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Lubricant film thickness between disc cams and followers

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The Mechanisms Committee, which first met in 1975, is continuing the work of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

LUBRICANT FILM THICKNESS BETWEEN DISC CAMS AND FOLLOWERS

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LUBRICANT FILM THICKNESS BETWEEN DISC CAMS AND FOLLOWERS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI. All angles are measured in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

		Units
A	viscosity parameter (see Equation (3.41))	–
a	pressure exponent of lubricant viscosity (see Equation (4.1))	m^2/N
B	follower arm crank length (see Sketch 4.3 for sign convention)	m
B	elasticity parameter (see Equation (3.42))	–
C_1	intermediate calculation parameter	–
$C_2, C_3, \dots, C_6$	intermediate calculation parameters	m^{-1}
D	distance between cam rotational centre and follower pivot (see Sketch 4.3 for sign convention)	m
E	follower offset (see Sketch 3.1(b) for sign convention)	m
E_c	Young's modulus for cam material	N/m^2
E_{eff}	effective modulus (see Equation (3.37))	N/m^2
E_f	Young's modulus for follower material	N/m^2
F_E	external force acting on follower (positive to increase contact force)	N
F_{xi}, F_{yi}	components of resultant of external forces acting on part i in x - and y -directions, respectively	N
H	follower lift corresponding to one motion segment	m
h	lubricant film thickness	m
$\bar{h}$	non-dimensional film thickness (see Sketch 3.3)	–
I_f	mass moment of inertia of follower system about follower pivot	kg m^2
k_{SC}	compression spring rate	N/m
k_{ST}	torsion spring rate	$\text{N m}/\text{rad}$
L	distance between follower pivot and centre of follower radius	m
m_f	mass of follower system	kg
P_C	contact force normal to cam profile	N

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$(P_C)_{mod}$	modified contact force for equivalent line contact (see Equation (3.38))	N/m
P_{ij}	force exerted by part i on part j	N
P_S	compression spring force	N
P_{SP}	compression spring preload	N
P_{xij}, P_{yij}	components of joint force, P_{ij} , in x - and y -directions, respectively	N
p	pressure of lubricant	N/m ²
p_0	maximum Hertzian pressure (see Equation (3.39))	N/m ²
R_b	reference circle radius of cam	m
R_c	crown (or barrel) radius of follower (see Sketch 4.4)	m
R_{eff}	effective radius of curvature (see Equation (3.35))	m
R_{equiv}	equivalent effective radius of curvature for line contact (see Equation (3.36))	m
R_f	radius of follower	m
R_{qc}	r.m.s. (root-mean-square) roughness of cam surface	m
R_{qf}	r.m.s. (root-mean-square) roughness of follower surface	m
R_{qt}	composite roughness (see Equation (3.45))	m
R_r	radius of roller	m
R'	radius ratio (see Equations (3.34a) to (3.34c))	—
R'_{calc}	intermediate calculation parameter (see Equation (3.33))	—
T_E	external moment acting on follower (positive to increase contact force)	N m
T_{SP}	torsion spring preload	N m
T_{ij}	moment exerted by part i on part j	N m
V_c	velocity of cam surface at contact point relative to contact point	m/s
$(V_c)_x, (V_c)_y$	components of velocity of cam surface at contact point (relative to contact point) in x - and y -directions, respectively	m/s
V_{ent}	lubricant entrainment velocity	m/s
$(V_{ent})_x, (V_{ent})_y$	components of entrainment velocity in x - and y -directions, respectively	m/s
V_f	velocity of follower surface at contact point relative to contact point	m/s

$(V_f)_x, (V_f)_y$	components of velocity of follower surface at contact point (relative to contact point) in x - and y -directions, respectively	m/s
V_s	sliding velocity at contact point	m/s
W_c	facewidth of cam	m
W_f	width of follower	m
w	normalised follower displacement	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
X_m, Y_m	axes fixed in machine frame	—
$\dot{x}_c, \dot{y}_c$	components of velocity of point on cam surface at contact point (relative to X_m, Y_m axes)	m/s
$\dot{x}_f, \dot{y}_f$	components of velocity of point on follower surface at contact point (relative to X_m, Y_m axes)	m/s
x_p, y_p	coordinates of contact point (relative to X_m, Y_m axes)	m
$\dot{x}_p, \dot{y}_p$	components of velocity of contact point (relative to X_m, Y_m axes)	m/s
y	translational displacement of follower	m
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
$\dddot{y}$	translational jerk of follower	m/s ²
y_b	height of centre of follower radius above X_m -axis at inner dwell	m
y_m	instantaneous height of centre of follower radius above X_m -axis	m
y_0	height of centre of follower radius above X_m -axis at beginning of segment	m
z_1	force factor (see Equation (3.49))	—
z_2	preload factor (see Equation (3.50))	—
α	pressure angle	degree
η	dynamic viscosity of lubricant	N s/m ²
η_0	dynamic viscosity of lubricant at atmospheric pressure and operating temperature (see Equation (4.1))	N s/m ²
λ	specific film thickness (see Equation (3.44))	—
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m

σ_c	Poisson's ratio for cam material	—
σ_f	Poisson's ratio for follower material	—
Φ	total angle of cam rotation corresponding to one motion segment	rad
ϕ	cam angle of rotation (from start of Segment 1)	degree
Ψ	follower swing corresponding to one motion segment	rad
ψ	rotational displacement of follower	rad
$\dot{\psi}$	rotational velocity of follower	rad/s
$\ddot{\psi}$	rotational acceleration of follower	rad/s ²
ψ_b	angle between X_m -axis and line joining follower pivot to centre of follower radius at inner dwell	rad
ψ_m	instantaneous angle between X_m -axis and line joining follower pivot to centre of follower radius	rad
ω_c	angular velocity of cam	rad/s

Subscripts

e	refers to end value of external load
i	refers to value at a particular calculation point
q	refers to value when separating force is a maximum
s	refers to start value of external load
max	refers to maximum value
min	refers to minimum value

2. INTRODUCTION

A cam/follower system will fail to perform its intended function adequately if the surfaces of the cam and/or the follower are worn excessively. This type of early service failure can often be avoided if the system is designed such that an adequate lubricant film is present in the contact region throughout the life of the system. Therefore the prediction of the thickness of such a lubricant film under a variety of operating conditions is a vital step in the systematic design of a cam mechanism (see ESDU 94017, Reference 28).

