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Contact stress in disc cams with domed or flat faced followers

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THE PREPARATION OF THIS DATA ITEM

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The Mechanisms Committee, which first met in 1975, is continuing the work of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

CONTACT STRESS IN DISC CAMS WITH DOMED OR FLAT FACED FOLLOWERS

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CONTACT STRESS IN DISC CAMS WITH DOMED OR FLAT FACED FOLLOWERS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI. Units for the program input parameters are given in Sections 7.3.1 to 7.3.3.

		Units
A	body geometry parameter (see Equations (3.13) and (3.14))	m^{-1}
a	major semi-axis of contact ellipse (see Equation (3.23))	m
B	follower arm crank length (see Sketch 4.3 for sign convention)	m
B	body geometry parameter (see Equations (3.13) and (3.14))	m^{-1}
b	minor semi-axis of contact ellipse (see Equation (3.24)) or half width of rectangular contact (see Equation (3.27))	m
D	distance between cam rotational centre and follower pivot (see Sketch 4.3 for sign convention)	m
E	follower offset (see Sketch 3.1 for sign convention)	m
E_c	Young's modulus for cam material	N/m^2
E_f	Young's modulus for follower material	N/m^2
F_E	external force acting on follower (positive to increase contact force)	N
F_{xi}, F_{yi}	component of resultant of external forces acting on part <i>i</i> in <i>x</i> - and <i>y</i> -directions, respectively	N
$(f_z)_{max}$	maximum compressive contact stress	N/m^2
H	follower lift corresponding to one motion segment	m
H_v	Vickers hardness	N/m^2
I_f	mass moment of inertia of follower system about follower pivot	kg m^2
k_c	material constant for cam (see Equation (3.15))	m^2/N
k_f	material constant for follower (see Equation (3.16))	m^2/N
k_{SC}	compression spring rate	N/m
k_{ST}	torsion spring rate	N m/rad

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L	distance between follower pivot and centre of follower radius	m
m_f	mass of follower system	kg
P_C	contact force normal to cam profile	N
P_{ij}	force exerted by part i on part j	N
P_S	compression spring force	N
P_{SP}	compression spring preload	N
P_{xij}, P_{yij}	components of joint force P_{ij} in x - and y -directions, respectively	N
R_b	reference circle radius of cam	m
R_c	crown (or barrel) radius of follower (see Sketch 4.4)	m
R_f	radius of follower	m
T_E	external moment acting on follower (positive to increase contact force)	N m
T_{SP}	torsion spring preload	N m
T_{ij}	moment exerted by part i on part j	N m
W_c	width of cam	m
W_f	width of follower	m
w	normalised follower displacement	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
y	translational displacement of follower	m
$\dot{y}$	translational velocity of follower	m/s
$\ddot{y}$	translational acceleration of follower	m/s ²
y_b	height of centre of follower radius above X_m -axis at inner dwell	m
y_m	instantaneous height of centre of follower radius above X_m -axis	m
y_0	height of centre of follower radius above X_m -axis at beginning of segment	m
Z_1, Z_2, Z_3	intermediate calculation parameters	—
z_1	force factor (see Equation (3.31))	—

z_2	preload factor (see Equation (3.32))	—
α	pressure angle	degree
β	semi-axis ratio of contact ellipse	—
δ	normal approach between cam and follower due to elastic deformation	m
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile	m
σ_c	Poisson's ratio for cam material	—
σ_f	Poisson's ratio for follower material	—
Φ	total angle of cam rotation corresponding to one motion segment	rad
ϕ	cam angle of rotation (from start of Segment 1)	degree
Ψ	follower swing corresponding to one motion segment	rad
ψ	rotational displacement of follower	rad
$\dot{\psi}$	rotational velocity of follower	rad/s
$\ddot{\psi}$	rotational acceleration of follower	rad/s ²
ψ_b	angle between X_m -axis and line joining follower pivot to centre of follower radius at inner dwell	rad
ψ_m	instantaneous angle between X_m -axis and line joining follower pivot to centre of follower radius	rad
ω_c	angular velocity of cam	rad/s

Subscripts

e	refers to end value of external load
i	refers to value at a particular calculation point
max	refers to maximum value
min	refers to minimum value
q	refers to value when separating force is a maximum
s	refers to start value of external load

2. INTRODUCTION

Cam/follower systems are liable to fail if the surface of the cam (or the follower) deteriorates under the influence of excessive load. These service failures can often be avoided if the system is designed such that stress in the contact region does not exceed an allowable value associated with the materials of the cam and the follower and the required life of the system. Therefore the prediction of contact stress under a variety of operating conditions is a vital step in the systematic design of a cam mechanism (see Reference 23).

