Angewandte Mathematik Physik (ZAMP)*, Vol. 33, pp. 229-240, 1982.

1.4 Application of the Program of ESDUpac A9407 (see Section 2)

The program can be used to obtain the axial elastic buckling load per unit width, N_{xB} , on the panel for any specified circumferential load per unit length, N_y .

As noted in Sections 1.1 and 1.3, the program is strictly applicable to panels having lay-ups which produce the specially orthotropic form, AsBoDs. (Type 0 in Table 7.1 of Data Item No. 94003.) If the program is applied to other laminates it will ignore the **B** matrix and certain terms of the **A** and **D** matrices as indicated in Equation (1.2).

Three examples are given to show applications of the program. The first example (Sketch 1.3) shows the typical variation of N_x , obtained using Equation (1.4) for various values of m and n with increasing panel length and indicates the values selected as the panel buckling load, N_{xB} . The second example (Sketch 1.4) shows the typical variation on the panel buckling load, N_{xB} , under different circumferential loading, N_y . The third example compares results from the program with data from Item No. Struct 02.01.50* for an isotropic curved panel, as shown in Table 1.1.

1.4.1 Variation of N_{xB} with increasing length

Sketch 1.3 gives curves showing the values of N_x obtained using Equation (1.4) for a range of 15 layer composite curved panels. The minimum values of N_x which are selected and output by the program as values of N_{xB} are represented by the full line portion of the curves. It can be seen that as the panel length is increased the panel buckles in an increasing number of longitudinal half-waves, m . For this particular panel configuration and circumferential load the buckling load at low values of a coincides with a single half wave in the circumferential direction, $n = 1$. However, for higher values of a it can be seen that the buckled form in the circumferential direction will later change to $n = 2$ with the panel eventually reverting to a single buckle in each direction, $m = 1, n = 1$.

The input data for the panels are as follows:

Length	a	=	Varies
Width	b	=	420 mm
Radius	R	=	1000 mm
Layer thickness	t_k	=	0.125 mm

* Data Item No. Struct 02.01.50 "Initial buckling of slightly curved plates under combined longitudinal and circumferential direct stress". (All edges simply-supported.)