

EXAMPLES OF FLIGHT PATH OPTIMISATION USING A MULTIVARIATE, GRADIENT-SEARCH METHOD

ADDENDUM A

VARIATION OF OPTIMUM FLIGHT PROFILE PARAMETERS WITH RANGE

1. INTRODUCTION

Data Item No. 93021 (Reference 1) provides examples of the solution of the problem of optimisation of a complete sortie using a constrained multivariate gradient-search method. The method is applied to a subsonic transport aircraft and the examples are given to demonstrate application of the method to optimisation of the flight profile for various objective functions, *i.e.* minimum fuel, minimum time, minimum direct operating cost (each for its own fixed range), and maximum range with fixed ramp mass and fuel mass.

The present Data Item gives a further example, showing the effect of varying the range on the optimum values for the flight parameters. A cruise-climb technique is adopted which, although seldom permissible in civil air traffic conditions, is used here because it shows greater sensitivity of the flight profile parameters to range than the more usual level cruise or cruise at stepped heights and so illustrates the results well.

The principles underlying Reference 1 and the present Item are presented in Data Item No. 94012 (Reference 2) which also describes the difficulties of formulating a problem for solution by the method of constrained multivariate optimisation.

2. EXAMPLE DETAILS

2.1 Aircraft

The example presented is concerned with the optimum performance in ISA, still-air conditions of the hypothetical subsonic transport aircraft used in Data Item No. 93021, which has the following characteristics.

Wing area	200 m ² ,
Aspect ratio	8,
Maximum ramp mass	120 000 kg,
Operating mass empty	64 000 kg,
Maximum zero fuel mass	92 000 kg,
Maximum fuel	40 000 kg,
Maximum design EAS	350 knots,
Maximum operating Mach number	0.86,
Engines	2 turbo fan,
Bypass ratio	4.4,
Thrust/weight	0.333 (sea level, static, maximum take-off rating; engine derates are not involved in these examples).

The aircraft has a payload of 16 000 kg (corresponding to zero fuel mass of 80 000 kg).

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2.2 Objective Function

The sorties are optimised to minimise fuel required (and, therefore, ramp mass).

2.3 Constraints

The constraints applied correspond to those in Case 4 in the examples in Data Item No. 93021, *i.e.* they are as follows.

Airspeed never below 1.3 times stalling speed,
airspeed never below 1.05 times minimum drag speed,
never less than 0.3 margin in load factor to buffet onset,
cabin inclination between 10 degrees (in climb) or 5 degrees (in cruise) nose-up and 5 degrees nose-down (in descent),
maximum rate of change of cabin pressure limited to $0.16 \text{ (lbf/in}^2\text{)/minute}$ ($1\,100 \text{ (N/m}^2\text{)/minute}$; corresponds to 300 ft/minute at sea level) in the descent,
maximum rate of change of cabin pressure limited to $-0.213 \text{ (lbf/in}^2\text{)/minute}$ ($-1\,465 \text{ (N/m}^2\text{)/minute}$; corresponds to 400 ft/minute at sea level) in the climb.

2.4 Fuel Reserves

The same fuel reserves are used as in the examples in Data Item No. 93021. The reserves policy assumed is as follows

30 minutes hold at 1 500 ft altitude, plus

200 n.miles diversion.

A 5% margin on range fuel was also imposed in Data Item No. 93021 but, since this is dependent on ramp mass, its calculation is included in the present optimisation work.

2.5 Flight Profile

The sortie is assumed to consist of the following segments (see Sketches 3.1 and 3.2 in Data Item No. 93021),

acceleration to climb speed,
climb to start of cruise height at constant calibrated airspeed (CAS) / constant Mach number,
acceleration to cruise speed,
cruise using constant Mach number cruise-climb technique,
acceleration to descent speed,
descent from end of cruise height at constant Mach number / constant calibrated airspeed (CAS).

The flight profile parameters to be selected to minimise the fuel mass are, therefore,

climb speed (CAS),
climb Mach number,
cruise Mach number,
height at start of cruise,
descent Mach number,
descent speed (CAS).

2.6 Discussion of Results

Figure 1 shows height and distance profiles for sorties with various ranges (150, 250, 600, 1 000, and 3 900 n.miles). The 3 900 n.miles sortie consists of a 100 n. miles climb to about 33 000 ft, a cruise climb to about 39 000 ft, and a 140 n.mile descent. The 1 000 n. miles sortie climbs to about 38 000 ft, and then joins virtually the same cruise-climb profile at a later (and, therefore, higher) stage* than its starting point for the 3 900 n.miles case. For intermediate ranges the pattern is similar in that as range decreases the start of cruise height increases. If this process were to continue down to a range of about 230 n.miles, the climb would just be completed in time to start the descent, and the sortie would consist simply of a climb followed by a descent. However, analysis of the results shows that the flight profile differs from this pattern at ranges below about 1 000 n.miles (although the effect is only obvious at much lower ranges). Below a range of about 1 000 n.miles the height at the start of cruise is less than the cruise-climb value for the longer ranges and a cruise segment is still present down to a range of about 150 n. miles.

Figure 2 shows how the heights at the start and end of cruise vary with range. The height at the end of cruise is virtually constant (rising only very slightly as range increases) for ranges above 1 000 n.miles whilst the height at the start of cruise decreases with increase of range. At ranges below 150 n.miles the flight consists simply of a climb and descent and, consequently, heights at the start and end of cruise are identical.

Variation of climb, cruise, and descent Mach numbers, and of climb and descent speeds with range is shown in Figures 3 and 4. The variation is very slight, except at ranges below 1 000 n.miles, where the sortie profile is changing significantly – as already explained. The constancy of the descent Mach number for ranges below 300 n.miles is irrelevant; at the top of descent heights involved, the descents are at constant CAS only – without any constant Mach number phase.

The overall result, in terms of ramp mass is shown in Figure 5. The variation in mass with range shown corresponds to a constant value of the Breguet factor for the cruise phase at ranges where a cruise-climb is followed.

3. REFERENCES

1. ESDU Examples of flight path optimisation using a multivariate gradient-search method. ESDU Data Item No. 93021, December 1993.
2. ESDU Application of multivariate optimisation techniques to determination of optimum flight path trajectories. ESDU Data Item No. 94012, June 1994.

* The cruise-climb phase for 1 000 n.miles range is, in fact, flown slightly higher than that for 3 900 n.miles range since the 5% cruise fuel allowance in the reserves is a lower value for the 1 000 n.miles range than for the 3 900 n.miles range and, in consequence, results in slightly lower masses at the end of the sortie and, consequently, at the end of cruise.