This Data Item presents a procedure for calculating values of compressive stress in the contact region between a disc cam and a domed or flat-faced follower at regular intervals during one complete rotation of the cam. To determine these stress values it is necessary to find, for a series of angular positions of the cam, the radius of curvature of the cam profile and the contact force at the interface between the cam and the follower. This analysis is time consuming and is best carried out using the computer program (see Section 7) supplied with this Data Item. The program determines values of maximum compressive contact stress at intervals of one degree of cam rotation (assuming constant angular velocity of the cam) and finds the peak value of stress during the cycle. The dimensions of the contact zone and the normal approach between the cam and the follower due to elastic deformation in the contact zone are also calculated. The program can be used to determine suitable values of rate and preload for a return spring designed to ensure that the cam and follower remain in contact. Alternatively, if a spring of known rate and preload is specified the program checks that contact is maintained. A check to ensure that profile undercutting (see Reference 4) does not occur is also included.

Correct implementation of the calculation procedure suggested in this Item will result in a decrease in design time together with improvements in machine reliability.

2.1 Assumptions

The procedure is applicable to both translating and swinging followers driven by disc cams with spring force closure. The follower may have a nominally flat[†] face or a domed[†] face. During the derivation of contact force it is assumed that:

- (a) contact force at the cam/follower interface is due only to follower system inertia, the action of the return spring and the externally-applied load acting on the follower system (friction in the follower system and gravitational force may be modelled as part of the external load if required),
 - (b) the cam and follower are designed and manufactured so that the follower can accurately contact the cam profile at all times, *i.e.* undercutting does not occur,
 - (c) the axes of the cam and follower are aligned such that the shape of the contact zone is not modified by edge effects,
 - (d) the widths of the cam and follower are chosen such that the shape of the contact zone is not modified by edge effects,
 - (e) the cam and follower remain in contact throughout the operating cycle (see Section 3.3)
- and (f) the follower system is sufficiently stiff to be considered rigid.

[†] The calculation procedure presented in this Data Item is strictly applicable only to "crowned" followers (see Sketch 4.4) in which some misalignment of the cam and follower axes is tolerated. It is regarded as good design practice to include such a feature even with a nominally flat-faced follower.

A number of further assumptions are implicit in the method adopted for calculation of contact stress (see Section 3.2). These are:

- (g) the materials of the cam and the follower are isotropic and linearly elastic,
 - (h) the dimensions of the contact area are very small in comparison with the radii of the cam profile and the follower in the vicinity of the contact,
 - (i) the shape of the deformed zone adjacent to the contact zone can be described with sufficient accuracy by a second order equation, *i.e.* surfaces are smooth,
 - (j) the contact force acts normal to the tangent to the cam surface at the contact point
- and (k) surface shear stresses resulting from traction are ignored (see Section 2.2).

Discussion of the implications of Assumptions (g) to (k) is presented in full in ESDU 78035 (Derivation 1) and is not repeated here. Particular comments regarding the suitability of the algorithm for calculation of contact stress at the cam/follower interface are given in Section 2.2.1.

