

THROUGH-THE-THICKNESS STRESSES AND FAILURE IN THE CORNER RADIUS OF A LAMINATED COMPOSITE SECTION

1. INTRODUCTION, THEORY AND SOME NUMERICAL RESULTS

1.1 Introduction

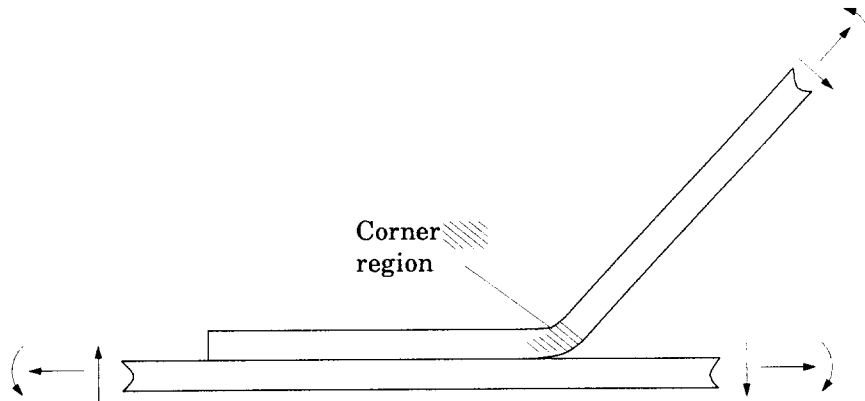
This Data Item together with ESDUpac A9419 presents a Fortran computer program for the calculation of through-the-thickness radial and shear stresses in the corner radius of a laminated composite section which is subjected to a bending moment, radial and tangential forces, M_0 , S_0 , N_0 . The corresponding moment and forces to cause failure are also calculated.

The Item consists of two parts. The first part describes the application of the program and its theoretical basis. The second part describes the use of the program, its input and output, and gives example calculations using the program. The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable program for PCs. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

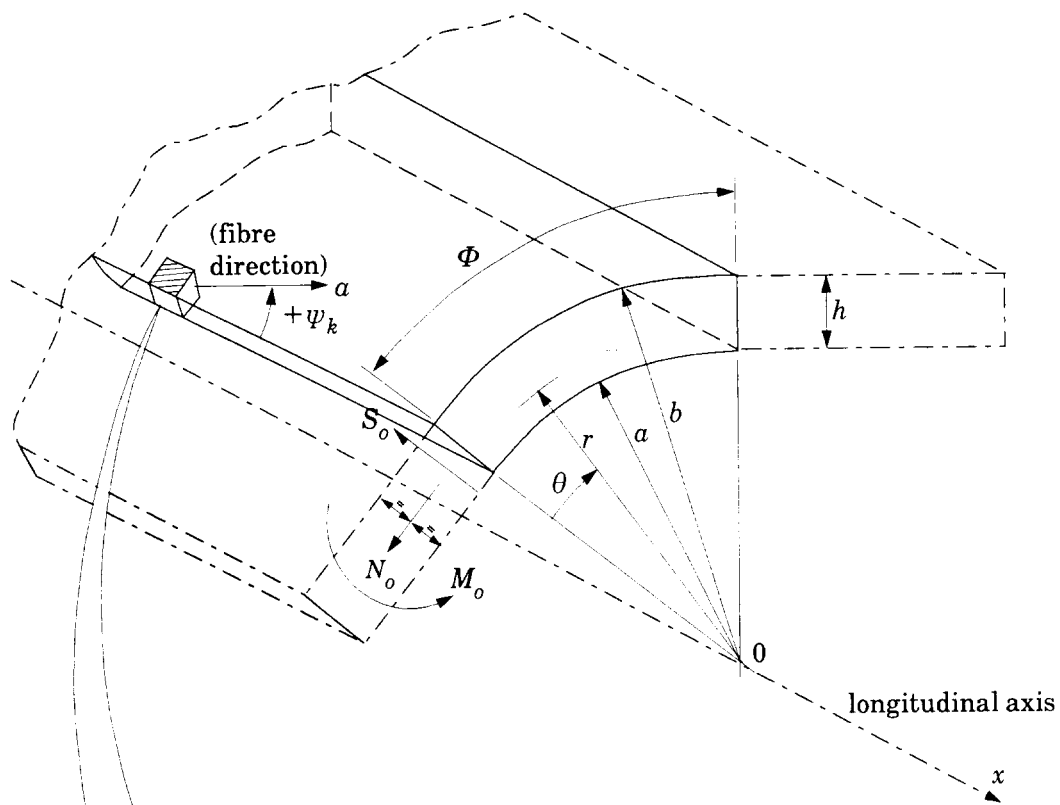
A common practical configuration is shown in Sketch 1.1 where an angled composite flange is shown attached to an adjacent plate. The straight portion that leads into the corner can carry a bending moment, shear and direct forces. These forces are carried over into the corner region and, because of the curvature, can induce relatively high through-the-thickness stresses which can cause failure in the radial direction. For example, if a corner is subjected to a bending moment that has the effect of increasing the included angle of the corner, then this would induce radial tensile stresses across the section. A typical stress distribution is shown in Figure 1 where it is seen that the stresses increase from zero at each of the free edges to reach a maximum value near the centre. Of course, if the sense of the applied moment is reversed, the radial stress distribution would have the same form as before, but would be compressive. This would give rise to a different type of failure mode as discussed in Section 1.3.3. The analysis does not consider the possibility of in-plane (tangential) failure.

