

SYMMETRIC STEADY MANOEUVRE LOADS ON RIGID AIRCRAFT OF CLASSICAL CONFIGURATION AT SUBSONIC SPEEDS

1. INTRODUCTION

The structural strength design procedures for an aircraft in manoeuvring flight involve the following prerequisite information:

- (i) specification of the types of manoeuvres appropriate to the intended use of the aircraft,
- (ii) the mass distribution of all the parts of the aircraft
- and (iii) the aerodynamic load distribution for the specified manoeuvres.

The limiting manoeuvres required to be performed by a particular aircraft are laid down by the airworthiness authorities, but the estimation of the associated load distributions is the concern of the manufacturer. The engineer responsible for load evaluation will require specialist information from the aerodynamicist, concerning lift distributions on the various elements of the aircraft, *e.g.* wings and fuselage, which will need to be combined with the mass data for the structure supplied by the structural engineer.

The aim of this Data Item is to describe the design process involved in (ii) and (iii) above for the case of a rigid aircraft of classical configuration executing steady manoeuvres restricted to the plane of symmetry of the aircraft. An example of an aircraft executing a steady normal acceleration manoeuvre illustrates the use of the Item.

An aircraft of classical configuration is assumed to have a wing of medium to large aspect ratio and low to moderate sweep angles. The tailplane is assumed to have an area that is small compared with the primary lifting surface of the wing and to lie behind the centre of mass of the aircraft with a moment arm that is large compared with the distance between the centre of mass and the aerodynamic centre of the aircraft. The centre of mass of the aircraft usually lies in the region of the wing-fuselage junction.

1.1 Mass and Aerodynamic Load Data

In order to undertake a calculation of structural strength, namely the evaluation of the distributions of shear force, bending moments and torques along the wing span and along the fuselage, the mass, lift and moment data for the various elements of the aircraft need to be evaluated.

The overall mass distribution of the aircraft must be known, including the fuel and payload for the flight condition to be investigated, from which the overall centre of mass can be determined. In particular the wing spanwise and chordwise mass distributions and the longitudinal mass distribution along the fuselage are required. The fin and tailplane assembly is often regarded as a concentrated mass with its own mass centre. Each engine nacelle may be treated as a single concentrated mass with a given centre of mass.

For project feasibility calculations, particularly in the early stages of an aircraft design, empirical estimates may be made concerning the breakdown of mass distribution, often based on experience. Table 1.1 shows a fairly typical mass breakdown for a conventional large transport aircraft which may be adapted for particular aircraft of similar configuration. The fuel and payload mass vary approximately in the range 20-30% and 10-20%, respectively, and are included in Table 1.1 as part of the wing and fuselage. The particular flight sortie allows determination of the particular fuel and payload requirements for the given flight condition. A reasonable assumption for mass distribution is that the wing mass varies spanwise as the square of the chord and that the fuselage mass distribution is essentially constant along the majority of its length. Such estimates are adapted to accommodate fuel and payload effects.

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TABLE 1.1 Typical mass breakdown for a large transport aircraft

Aircraft element	Element mass as % of gross mass
Wing { structure	14
{ fuel	21
Fuselage { structure	18
{ payload	12
Undercarriage	4
Tail + fin	3
Powerplant	12
Equipment + systems	16

For the purposes of aerodynamic load calculations the aircraft configuration is conceptually separated into two parts, that part of the aircraft excluding the tail unit, known as the wing-fuselage combination, and the tail unit itself. For mass considerations the tail unit includes the fin but for aerodynamic purposes during symmetric manoeuvres only the tailplane need be considered.

The aerodynamic load data required for the wing-fuselage combination consist of the following information:

- (i) the wing lift and pitching moment distributions across the wing span covering the effects of wing camber, design wing twist, incidence, pitch rate and any appropriately deployed lift or drag devices,
 - (ii) the carry-over lift from the wing on the fuselage in the region of the wing-fuselage junction,
 - (iii) the lift distribution along the fuselage length, particularly in the fuselage nose region
- and (iv) the lift on the engine nacelles.