This Data Item presents a procedure for calculating values of lubricant film thickness in the contact region between a disc cam and its follower (either rolling or sliding contact). To determine film thickness it is necessary to find the radius of curvature of the cam profile, the lubricant entrainment velocity and the contact force at the interface between the cam and the follower for a series of angular positions of the cam. This analysis is time consuming and is best carried out using the computer program (see Section 7) supplied with this Data Item. The program determines values of minimum film thickness at intervals of one degree of cam rotation (assuming constant angular velocity of the cam) and finds the smallest value of film thickness during the cycle. The program can be used to determine suitable values of rate and preload for a return spring designed to ensure that the cam and follower remain in contact. Alternatively, if a spring of known rate and preload is specified the program checks that contact is maintained. A check to ensure that profile undercutting (see ESDU 92005, Reference 23) does not occur is also included.

The calculation method presented in this Data Item is based on the theory of elastohydrodynamic lubrication (see Taylor, Reference 38, or Zhu and Taylor, Reference 39) in which changes in the viscosity of the lubricant and elastic deformation of the components in contact are significant. The ESDU Tribology Series contains procedures for calculating film thicknesses in lubricated Hertzian contacts and these are given in ESDU 85027 and ESDU 89045 for two dimensional line contacts and three dimensional point contacts, respectively (Derivations 2 and 3). Computer programs that implement these procedures are also available (see ESDU 91037 and ESDU 91038, References 21 and 22, respectively). The algorithm used to calculate film thickness in the program (see Section 7) supplied with the current Data Item is exactly the same as that used in ESDU 91037 (for the reasons discussed in Appendix A).

2.1 Assumptions

The film thickness calculation procedure is applicable to both translating and swinging followers driven by disc cams with spring force closure. The follower may have a nominally flat[†] face or a domed[†] face or it may be a roller[†]. During the derivation of contact force it is assumed that:

- (a) the contact force at the cam/follower interface is due only to follower system inertia, the action of the return spring and the externally-applied load acting on the follower system (friction in the follower system and gravitational force may be modelled as part of the external load if required),
- (b) the cam and follower are designed and manufactured so that the follower can accurately contact the cam profile at all times, *i.e.* undercutting does not occur,
- (c) the axes of the cam and the follower are aligned such that the shape of the contact zone is not modified by edge effects,
- (d) the widths of the cam and the follower are chosen such that the shape of the contact zone is not modified by edge effects,

[†] The calculation procedure presented in this Data Item is strictly applicable only to "crowned" followers (see Sketch 4.4) in which some misalignment of the cam and follower axes is tolerated. It is regarded as good design practice to include such a feature even with a nominally flat-faced sliding follower.

- (e) the cam and follower remain in contact throughout the operating cycle (see Section 3.3)
- and (f) the follower system is sufficiently stiff to be considered rigid.

If the follower is a roller type it is also assumed that:

- (g) friction torque at the roller pivot is negligible
- and (h) slip between the cam and the roller is negligible (see Section 2.2.1).

A number of further assumptions are implicit in the method adopted for calculation of lubricant film thickness (see Section 3.2). These are that:

- (i) the materials of the cam and the follower are isotropic and linearly elastic,
- (j) the surface roughnesses of both components are not considered (see Section 2.2.3),
- (k) the dimensions of the contact area are very small in comparison with the radii of the cam profile and the follower in the vicinity of the contact,
- (l) the lubricant can be described as Newtonian, *i.e.* viscosity is independent of shear rate,
- and (m) isothermal conditions prevail, *i.e.* inlet shear heating does not occur (see ESDU 85027, Derivation 2).

Discussion of the implications of Assumptions (i) to (m) is presented in ESDU 85027 (Derivation 2) and ESDU 89045 (Derivation 3) and is not repeated here.