2.2 Scope

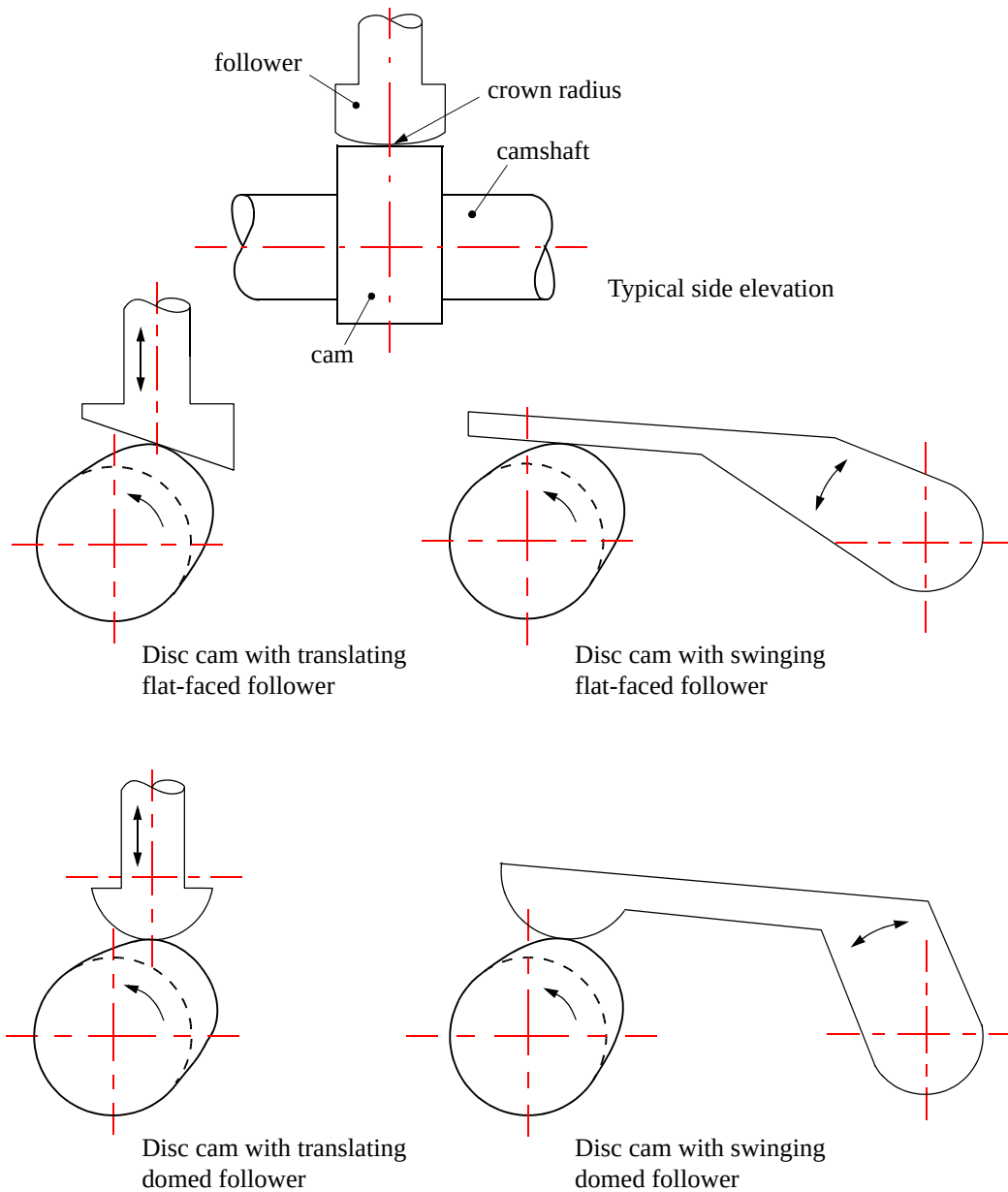
2.2.1 Follower type

The method presented in this Item is applicable to the design of disc cams with any one of the four types of crowned follower illustrated in Sketch 2.1. The algorithm used to calculate the compressive contact stress at the surface (taken from ESDU 78035, Derivation 1) is strictly applicable only to static contacts where no traction forces act. In the contact zone between a cam and a domed or a flat-faced follower traction forces (due to friction) are present. However, if the cam and follower are separated by a film of lubricant, the coefficient of friction is likely to be very low (less than 0.1) and the values of compressive contact stress predicted by the algorithm will be sufficiently accurate. Therefore this Item is applicable to the analysis of sliding contact (domed or flat-faced) followers only if the lubrication regime can be described as “mixed” or “elastohydrodynamic” as described in ESDU 94008 and ESDU 94017 (References 22 and 23). If “boundary” lubrication is predicted (ESDU 94008 and ESDU 94017) it is more difficult to be certain that the coefficient of friction is less than 0.1 and so results obtained using the computer program (see Section 7) should be treated with caution.

2.2.2 Follower motion

The method presented in this Item can be used to analyse cams with a wide range of profiles. In cam design the relationship between the motion of the cam and the motion of the follower, during a dwell-rise-dwell (DRD) or a dwell-return-dwell (DRD) motion, is usually governed by a mathematical equation known as a cam law (see ESDU 82006, Reference 6). Although the theory (presented in Section 3 of this Item) is general, in that any mathematical cam law can be specified, the computer program (see Section 7) contains only six commonly-used cam laws (identified in Section 7.3.1). If the required cam law for a particular motion segment is not included in the current version of ESDUpac A9411, then the data can be entered as a motion data file.

This motion data file must contain values of follower displacement, follower velocity, follower acceleration and follower jerk at intervals of one degree of cam rotation. The required format of the file is described in Section 7.3.2 and the reasons for specifying motion in this way are explained in Section 4.1.1.



Sketch 2.1 Disc cams with four commonly-used followers

2.3 Use of This Data Item

2.3.1 General guidance

The theoretical background is presented in Section 3 of this Data Item but it is not necessary to have a deep understanding of this theory to carry out the required calculation using the accompanying computer program described in Section 7. A guide to the choice of input values for this program and a description of the program output, including design guidance, is presented in Section 4. The examples in Section 6 also provide valuable hints on the effective use of the program.