The distribution of aerodynamic load over the tailplane is ignored in this Item and the tailplane contribution to lift and moment is represented by a single force acting at the aerodynamic centre of the tailplane.

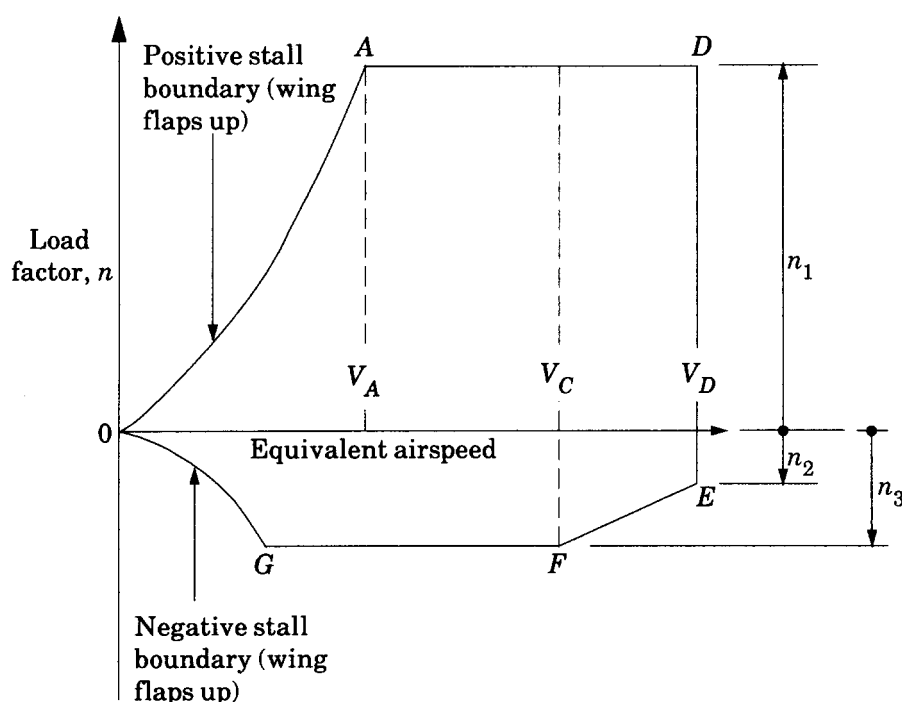
The drag distribution on the total aircraft will also be required if thrust effects are to be included in the loading calculations.

1.2 Manoeuvre and Flight Envelopes

An aircraft is subject to a wide variety of normal accelerations, ng , and flight speeds, V , and flies over a range of altitudes. The manoeuvre envelope or $V - n$ diagram specifies the limiting envelope of velocity, V , and normal load factor, n , within which a given aircraft must fly.

Sketch 1.1 shows a typical manoeuvre envelope as specified, for example, by the Civil Aviation Authority and the Ministry of Defence. The sketch gives the maximum positive and negative values of n and their relation to various design speeds which include the maximum diving speed, V_D , the cruising speed, V_C , and the design manoeuvre speed, V_A . The corners on the boundary ADEFG dictate the most severe loading conditions which need to be investigated for the calculation of maximum stresses. The values of the load factors n_1 , n_2 and n_3 are specified for the type and role of the aircraft.

For an aircraft in the large aircraft category the load factors specified in the Joint Airworthiness Requirements (JAR-25) are $n_1 = 2.5$, $n_2 = 0$ and $n_3 = -1$.



Sketch 1.1 Manoeuvre envelope

For a light aircraft the load factors are specified by Section K of the British Civil Airworthiness Requirements or JAR-23 and give load factors for the three aeroplane types, Normal, Utility and Aerobatic. For the Normal aircraft type, n_1 is specified by either the expression

$$n_1 = 2.1 + [24000(\text{lbf}) / \{W(\text{lbf}) + 10000(\text{lbf})\}],$$

or 3.8 whichever is the lesser, $n_2 = 0$ and $n_3 = -0.4 \times n_1$. For the Utility type aircraft $n_1 = 4.4$, $n_2 = 0$ and $n_3 = -1.8$. The Aerobatic type aircraft has load factor values of $n_1 = 6$, $n_2 = -1$ and $n_3 = -3$.