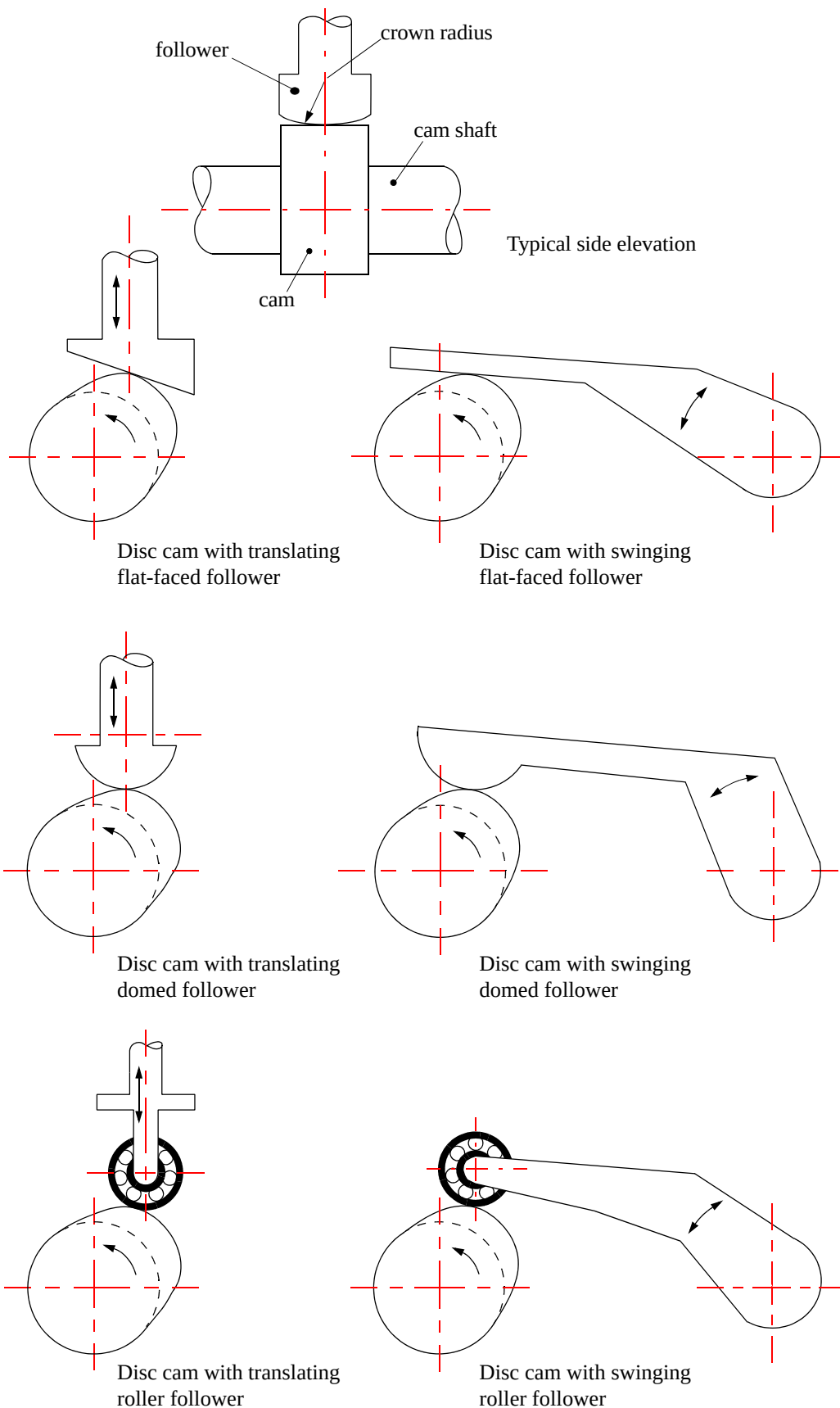
2.2 Scope

2.2.1 Follower type

The method presented in this Item is applicable to the design of disc cams with any one of the six main types of crowned follower illustrated in Sketch 2.1.

The results are much more important in terms of the overall design when there is sliding contact between the cam and the follower. This is because the successful operation of these types of mechanism depends on the separation of the contacting surfaces by a lubricant film or, in the absence of an adequate film, on careful choice of surface finish and surface treatments. However, for systems with these types of follower, when accurate prediction is more critical, the method has proved to be more reliable.

The contact between a disc cam and a roller follower is likely to involve a combination of rolling and sliding motion where the degree of sliding increases with the angular velocity of the cam shaft. The method for predicting film thickness between cams and roller followers in this Item assumes perfect rolling throughout the cycle (see ESDU 94017, Reference 28) and so the results for this case should be treated with caution (see Example 3 in Section 6.3).



Sketch 2.1 Disc cams with six commonly-used followers

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2.2.2 Follower motion

The method presented in this Item can be used to analyse cams with a wide range of profiles. In cam design the relationship between the motion of the cam and the motion of the follower is usually governed by a mathematical equation known as a cam law (for DRD motions see ESDU 82006, Reference 7). The theory (presented in Section 3 of this Item) is general, in that any mathematical cam law can be specified; however the computer program has embedded within its code the equations defining only six commonly-used cam laws (see Entry 12, Section 7.3.1).

If the required cam law for a particular motion segment is not included in the current version of ESDUpac A9408 then the data can be entered via a motion data file.

This motion file must contain values of follower displacement, follower velocity, follower acceleration and follower jerk at intervals of one degree of cam rotation. The required format of the file is described in Section 7.3.2 and some of the reasons for specifying motion in this way are explained in Section 4.1.1.

2.2.3 Materials and surface finishes

The method used to calculate film thickness has been validated for use with contacts between bodies made from many common engineering materials including steel, cast iron and ceramics. In the calculation of lubricant film thickness the surfaces roughnesses of the cam and follower are not considered. However to determine the lubrication regime, and so to evaluate the design, a value of specific film thickness (in which the thickness of the lubricant film is divided by a composite value of surface roughness – see Equation (3.44)) is required. Thus the effect on lubrication of various manufacturing methods and different standards of surface finish of both the cam and the follower can be assessed.

2.2.4 Lubricants

The method is valid for the full range of oils (mineral and synthetic) commonly used to lubricate machinery.

2.3 Use of This Data Item

2.3.1 General guidance

The theoretical background is presented in Section 3 of this Data Item but it is not necessary to have a deep understanding of this theory to carry out the required calculation using the accompanying computer program described in Section 7. A guide to the choice of input values for this program and a description of the program output, including design guidance, is presented in Section 4. The examples in Section 6 also provide valuable hints on the effective use of the program. Calculation of lubricant film thickness may be useful:

- (a) during a feasibility study,
 - (b) as part of the detail design, especially during the selection of a suitable combination of materials, surface treatments, surface roughnesses and lubricant type (see ESD 94017, Reference 28)
- or
- (c) to investigate the cause of deterioration of the surface of a cam or a follower (see Tallian, Reference 37).