Calculation of contact stress may be useful:

- (a) during a feasibility study,
 - (b) as part of a detail design, especially during the selection of a suitable cam material (see ESDU 94017, Reference 23),
- or
- (c) to investigate the cause of deterioration of the surface of a cam or a follower (see Tallian, Reference 34) .

It is likely that the nature and extent of the information available in these cases will vary considerably. For instance during a feasibility study the mass properties of the follower and the externally-applied loads may not be known accurately. The analytical procedure is applicable when uncertainty surrounds values of inputs but the accuracy of the results is entirely dependent on the quality of the input information.

2.3.2 Sources of information regarding values of allowable contact stress

Values of compressive contact stress obtained using the procedure in this Item should be used in conjunction with a corresponding “allowable” value of contact stress. However, there are a number of difficulties in obtaining reliable values of this limiting stress (or surface strength).

Firstly, there are several different modes by which a surface can fail and each mode has different criteria associated with it. Surfaces can deteriorate as a result of plastic deformation, brittle cracking, fatigue cracking or wear-related processes such as scuffing and polishing. ESDU 84017 (Reference 15) includes a detailed discussion of the failure criteria usually associated with surfaces in contact. For cams with domed or flat-faced followers a significant cause of surface failure is a fatigue phenomenon caused by the cyclical nature of the load and known as “pitting”. Initiation of fatigue cracks below the surface is believed to be associated with cyclic variation of the orthogonal shear stress. The magnitude and location of this quantity could be found as described in ESDU 85007 (Reference 16). However limiting values of the range of orthogonal shear stress are not readily available.

A more pragmatic approach is to make use of allowable values of compressive contact stress that have been obtained by experiment. Such values are given in References 4, 23 and 31 for a range of commonly-used cam materials. The reliability and generality of these sources is unknown and in practice there is no substitute for data obtained in service using identical materials in a similar application.

The allowable stress is heavily dependent on the hardness of the surface which can usually be increased by surface treatment. Advice regarding the improvement of resistance to pitting is given in ESDU 86040 and ESDU 94017 (References 18 and 23). As fatigue cracks are often initiated below the surface it is advisable to ensure that surface hardness is maintained to an appropriate depth.

3. THEORETICAL BACKGROUND

This section presents details of the mathematical model on which the calculation procedure is based. It is strongly recommended that the calculations are carried out using the computer program described in Section 7. The equations used to evaluate contact force and cam profile radius of curvature in this program are derived in Section 3.1 for the case of a translating domed follower. For other follower types covered by this Item the equations for these quantities are given in ESDU Technical Memorandum No. 102 (Reference 3). Section 3.2 describes the algorithm for calculating contact stress and Section 3.3 presents the formulae used for initial selection of a spring. Recommended values of the quantities needed as inputs to the computer program are given in Section 4.1.

The convention adopted for forces and moments in this Item is such that P_{x23} is the component of force in the x-direction exerted by Part 2 on Part 3. Thus

$$P_{x23} = -P_{x32}.$$

The magnitude of the force exerted by Part 2 on Part 3 is given by

$$P_{23} = \left(P_{x23}^2 + P_{y23}^2 \right)^{1/2}.$$

Similarly, T_{34} is the moment (positive in the anticlockwise sense) exerted by Part 3 on Part 4. Thus

$$T_{34} = -T_{43}.$$

3.1 Translating Domed Follower

Sketch 3.1(a) shows a typical sub-system composed of a disc cam with an offset translating domed follower. A compression spring is used to provide force closure. The convention used is shown in Sketch 3.1(b). Diagrams showing the forces acting on each part are shown in Sketch 3.1(c).