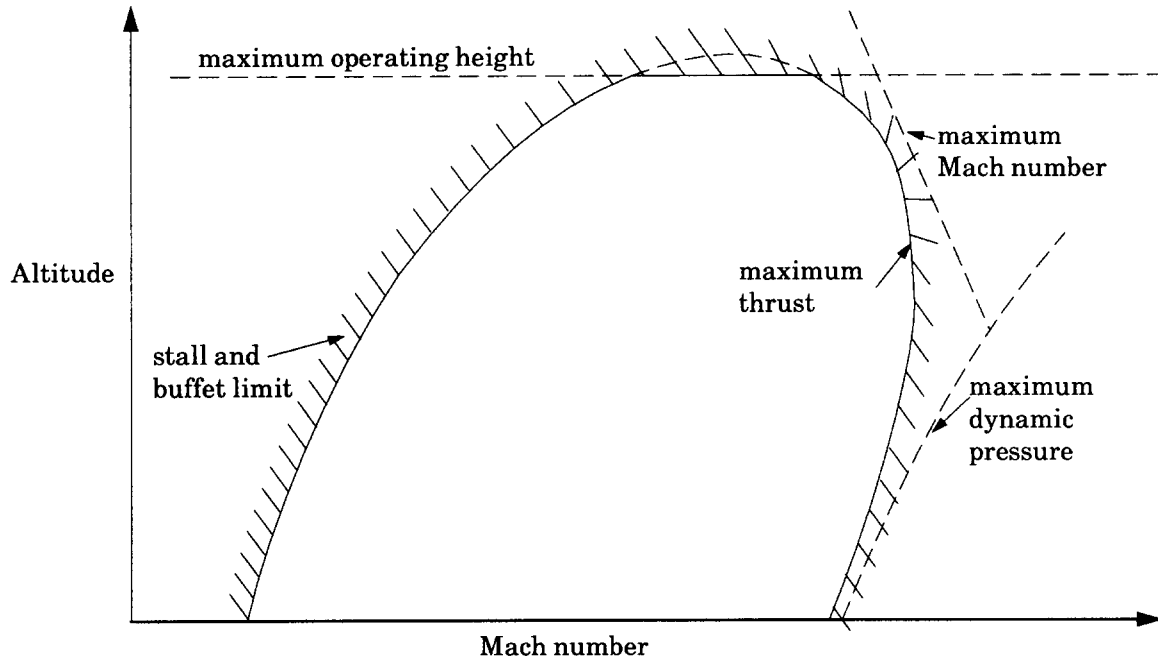
The flight manual for a particular aircraft will specify the actual design capabilities in terms of manoeuvre envelopes, where the points A, D, E, F and G on Sketch 1.1 will vary with both altitude and aircraft weight. The magnitude of aircraft loads depends on the level of normal acceleration and the equivalent airspeed. Equivalent airspeed is related to true airspeed (see Data Item 78017*) by the relation $V\sqrt{\sigma}$, where σ is the ratio of density of air to density of air at sea level in the International Standard Atmosphere.

An aircraft's flight capability is expressed in terms of a region on an airspeed-altitude plot known as a flight envelope, within which the operation of an aircraft is restricted as a result of possible design constraints, performance constraints and operational constraints. A typical example of a flight envelope for a subsonic aircraft is illustrated in Sketch 1.2.

Within Sketch 1.2 the left-hand boundary indicates the limiting speed of the aircraft for straight and level flight, determined by the stall characteristics of the wing. At high subsonic speeds buffet can further limit this boundary to flight speeds higher than the stalling speeds. The limiting boundary at the higher speeds, apart from a possible maximum operating height requirement and a maximum permissible speed or Mach

* True airspeed in the International Standard Atmosphere and equivalent airspeed corresponding to Mach number and pressure altitude. Data Item No. 78017, ESDU International, London 1978.

number, is essentially determined by the thrust available. A limit on the maximum dynamic pressure that can be tolerated, arising from structural considerations such as flutter or torsional divergence, may also provide a constraint for flight operation.