It is likely that the nature and extent of the information available in these cases will vary considerably. For instance during a feasibility study the mass properties of the follower and the externally-applied loads may not be known accurately. The analytical procedure is applicable when uncertainty surrounds values of inputs but the accuracy of the results is entirely dependent on the quality of the input information.

2.3.2 Allowable values of lubricant film thickness

The series of values of specific film thickness, λ , is the most important result obtained from the procedure in this Item. Each value gives a measure of the degree of surface asperity contact at a different point on the cam and the follower surfaces throughout one rotation of the cam (see Equation (3.44)). The conditions in the contact zone at each point can be loosely classified in terms of a lubrication regime which is defined by a range of values of specific film thickness. These regimes are (in order of increasing asperity contact):

- (1) fluid film lubrication ($\lambda > 3$),
 - (a) hydrodynamic lubrication ($\lambda > 10$),
 - (b) elastohydrodynamic lubrication ($3 < \lambda < 10$),
- (2) mixed lubrication ($0.5 < \lambda < 3$)
- and (3) boundary lubrication ($\lambda < 0.5$).

A full explanation of the phenomena associated with each regime is given in ESDU 94017 (Reference 28). The numerical values chosen to define each regime should not be regarded as firm limits because of the statistical nature of the knowledge of the surface roughness. Each type of lubrication may be experienced by different points on the surfaces of a cam and a follower during a single rotation at constant cam shaft angular velocity. The possibility of wear by scuffing and polishing is greatly increased if boundary lubrication occurs and the use of extreme pressure additives with the lubricant and/or special surface treatments may be necessary (for a full explanation and further guidance see ESDU 94017, Reference 28). Accelerated wear can be expected at those locations on the surface of the cam and/or the follower at which the value of specific film thickness falls below about 0.5. It may be possible to reduce wear by making changes to the geometry of the cam or follower, to the lubricant specification, to the materials properties and/or to the surface roughness that result in an increase in the lowest value of specific film thickness. Further advice regarding the changes most likely to result in a reduction of wear is given in Section 4.3.2 and, more fully, in ESDU 94017 (Reference 28).

3. THEORETICAL BACKGROUND

This section presents details of the mathematical model on which the calculation procedure is based. It is strongly recommended that the calculations are carried out using the computer program described in Section 7. The equations used to evaluate contact force, cam profile radius of curvature and lubricant entrainment velocity in this program are derived in Section 3.1 for the case of a domed translating follower. For the other follower types covered by this Item the equations for these quantities are given in ESDU Technical Memoranda Nos. 101 and 102 (Derivations 4 and 5). Section 3.2 describes the algorithm for calculating lubricant film thickness and Section 3.3 presents the formulae used for the initial selection of a spring. Recommended values of the quantities needed as inputs to the computer program are given in Section 4.1.

The convention adopted for forces and moments in this Item is such that P_{x23} is the component of force in the x-direction exerted by Part 2 on Part 3. Thus

$$P_{x23} = -P_{x32}.$$

The magnitude of the force exerted by Part 2 on Part 3 is given by

$$P_{23} = \left(P_{x23}^2 + P_{y23}^2 \right)^{1/2}.$$

Similarly, T_{34} is the moment (positive in the anticlockwise sense) exerted by Part 3 on Part 4. Thus

$$T_{34} = -T_{43}.$$

3.1 Translating Domed Follower

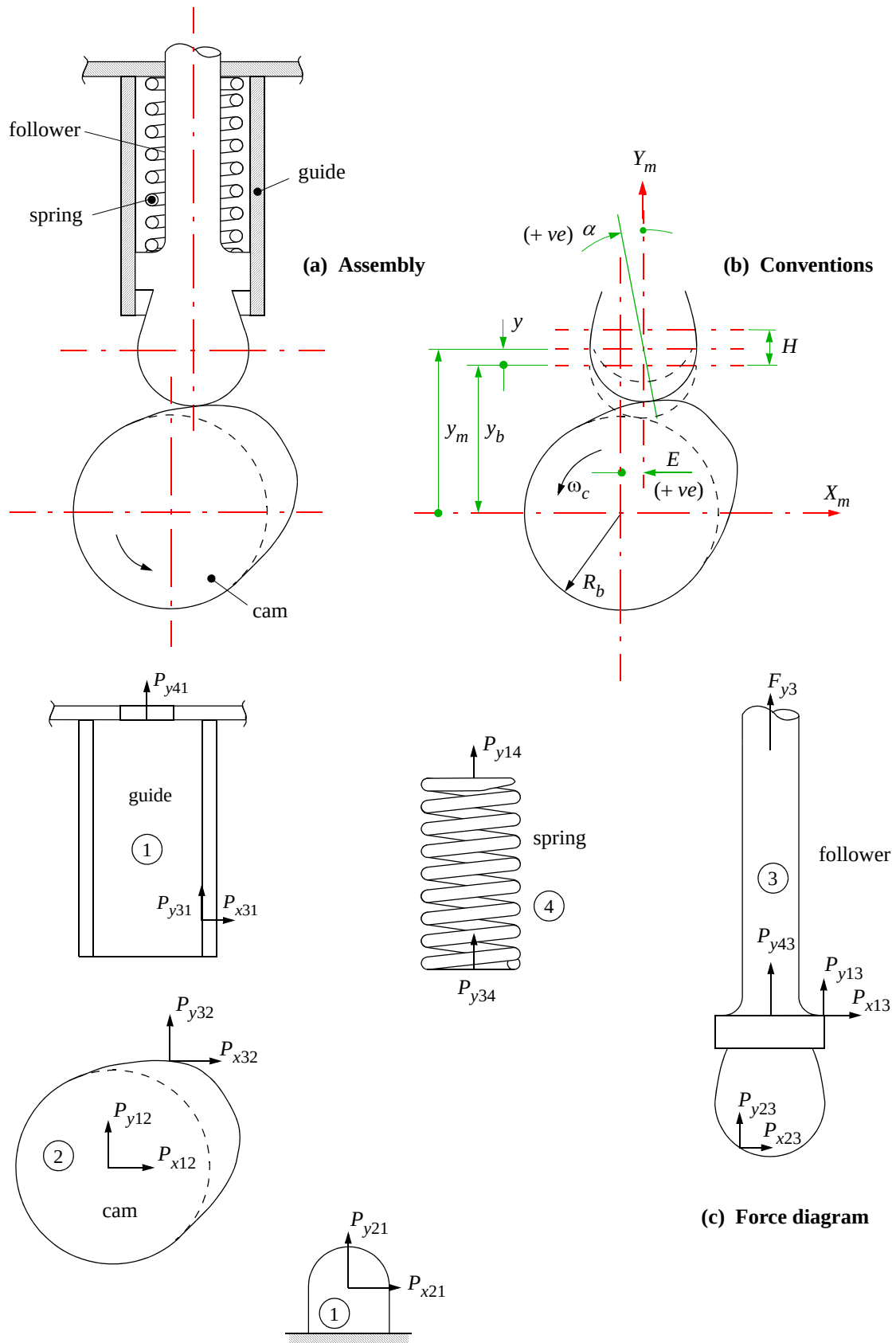
Sketch 3.1(a) shows a typical sub-system composed of a disc cam with an offset translating domed follower. A compression spring is used to provide force closure. Sketch 3.1(b) shows the conventions chosen for the positive signs of pressure angle, offset and angular rotation. Diagrams showing the forces acting on each part are given in Sketch 3.1(c).