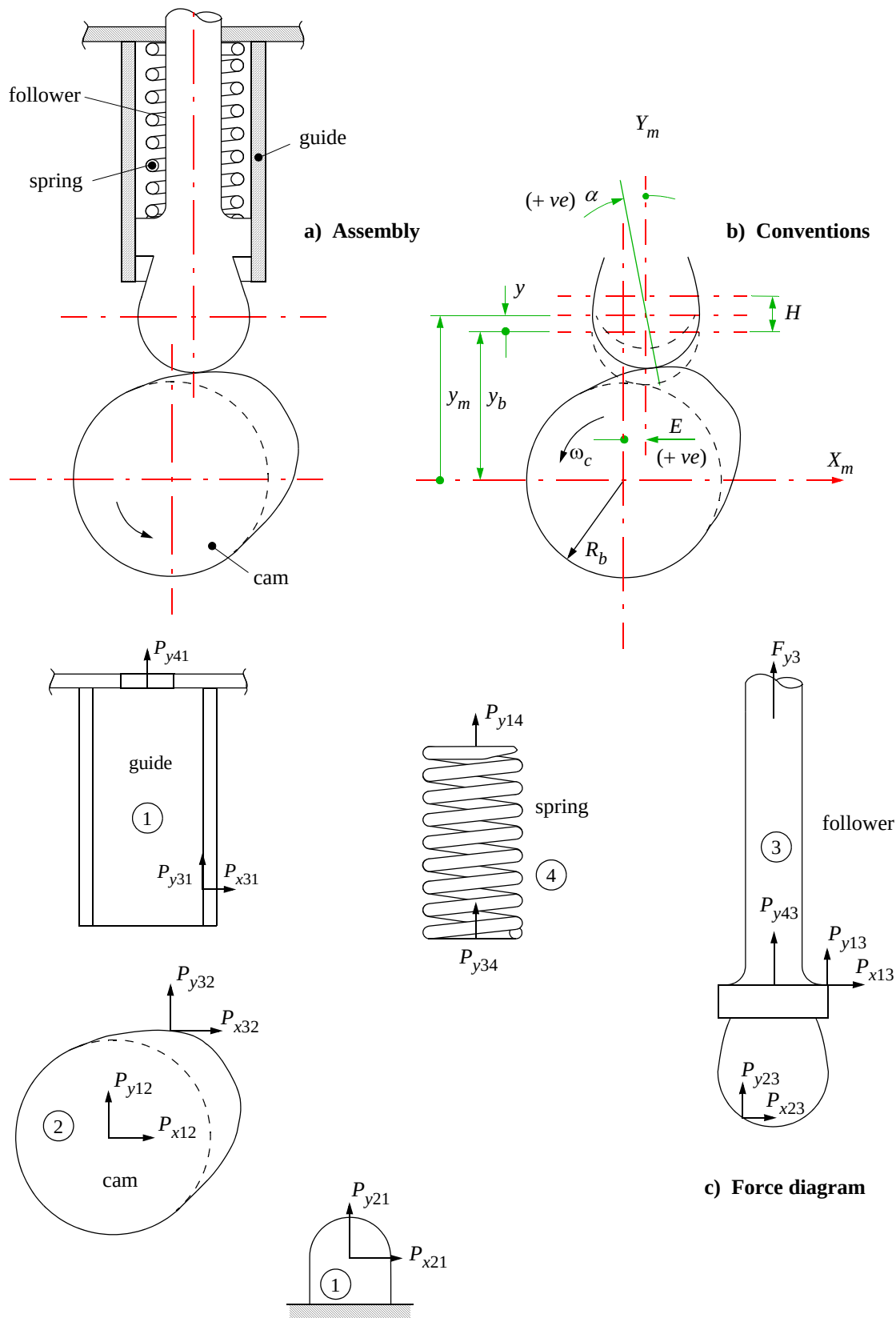
3.1.1 Derivation of contact force

It is required to derive an expression for the contact force, P_C , between the follower and the cam due to the action of the spring, the inertia of the follower system and the external force acting on the follower. The follower is constrained to move only in the y-direction and it is assumed that friction force, P_{y13} , between the follower and the guide is zero. However, in practice, friction can be included by considering it as part of the external force, F_{y3} . The sum of the components of force acting on the follower in the y-direction is equal to the product of the mass of the follower (see Section 4.1.1) and the acceleration of its centre of mass in the y-direction. Thus for equilibrium of the follower

$$P_{y23} + P_{y43} + F_{y3} = m_f \ddot{y}, \quad (3.1)$$

where

P_{y23}	is the component of force exerted by the cam on the follower in the y-direction,
P_{y43}	is the force exerted by the spring on the follower,
F_{y3}	is the external force exerted on the follower
and m_f	is the mass of the follower system.



Sketch 3.1 Cam and follower system

The instantaneous motion (displacement, y , velocity, $\dot{y}$, and acceleration, $\ddot{y}$) of the centre of the follower centre radius during a rise or return segment is required. These quantities are given by

$$y = wH, \quad (3.2a)$$

$$y_m = y_0 + wH, \quad (3.2b)$$

$$\dot{y} = \frac{w' \omega_c H}{\Phi} \quad (3.3)$$

and

$$\ddot{y} = \frac{w'' \omega_c^2 H}{\Phi^2}, \quad (3.4)$$

where y_0 is the height of the centre of the follower radius above the X_m -axis at the start of the motion segment (rise or return),

H is the total follower lift corresponding to one motion segment (in this Data Item it is taken to be a positive scalar quantity for both rise and return motion segments),

Φ is the total cam angle corresponding to one motion segment (note that units are radians),

ω_c is the angular velocity of the cam

and w, w', w'' are the dimensionless follower displacement, velocity and acceleration, respectively (expressions for these parameters corresponding to commonly-used cam laws are given in ESDU 82006 (Reference 6)).