Sketch 1.2 Flight envelope

2. NOTATION

Any coherent system of units may be used.

A	acceleration
ac	aerodynamic centre of wing-fuselage combination
b	wing span
C_L	lift coefficient, $L/(\frac{1}{2}\rho V^2 S)$
C_{L0}	lift coefficient at zero incidence
C_{Lq}	rate of change of C_L with q , dC_L/dq
$C_{L\alpha}$	lift-curve slope, $dC_L/d\alpha$
C_M	moment coefficient, $M/(\frac{1}{2}\rho V^2 S \bar{c})$
C_{M0}	moment about ac when $q = 0$
C_Q	dynamic pressure coefficient, $W/(\frac{1}{2}\rho V^2 S)$

c	local wing chord
$\bar{c}$	aerodynamic mean chord
cm	centre of mass of total aircraft
D	drag
d	fuselage diameter
g	acceleration due to gravity
h	distances in z-direction measured as positive from cm position
i	general integer
K	number of engine nacelles
k	factor used for wing carry-over lift estimation
L	lift
L_0	lift at zero incidence
L_q	rate of change of L with q , dL/dq
L_α	lift-curve slope, $dL/d\alpha$
l	distances in x-direction measured as positive from cm position
ℓ	local lift per unit span
ℓ_q	rate of change of local lift per unit span with q , $d\ell/dq$
ℓ_0	local lift per unit span at zero incidence
ℓ_α	local lift curve slope per unit span, $d\ell/d\alpha$
M	Mach number
M	pitching moment about ac
M_q	rate of change of pitching moment with q , dM/dq
M_0	pitching moment about ac when $q = 0$
m	local pitching moment per unit span
m_q	rate of change of local pitching moment per unit span with q , dm/dq
m_0	local pitching moment coefficient about local aerodynamic centre when $q = 0$
m	mass of body (Section 3.2 only)

n	load factor, L/W
p	atmospheric pressure
Q	torque
q	steady pitch rate
R	radius of circle
S, S_T	wing area and tailplane area, respectively
T	engine thrust
t	time
V	true airspeed of aircraft
V_A, V_C, V_D	equivalent airspeeds associated with manoeuvre envelope
W	weight
$w(x)_{Fus}$	weight per unit section of fuselage at fuselage station x
$w(y)_W$	weight per unit span of wing at span position y
x, y, z	axis directions
y	general span position measured from fuselage centre line
y_r	distance in y direction of wing root from fuselage centre line
α	incidence
$\alpha(y)$	local incidence of wing at span position y
γ	ratio of specific heat capacities of air
γ	flight path angle
$\dot{\gamma}$	rate of change of γ with time, $d\gamma / dt (\equiv q)$
Δ, δ	increments
θ	pitch angle
ρ	density of air
σ	ratio of density of air to density of air at sea level
ϕ	bank angle
Ω	steady turn rate