3.1.1 Derivation of contact force

It is required to derive an expression for the contact force, P_C , between the follower and the cam due to the action of the spring, the inertia of the follower system and the external force acting on the follower. The follower is constrained to move only in the y-direction and it is assumed that the friction force, P_{y13} , between the follower and the guide is zero. However, in practice, friction can be included by considering it as part of the external force, F_{y3} . The sum of the components of force acting on the follower in the y-direction is equal to the product of the mass of the follower (see Section 4.1.1) and the acceleration of its centre of mass in the y-direction. Thus for equilibrium of the follower

$$P_{y23} + P_{y43} + F_{y3} = m_f \ddot{y}, \quad (3.1)$$

where P_{y23} is the component of force exerted by the cam on the follower in the y-direction,
 P_{y43} is the force exerted by the spring on the follower,
 F_{y3} is the external force exerted on the follower
 and m_f is the mass of the follower system.



Sketch 3.1 Cam and follower system

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The instantaneous motion (displacement, y , velocity, $\dot{y}$, and acceleration, $\ddot{y}$) of the centre of the follower radius during a rise or return segment is required. These quantities are given by

$$y = wH, \quad (3.2a)$$

$$y_m = y_0 + wH, \quad (3.2b)$$

$$\dot{y} = \frac{w' \omega_c H}{\Phi} \quad (3.3)$$

and

$$\ddot{y} = \frac{w'' \omega_c^2 H}{\Phi^2} \quad (3.4)$$

where y_0 is the height of the centre of the follower radius above the X_m -axis at the start of the motion segment (rise or return),

H is the total follower lift corresponding to one motion segment (in this Data Item it is taken to be a positive scalar quantity for both rise and return motion segments),

Φ is the total cam angle corresponding to one motion segment (note that the units are radians),

ω_c is the angular velocity of the cam

and w, w', w'' are the dimensionless follower displacement, velocity and acceleration, respectively (expressions for these parameters corresponding to commonly-used cam laws are given in ESDU 82006, Reference 7).

Note that:

(a) w and w' are positive during a rise motion and negative during a return motion

and (b) w'' changes sign during every motion segment (it is positive during the first part of a rise segment and negative during the first part of a return segment).

The pressure angle, α (see Derivation 1), is given by

$$\alpha = \tan^{-1} \left[\frac{\left(\frac{\dot{y}}{\omega_c} \right) - E}{y_m} \right]. \quad (3.5)$$

The spring force is composed of the preload, P_{SP} , with the product of the spring rate, k_{SC} , and the follower displacement from inner dwell ($y_m - y_b$), that is

$$P_S = -P_{y43} = P_{SP} + k_{SC} (y_m - y_b), \quad (3.6)$$

where

$$y_b = \left((R_b + R_f)^2 - E^2 \right)^{1/2}. \quad (3.7)$$

The external force, F_E , and the spring force, P_S , are considered as positive (by convention) when they act to increase the contact force on the cam. So

$$F_E = -F_{y3} \quad (3.8)$$

$$\text{and} \quad P_S = -P_{y43} \quad (3.9)$$

Thus the expression for contact force, as used in the program, is obtained by substituting Equations (3.8), (3.9) and (3.6) in Equation (3.1) to give

$$P_C = P_{23} = \frac{P_{y23}}{\cos \alpha} = \frac{[P_{SP} + k_{SC} (y_m - y_b)] + F_E + m_f \ddot{y}}{\cos \alpha} \quad (3.10)$$

Contact force is calculated in the computer program at intervals of one degree of cam angle during one full rotation of the cam.