Note that: (a) w and w' are positive during a rise motion and negative during a return motion

and (b) w'' changes sign during every motion segment (it is positive during the first part of a rise segment and negative during the first part of a return segment).

The pressure angle, α , is given (see Derivation 2) by

$$\alpha = \tan^{-1} \left(\frac{\left(\frac{w' H}{\Phi} \right) - E}{y_m} \right). \quad (3.5)$$

The spring force is composed of the preload, P_{SP} , with the product of the spring rate, k_{SC} , and the follower displacement from inner dwell ($y_m - y_b$), that is

$$P_S = -P_{y43} = P_{SP} + k_{SC} (y_m - y_b), \quad (3.6)$$

where

$$y_b = \left((R_b + R_f)^2 - E^2 \right)^{1/2}. \quad (3.7)$$

The external force, F_E , and the spring force, P_S , are considered as positive (by convention) when they act to increase the contact force on the cam. So

$$F_E = -F_{y3} \quad (3.8)$$

and $P_S = -P_{y43}$. (3.9)

Thus the expression for contact force, as used in the program, is obtained by substituting Equations (3.8), (3.9) and (3.6) in Equation (3.1) and rearranging to give

$$P_C = P_{23} = \frac{p_{y23}}{\cos \alpha} = \frac{[P_{SP} + k_{SC}(y_m - y_b)] + F_E + m_f \ddot{y}}{\cos \alpha}. \quad (3.10)$$

Contact force is calculated in the computer program (see Section 7) at intervals of one degree of cam angle during one full rotation of the cam.

3.1.2 Radius of cam profile curvature

The radius of curvature of the cam profile is required for the calculation of contact stress. The derivation of an expression for this quantity is given in ESDU ME2 (Derivation 2) where the radius of curvature of the pitch curve for a cam driving a translating roller follower is given by

$$\rho = \frac{y_m^2}{\frac{H}{\Phi} \cos^2 \alpha \left(w' \sin \alpha - \frac{w''}{\Phi} \cos \alpha \right) + y_m \cos \alpha} \quad (3.11)$$

and the radius of curvature of the cam profile (with a domed follower) is given by

$$\rho_p = \rho - R_f. \quad (3.12)$$

This quantity is determined in the computer program at intervals of one degree of cam angle during one full rotation of the cam. The determination of contact stress is presented in Section 3.2.

3.2 Evaluation of Contact Stress

This section comprises a statement of the equations used to calculate values of maximum compressive contact stress in the computer program (see Section 7). The approach used to determine these values for an elliptical contact zone is that described in Table 9.5 of ESDU 78035 (Derivation 1) while for a rectangular contact zone the relevant equations are given in Table 9.6 of the same Data Item. The equations are simplified because of the relatively simple geometry of the contact zone between a cam and follower and the notation used is that of this Item. Derivations of the following equations are not given in this Item.

The first step is to evaluate the body geometry parameters, **A** and **B**.

If $\frac{1}{2R_c} < \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_f} \right)$ then $\mathbf{A} = \frac{1}{2R_c}$ and $\mathbf{B} = \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_f} \right)$ (3.13)

or if $\frac{1}{2R_c} \geq \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_f} \right)$ then $\mathbf{A} = \frac{1}{2} \left(\frac{1}{\rho_p} + \frac{1}{R_f} \right)$ and $\mathbf{B} = \frac{1}{2R_c}$. (3.14)

The material constants, k_c and k_f , for the cam and follower respectively, are given by

$$k_c = \frac{(1 - \sigma_c^2)}{\pi E_c} \quad (3.15)$$

and
$$k_f = \frac{(1 - \sigma_f^2)}{\pi E_f} . \quad (3.16)$$

For an elliptical contact region (where $\mathbf{A/B} \geq 2.0 \times 10^{-6}$) the semi-axis ratio for the contact zone is given by

$$\beta = \left(\frac{\mathbf{A}}{\mathbf{B}} \right)^{1/2} \left(0.555 \log_e \left(\frac{\mathbf{B}}{\mathbf{A}} \right) + 0.755 \right)^{-1/2} \quad \text{if } \mathbf{A/B} < 0.155 \quad (3.17a)$$

$$\beta = \left(\frac{\mathbf{A}}{\mathbf{B}} \right)^{2/3} \left(1 + \frac{1}{324} \left(\log_e \left(\frac{\mathbf{B}}{\mathbf{A}} \right) \right)^3 \right) \quad \text{if } \mathbf{A/B} \geq 0.155 \quad (3.17b)$$

and the intermediate calculation parameters (corresponding to the expressions for elliptic integrals in ESDU 78035, Derivation 1), are given by

$$Z_1 = 1 + \frac{\mathbf{A}}{\mathbf{B}} \left(\frac{\pi}{2} - 1 \right) \quad (3.18)$$

and
$$Z_2 = \frac{\pi}{2} \left(1 + 0.36 \log_e \left(\frac{\mathbf{B}}{\mathbf{A}} \right) \right) . \quad (3.19)$$