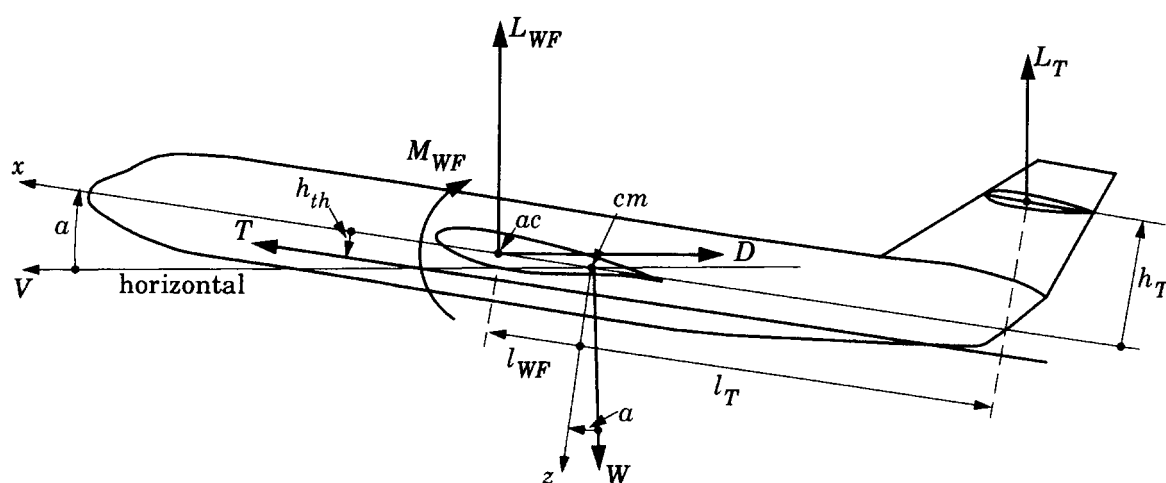
Subscripts

F	fuselage nose
Fus	complete fuselage
i	general integer
le	refers to leading edge apex of net wing measured along the fuselage centre line
m	relates to mass quantity
Nac	nacelle
$nose$	front of fuselage
$rear$	rear of fuselage
T	tailplane and fin assembly
th	thrust
W	wing
WF	wing-fuselage combination
y	span position

3. BACKGROUND DESCRIPTION

3.1 Steady Level Flight

Before introducing symmetric manoeuvres in general terms, the steady level flight condition will be introduced to help define the terms and notation to be used. Consider Sketch 3.1 which shows a schematic of an aircraft in level flight complete with the overall forces and moments illustrated.



Sketch 3.1 Aircraft in steady level flight

In this sketch the longitudinal fuselage axis, x , is at a steady aircraft incidence angle, α , to the horizontal with a wing inclined to this x -axis at an angle known as the wing setting angle. The thrust, T , is assumed to be in a direction parallel to the x axis and to be below it at a distance h_{th} from the centre of mass, cm , in the z -direction.

The lift of the wing-fuselage combination, L_{WF} , and the total drag, D , are shown acting through the aerodynamic centre, ac , of the wing-fuselage, which is the point on the aircraft at which the pitching moment of the wing-fuselage configuration is independent of incidence. The aerodynamic pitching moment about the ac of the wing-fuselage combination, M_{WF} , is shown as a nose-up positive moment. The distance l_{WF} defines the aerodynamic centre position of the wing-fuselage combination ahead of the centre of mass of the aircraft.

As noted in Section 1.1 the effects of the distribution of load over the tailplane are ignored and the tailplane contributions to the pitching moment are those that arise solely from a single force, L_T , acting at the aerodynamic centre of the tailplane, a distance l_T aft of the centre of mass position.

In equilibrium, the horizontal and vertical force equations and the moment equation about the centre of mass of the aircraft are, (in dimensional form)

$$T \cos \alpha - D = 0 \quad (3.1a)$$

$$L_{WF} + L_T + T \sin \alpha - W = 0 \quad (3.1b)$$

$$\text{and} \quad M_{WF} + T h_{th} + L_{WF} l_{WF} \cos \alpha + D l_{WF} \sin \alpha - L_T [l_T \cos \alpha + h_T \sin \alpha] = 0, \quad (3.1c)$$

where all distances are measured in the x - or z -directions from the centre of mass, cm , of the aircraft.

Taking α to be small, *i.e.* $\cos \alpha \approx 1$ and $\sin \alpha \approx \alpha$, Equations (3.1a) to (3.1c) reduce to

$$T - D = 0, \quad (3.2a)$$

$$L_{WF} + L_T - W = 0 \quad (3.2b)$$

$$\text{and} \quad M_{WF} + T h_{th} + L_{WF} l_{WF} - L_T l_T = 0, \quad (3.2c)$$

in which the terms $T\alpha$ and $D l_{WF} \alpha$ are small compared with the other terms in Equations (3.2b) and (3.2c), respectively, and $h_T \alpha$ has been neglected in comparison with l_T in the tailplane lift term of Equation (3.2c).