3.1.2 Radius of cam profile curvature

The radius of curvature of the cam profile is required for the calculation of lubricant film thickness. The derivation of an expression for this quantity is given in ESDU ME2 (Derivation 1) where the radius of curvature of the pitch curve for a cam driving a translating domed follower is given by

$$\rho = \frac{y_m^2}{\frac{H}{\Phi} \cos^2 \alpha \left(w' \sin \alpha - \frac{w''}{\Phi} \cos \alpha \right) + y_m \cos \alpha} \quad (3.11)$$

and the radius of curvature of the cam profile is given by

$$\rho_p = \rho - R_f \quad (3.12)$$

This quantity is determined in the computer program at intervals of one degree of cam angle during one full rotation of the cam.

3.1.3 Derivation of lubricant entrainment velocity

The velocity with which the lubricant is entrained into the contact region has a very important effect on lubricant film thickness and must be calculated. The lubricant entrainment velocity is defined in Derivations 2 and 3 as

$$V_{ent} = 0.5 |V_c + V_f| \quad (3.13a)$$

and the sliding velocity (also calculated in the computer program) is given by

$$V_s = V_c - V_f \quad (3.13b)$$

where V_c is the velocity of the cam surface at the contact point relative to the contact point and V_f is the velocity of the follower surface at the contact point relative to the contact point. It is therefore necessary to derive expressions for V_c and V_f .

The diagram illustrates the geometry of a contact point P on a curved surface. Key parameters and labels include:

- contact normal**: A line perpendicular to the surface at the contact point.
- line of action**: A line passing through the contact point, making an angle α with the vertical.
- contact point, P** : The point of contact between the gripper and the surface.
- α** : The angle between the line of action and the vertical.
- α_c** : The contact angle, indicated by a curved arrow.
- Y_m** : The vertical distance from the gripper's center to the contact point.
- E** : The horizontal distance from the gripper's center to the contact point.
- R_f** : The radius of the gripper's finger.
- x_p** : The horizontal distance from the gripper's center to the contact point.
- y_p** : The vertical distance from the gripper's center to the contact point.
- y_m** : The vertical distance from the gripper's center to the contact point.
- X_m** : The horizontal distance from the gripper's center to the contact point.

By observation of Sketch 3.2 it can be seen that the instantaneous velocity of a particle on the cam surface at the contact point (relative to the fixed axes X_m, Y_m) in the x -direction is given by

and similarly in the y -direction by

The negative sign in Equation (3.14) arises because of the convention regarding the direction of rotation of the cam and the fact that y_p is always positive.

$$\text{and} \quad y_p = y_m - R_f \cos \alpha . \quad (3.17)$$

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The instantaneous velocity of a particle on the follower surface at the contact point (relative to the fixed axes X_m, Y_m) is known. The x -component is given by

$$\dot{x}_f = 0 \quad (\text{due to constraint provided by follower guide}) \quad (3.18)$$

and the y -component is given by

$$\dot{y}_f = \dot{y} . \quad (3.19)$$

Note that $\dot{y} = \dot{y}_m$ because $y_m = y + \text{constant}$.

The next step is to find the x - and y -components of velocity of the contact point relative to the fixed machine axes. To find these quantities it is necessary to differentiate the expressions given in Equations (3.16) and (3.17) with respect to time. Differentiating Equation (3.16) gives

$$\dot{x}_p = R_f \dot{\alpha} \cos \alpha . \quad (3.20)$$

An expression for the rate of change of pressure angle with respect to time must be found and by observation of Equation (3.5) it is convenient to use the substitution:

$$C_1 = \frac{(\dot{y}/\omega_c) - E}{y_m} . \quad (3.21)$$

Thus
$$\dot{\alpha} = \frac{d}{dt} \tan^{-1}[C_1]$$

or
$$\dot{\alpha} = \frac{1}{1 + [C_1]^2} \dot{C}_1 \quad (3.22)$$

and by the quotient rule of differentiation

$$\dot{C}_1 = \frac{\frac{y_m \ddot{y}}{\omega_c} - \dot{y} \left(\frac{\dot{y}}{\omega_c} - E \right)}{y_m^2} . \quad (3.23)$$

To obtain the velocity of the contact point in the x -direction relative to the fixed axes substitute Equation (3.23) in Equation (3.22) and then substitute Equations (3.21) and (3.22) in Equation (3.20). Thus

$$\dot{x}_p = R_f \left[\frac{\frac{y_m \ddot{y}}{\omega_c} - \dot{y} \left(\frac{\dot{y}}{\omega_c} - E \right)}{y_m^2 + \left(\frac{\dot{y}}{\omega_c} - E \right)^2} \right] \cos \alpha . \quad (3.24)$$

By a similar process the velocity of the contact point in the y -direction can be shown to be

$$\dot{y}_p = \dot{y} + R_f \left[\frac{\frac{y_m \ddot{y}}{\omega_c} - \dot{y} \left(\frac{\dot{y}}{\omega_c} - E \right)}{y_m^2 + \left(\frac{\dot{y}}{\omega_c} - E \right)^2} \right] \sin \alpha . \quad (3.25)$$

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The components of velocity of the cam surface at the contact point relative to the contact point can now be calculated. They are given by

$$(V_c)_x = \dot{x}_c - \dot{x}_p \quad (3.26)$$

and
$$(V_c)_y = \dot{y}_c - \dot{y}_p . \quad (3.27)$$

Similarly the components of velocity of the follower surface relative to the contact point are given by

$$(V_f)_x = \dot{x}_f - \dot{x}_p \quad (3.28)$$

and
$$(V_f)_y = \dot{y}_f - \dot{y}_p . \quad (3.29)$$

Thus, using Equation (3.13a), the components of the entrainment velocity are given by

$$(V_{ent})_x = 0.5 |(V_c)_x + (V_f)_x| \quad (3.30)$$

and
$$(V_{ent})_y = 0.5 |(V_c)_y + (V_f)_y| . \quad (3.31)$$

The magnitude of the entrainment velocity is given by

$$V_{ent} = \left((V_{ent})_x^2 + (V_{ent})_y^2 \right)^{1/2} . \quad (3.32)$$

Entrainment velocity and sliding velocity are calculated in the computer program at intervals of one degree of cam angle during one full rotation of the cam. The determination of film thickness is described in Section 3.2.