A further intermediate calculation parameter is given by

$$Z_3 = - \left[\frac{3P_C \pi (k_c + k_f) (\mathbf{A} + \mathbf{B})^2}{4} \right]^{1/3} . \quad (3.20)$$

The maximum compressive surface contact stress is given by

$$(f_z)_{max} = - \left(\frac{\pi^2 \beta}{4 Z_1^2} \right)^{1/3} \left(\frac{2 Z_3}{\pi^2 (k_c + k_f)} \right) \quad (3.21)$$

and the normal approach between the cam and follower is given by

$$\delta = Z_2 \left(\frac{4 \beta^2}{\pi^2 Z_1} \right)^{1/3} \frac{Z_3^2}{(\mathbf{A} + \mathbf{B})} . \quad (3.22)$$

The contact ellipse major semi-axis is given by

$$a = \left(\frac{2Z_1}{\pi\beta^2} \right)^{1/3} \frac{Z_3}{(\mathbf{A} + \mathbf{B})} \quad (3.23)$$

and the minor semi-axis is given by

$$b = a\beta. \quad (3.24)$$

If $\mathbf{A}/\mathbf{B} < 2.0 \times 10^{-6}$ then the contact zone is regarded as rectangular and Equations (3.25) to (3.27) are used. The maximum compressive contact stress is given by

$$(f_z)_{max} = - \left(\frac{P_C}{W\pi^2 k_c + k_f} \left(\frac{\rho_p + R_f}{\rho_p R_f} \right) \right)^{1/2}, \quad (3.25)$$

where W is the smaller of the cam facewidth W_c and the follower width W_f . The normal approach between the cam and a domed follower is given by

$$\delta = \frac{2P_C k_c}{W} \left(\log_e \left(\frac{4\rho_p}{b} \right) - 0.5 \right) + \frac{2P_C k_f}{W} \left(\log_e \left(\frac{4R_f}{b} \right) - 0.5 \right), \quad (3.26)$$

where half of the width of the contact zone is given by

$$b = \left(\frac{4P_C(k_c + k_f)}{W} \left(\frac{\rho_p + R_f}{\rho_p R_f} \right) \right)^{1/2}. \quad (3.27)$$

Note that for a flat-faced follower (where R_f is infinite) when the contact zone is rectangular the normal approach cannot be calculated using Equation (3.26). The computer program implements an algorithm using the above equations at intervals of one degree of cam angle and the maximum value of contact stress during one complete rotation of the cam is determined.

3.3 Determination of Suitable Spring Rate and Preload

The function of the spring is to maintain contact between the cam and follower throughout the motion. For the purposes of this Item it is sufficient to characterise a spring by values of spring rate and preload that ensure contact throughout one rotation of the cam under the worst combination of operating conditions. In many practical design situations these appropriate values of spring rate and preload are unknown. The program contains an option that allows the determination of values of spring rate and preload that will ensure positive contact force throughout the cycle. The way that these values are determined is described in this section.

To ensure contact for both translating followers and swinging followers, the condition

$$P_C > 0 \quad (3.28)$$

must hold at every calculated cam position throughout the cycle. For translating followers substituting Equation (3.10) into Expression (3.28) gives

$$\frac{(P_{SP} + k_{SC} (y_m - y_b)) + F_E + m_f \ddot{y}}{\cos \alpha} > 0 . \quad (3.29)$$

Since $\cos \alpha$ is always positive this condition becomes

$$(P_{SP} + k_{SC} (y_m - y_b)) + F_E + m_f \ddot{y} > 0 . \quad (3.30)$$

The spring force $(P_{SP} + k_{SC} (y_m - y_b))$ must exceed the separating force $(-F_E - m_f \ddot{y})$.

The program finds the cam angle, ϕ_q , at which the maximum separating force occurs and the corresponding follower motion at this position is defined by y_q , $\dot{y}_q$ and $\ddot{y}_q$ while the value of the external force is F_{Eq} . A user-specified factor of safety known as the force factor, z_1 , is then introduced and the inequality (Expression (3.30)) becomes

$$(P_{SP} + k_{SC} (y_{mq} - y_b)) + z_1 (F_{Eq} + m_f \ddot{y}_q) = 0 . \quad (3.31)$$

The force factor is then the ratio of spring force to separating force defined at cam angle ϕ_q , which is the point on the cam when the resultant force is a minimum. Since the spring force must be greater than the separating force for the follower to remain in contact with the cam, z_1 must be chosen to be greater than unity (see Section 4.1.2). Often the worst case of this minimum occurs when the cam is running under no-load conditions and the follower reaches a value of maximum acceleration towards the cam (see the example shown in Sketch 3.2).