Solving for L_{WF} and L_T in Equations (3.2b) and (3.2c)

$$L_{WF} = W \left(\frac{l_T}{l_{WF} + l_T} \right) - (M_{WF} + T h_{th}) \left(\frac{1}{l_{WF} + l_T} \right) \quad (3.3a)$$

$$\text{and} \quad L_T = W \left(\frac{l_{WF}}{l_{WF} + l_T} \right) + (M_{WF} + T h_{th}) \left(\frac{1}{l_{WF} + l_T} \right). \quad (3.3b)$$

For simplicity the thrust term is often ignored in Equations (3.3a) and (3.3b) and the quantities L_{WF} and

L_T may be evaluated knowing only W , M_{WF} , l_T and l_{WF} , i.e. from

$$L_{WF} = W \left(\frac{l_T}{l_{WF} + l_T} \right) - M_{WF} \left(\frac{1}{l_{WF} + l_T} \right) \quad (3.4a)$$

and
$$L_T = W \left(\frac{l_T}{l_{WF} + l_T} \right) + M_{WF} \left(\frac{1}{l_{WF} + l_T} \right). \quad (3.4b)$$

Writing M_{WF} in non-dimensional form, i.e. as

$$M_{WF} = \frac{1}{2} \rho V^2 S \bar{c} (C_M)_{WF}, \quad (3.5a)$$

$$M_{WF} = \frac{W \bar{c}}{C_Q} (C_M)_{WF}, \quad (3.5b)$$

where a non-dimensional quantity, $C_Q = W/(\frac{1}{2}\rho V^2 S)$, has been introduced and is a coefficient that is inversely proportional to dynamic pressure. Equations (3.4a) and (3.4b) then become

$$L_{WF} = W \left\{ \frac{l_T}{l_{WF} + l_T} - \frac{(C_M)_{WF}}{C_Q} \left(\frac{\bar{c}}{l_{WF} + l_T} \right) \right\} \quad (3.6a)$$

and
$$L_T = W \left\{ \frac{l_{WF}}{l_{WF} + l_T} + \frac{(C_M)_{WF}}{C_Q} \left(\frac{\bar{c}}{l_{WF} + l_T} \right) \right\}. \quad (3.6b)$$

In steady level flight $(C_M)_{WF} = (C_{M0})_{WF}$, where $(C_{M0})_{WF}$ is a function of Mach number only. These lift terms L_{WF} and L_T are, within the accuracy of the approximations used, independent of incidence and altitude but depend on dynamic pressure through C_Q and on Mach number through $(C_M)_{WF}$ and C_Q .

Note that C_Q and Mach number are related by

$$C_Q = \frac{W/S}{\frac{1}{2}\rho V^2} = \frac{W/S}{\frac{1}{2}\gamma p M^2}, \quad (3.7)$$

where γ , p and M are the ratio of specific heat capacities of air, the atmospheric pressure and the Mach number, respectively.

Typical values for large transport aircraft for the various quantities in Equations (3.6a) and (3.6b) are

$$\begin{aligned} l_{WF} &\approx 0.3\bar{c}, \\ l_T &\approx 3.0\bar{c}, \\ (C_{M0})_{WF} &\approx -0.05 \rightarrow -0.15 \end{aligned}$$

and
$$C_Q \approx 0.4 \rightarrow 1.0 \text{ (flaps up).}$$

With these values used within Equations (3.6a) and (3.6b), L_{WF} and L_T are approximately 90% and 10%, respectively, of all up weight, W . The distances l_{WF} and l_T are independent of the dynamic pressure, $\frac{1}{2}\rho V^2$, and Mach number, at subsonic speeds and for conventional transport aircraft $(C_{M0})_{WF}$ is negative.