3.2 Evaluation of Lubricant Film Thickness

This section comprises a statement of the equations used to calculate values of lubricant film thickness in the computer program (see Section 7) and which can also be used to perform a hand calculation. The method adopted is a modified form of the line contact solution presented in ESDU 85027 (Derivation 2). The modification involves calculating the load per unit width of the contact based on the assumption that the effective width of the contact is 70 per cent of the length of the contact ellipse. The reasons for choosing this procedure are explained in Appendix A

The first step is to evaluate the parameter, R'_{calc} , from

$$R'_{calc} = \frac{\frac{1}{R_c}}{\frac{1}{R_f} + \frac{1}{\rho_p}} . \quad (3.33)$$

If the follower is a roller type, use R_r in place of R_f or, if the follower is flat-faced, set R_f to infinity.

The radius ratio is given by

$$R' \begin{cases} = R'_{calc} & \text{if } R'_{calc} > 1 \\ = 1 & \text{if } R'_{calc} = 1 \\ = \frac{1}{R'_{calc}} & \text{if } R'_{calc} < 1. \end{cases} \quad \begin{matrix} (3.34a) \\ (3.34b) \\ (3.34c) \end{matrix}$$

If the radius ratio is less than 10 at any point during the cycle a warning message is written to the program output because the algorithm is not strictly valid for contacts that approach points rather than lines (see Table 4.3).

The effective radius of curvature, R_{eff} , of the contact is given by

$$\frac{1}{R_{eff}} = \frac{1}{2} \left(\frac{1}{R_f} + \frac{1}{R_c} + \frac{1}{\rho_p} \right). \quad (3.35)$$

An equivalent effective radius of curvature, R_{equiv} , for line contact is given by

$$R_{equiv} = \frac{2}{C_2 + C_3}, \quad (3.36)$$

where $C_2 = \frac{1}{R_c} + \frac{1}{R_f} + \frac{1}{\rho_p},$

$$C_3 = |C_4 - C_5|,$$

$$C_4 = \frac{1}{R_c} - \frac{1}{R_f}$$

and $C_5 = \frac{1}{\rho_p}.$

The effective modulus, E_{ff} , for the contact is given by

$$\frac{1}{E_{eff}} = \frac{1}{2} \left(\frac{1 - \sigma_c^2}{E_c} + \frac{1 - \sigma_f^2}{E_f} \right). \quad (3.37)$$

The load per unit width, $(P_C)_{mod}$, of the contact for a high aspect ratio contact ellipse is given by

$$(P_C)_{mod} = \left(\frac{1.07 P_C p_0}{(R')^{2/3}} \right)^{1/2}, \quad (3.38)$$

where the maximum Hertzian pressure, p_0 , is given by

$$p_0 = C_6 P_C^{1/3} \left(\frac{E_{eff}}{R_{eff}} \right)^{2/3} \quad (3.39)$$

and

$$C_6 = 0.4967 \left(2 + R' + \frac{1}{R'} \right)^{-2/9} \quad (3.40)$$

Two non-dimensional parameters are required. The first, the viscosity parameter, **A**, is given by

$$\mathbf{A} = \left(\frac{a^2 ((P_C)_{mod})^3}{\eta_0 V_{ent} (R_{equiv})^2} \right)^{1/2} \quad (3.41)$$

and the second, the elasticity parameter, **B**, is given by

$$\mathbf{B} = \left(\frac{((P_C)_{mod})^2}{\eta_0 V_{ent} E_{eff} R_{equiv}} \right)^{1/2} \quad (3.42)$$

The non-dimensional film thickness, $\bar{h}$, is found from Sketch 3.3 using these parameters. In the computer program, a Fortran subroutine (from ESDUpac A9137) is used to find this parameter. The routine does not exactly reproduce the values of Sketch 3.3, so some small discrepancies between the results obtained by hand and those from the program should be expected.