In addition, a user-specified preload factor, z_2 , is introduced. This defines the preload as a proportion of the maximum spring force and is given by

$$z_2 = \frac{P_{SP}}{k_{SC} [(y_m)_{max} - y_b]} . \quad (3.32)$$

An expression for the required spring rate is obtained by substituting Equation (3.32) into Equation (3.31) and rearranging to give

$$k_{SC} = \frac{-z_1 (F_{Eq} + m_f \ddot{y}_q)}{z_2 [(y_m)_{max} - y_b] + (y_{mq} - y_b)} \quad (3.33)$$

and the required spring preload is then given by

$$P_{SP} = z_2 k_{SC} ((y_m)_{max} - y_b) . \quad (3.34)$$

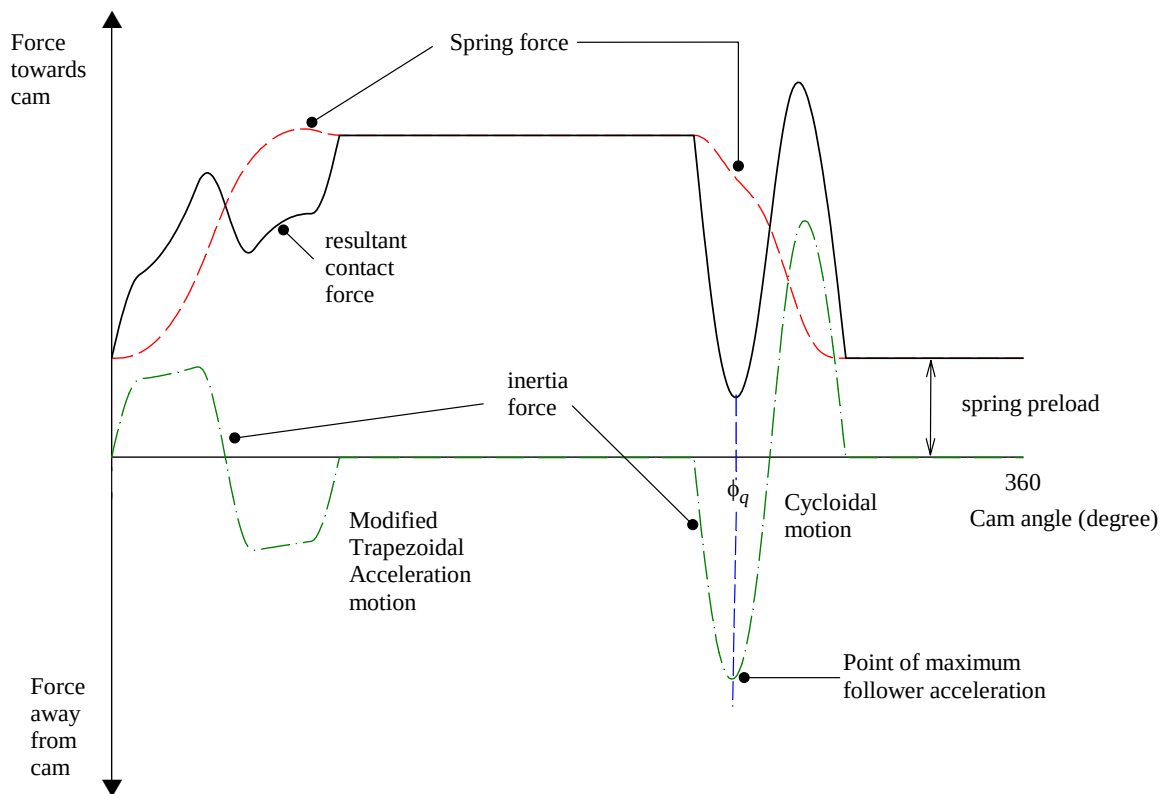
Similarly, for a swinging follower, the spring rate is given by

$$k_{ST} = \frac{-z_1(T_{Eq} - I_f \ddot{\psi}_q)}{z_2[\psi_b - (\psi_m)_{min}] + (\psi_b - \psi_{mq})} \quad (3.35)$$

and the preload is given by

$$T_{SP} = z_2 k_{ST} (\psi_b - (\psi_m)_{min}) \quad (3.36)$$

It should be noted that the above procedure may result in the specification of a spring that has unrealisable values of spring rate and preload (see Section 4.1.2).



Sketch 3.2 Example of contact force versus cam angle