The lubricant film thickness, h , is given by

$$h = \frac{\bar{h} \eta_0 V_{ent} R_{equiv}}{(P_C)_{mod}} \quad (3.43)$$

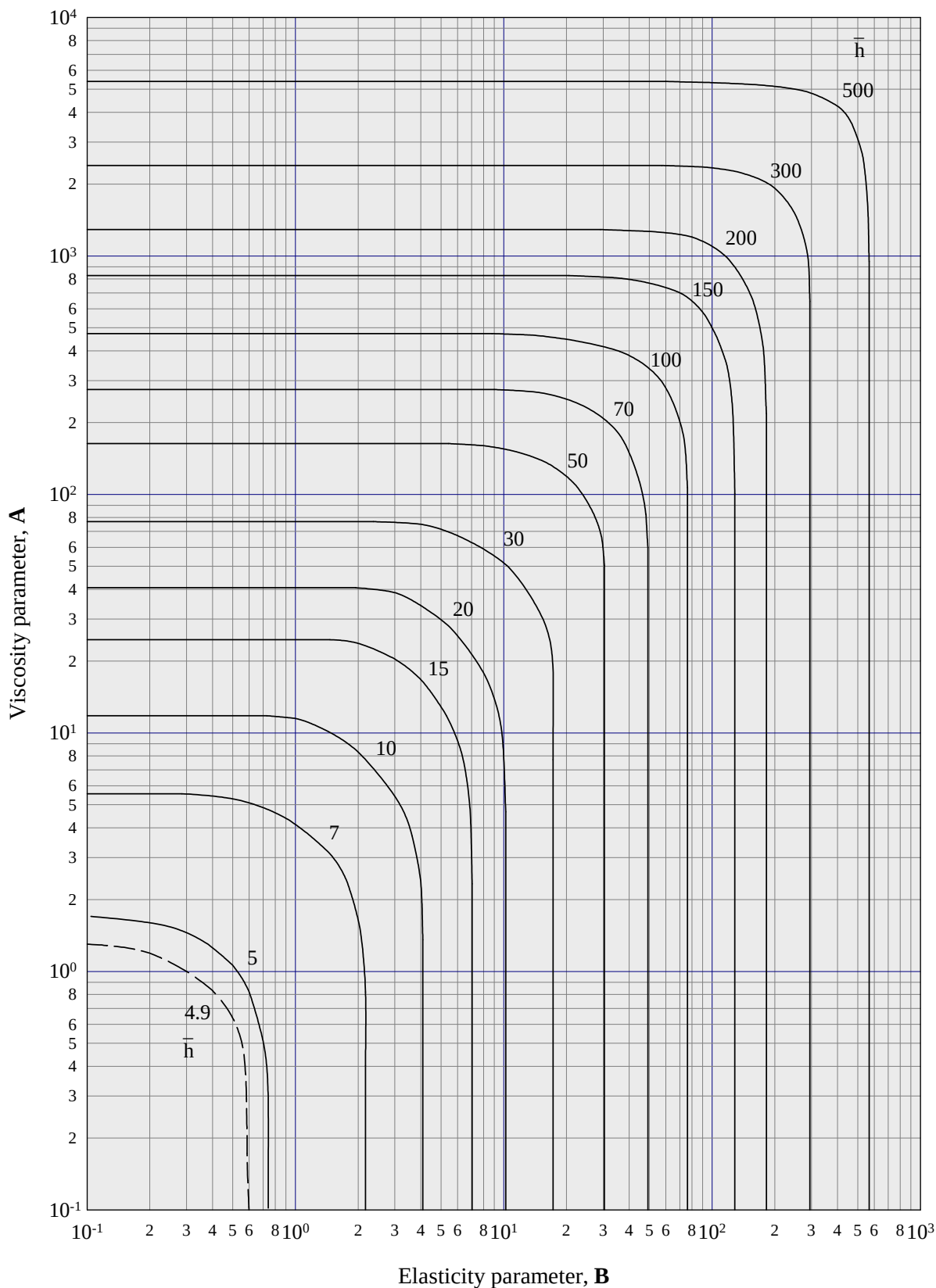
and the specific film thickness, λ , is given by

$$\lambda = \frac{h}{R_{qt}}, \quad (3.44)$$

where R_{qt} is the composite surface roughness given by

$$R_{qt} = \left(R_{qc}^2 + R_{qf}^2 \right)^{1/2} \quad (3.45)$$

The computer program implements an algorithm based on the above equations and the Subroutine from ESDUpac A9137 at intervals of one degree of cam angle and the minimum value of film thickness during one complete rotation of the cam is determined.



Sketch 3.3 Non-dimensional film thickness

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3.3 Determination of Suitable Spring Rate and Preload

The function of the spring is to maintain contact between the cam and follower throughout the motion. For the purposes of this Item it is sufficient to characterise a spring by values of spring rate and preload that ensure contact throughout one rotation of the cam under the worst combination of operating conditions. In many practical design situations these appropriate values of spring rate and preload are unknown. The program contains an option that allows the determination of values of spring rate and preload that will ensure positive contact force throughout the cycle. The way that these values are determined is described in this section.

To ensure contact for both translating followers and swinging followers, the condition

$$P_C > 0 \quad (3.46)$$

must hold at every calculated cam position throughout the cycle. For translating followers substituting Equation (3.10) into Expression (3.46) gives

$$\frac{[P_{SP} + k_{SC}(y_m - y_b)] + F_E + m_f \ddot{y}}{\cos \alpha} > 0 . \quad (3.47)$$

Since $\cos \alpha$ is always positive this condition becomes

$$[P_{SP} + k_{SC}(y_m - y_b)] + F_E + m_f \ddot{y} > 0 . \quad (3.48)$$

The spring force $[P_{SP} + k_{SC}(y_m - y_b)]$ must exceed the separating force $(-F_E - m_f \ddot{y})$. The program finds the cam angle, ϕ_q , at which the maximum separating force occurs and the corresponding follower motion at this position is defined by y_q , $\dot{y}_q$ and $\ddot{y}_q$ while the value of the external force is F_{Eq} . A user-specified factor of safety known as the force factor, z_1 , is then introduced and the inequality (Expression (3.48)) becomes

$$[P_{SP} + k_{SC}(y_{mq} - y_b)] + z_1(F_{Eq} + m_f \ddot{y}_q) = 0 . \quad (3.49)$$

The force factor is then the ratio of spring force to separating force defined at the cam angle ϕ_q , which is the cam angle corresponding to minimum resultant force. Since the spring force must be greater than the separating force for the follower to remain in contact with the cam, z_1 must be chosen to be greater than unity (see Section 4.1.2). Often the worst case of this minimum occurs when the cam is running under conditions of zero external load and the follower reaches a value of maximum acceleration (see the example shown in Sketch 3.4)

In addition, a user-specified preload factor, z_2 , is introduced. This defines the preload as a proportion of the maximum spring force and is given by

$$z_2 = \frac{P_{SP}}{k_{SC}[(y_m)_{max} - y_b]} . \quad (3.50)$$