The corner geometry and applied loading are shown in Sketch 1.2 and it is noted that the cross-sectional shape remains constant along the longitudinal axis. Also, the corner angle ϕ can be given any value from 0° up to 360° . The laminate corner and associated straight portions are constructed as shown in Sketch 1.2. They are made from a number of thin orthotropic layers of continuous fibre reinforced composite with perfect bonding assumed between the layers. The local fibre direction is inclined at an angle ψ_k to the longitudinal axis of the corner as shown in Sketch 1.2.

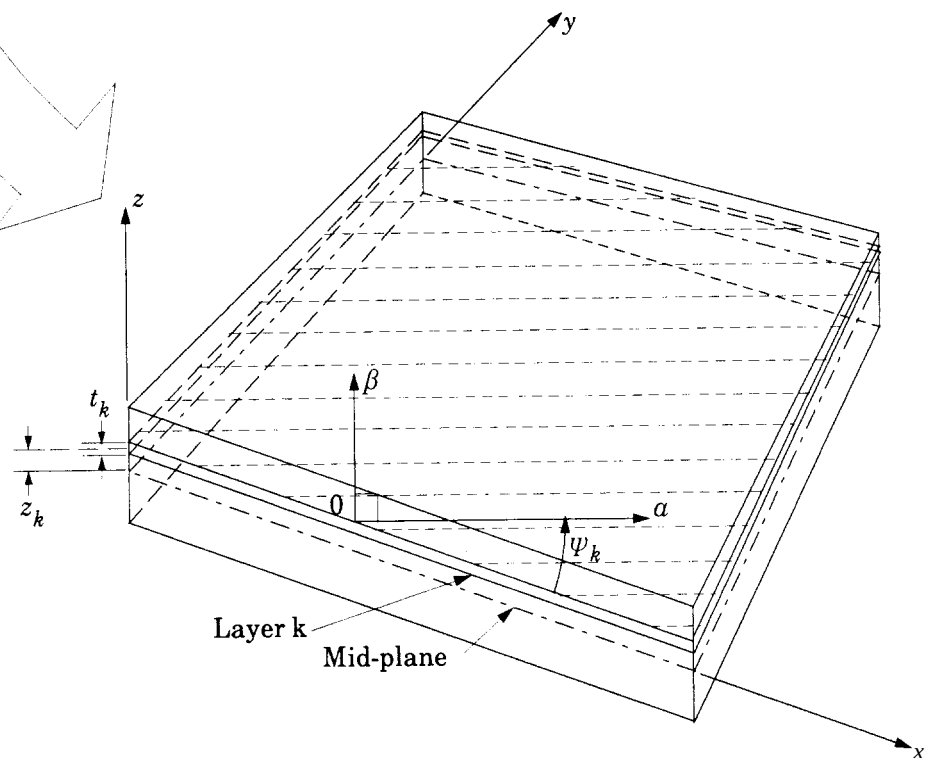
The computer program is based on the theory of elasticity for a homogeneous anisotropic material given in Derivation 1. For given forces N_0 and S_0 , and bending moment M_0 , the program calculates the radial direct stress and the transverse shear stress for a large number of coordinate positions (r, θ) within the corner. A failure factor is then calculated for each coordinate position and the most critical point is noted. The corresponding set of applied forces, in the same ratio as the input values, that is necessary to cause failure is calculated and compared with the applied forces.



Sketch 1.1



Corner geometry and loading
 Sign convention for applied forces and moments positive as shown



Sketch 1.2 Local coordinates and geometry for laminate

1.2 Notation

1.2.1 General notation

Both SI and British units are quoted but any coherent system of units may be used.

a	inner corner radius	m	in
b	outer corner radius, $b = a + h$	m	in
C	sequential identification number for a "layer type", see Section 2.3 (varies from 1 to C_{max})		
c	the ratio a/b		
E_r	effective modulus of elasticity of corner in radial, through-the-thickness direction, assuming material homogeneity	N/m ²	lbf/in ²
E_x, E_θ	effective moduli of elasticity of corner in x - and θ -directions, respectively, calculated from laminated plate theory	N/m ²	lbf/in ²
$E_{\alpha k}, E_{\beta k}$	moduli of elasticity of k th ply in α_k - and β_k -directions respectively	N/m ²	lbf/in ²
$G_{r\theta}$	effective shear modulus of corner in (r, θ) direction assuming material homogeneity	N/m ²	lbf/in ²
$G_{\alpha\beta k}$	shear modulus of k th ply in $(\alpha, \beta)_k$ direction	N/m ²	lbf/in ²
g, g_1, g_2	parameters in expressions for radial stress, defined by Equations (1.2), (1.3) and (1.8) respectively		
H_{22}	transverse shear stiffness with respect to yz -plane in Sketch 1.2	N/m	lbf/in
h	total corner thickness	m	in
h_1, h_2	distance of innermost and outermost radial points, respectively, defining stress distribution mesh, measured outwards from radius a , through the thickness of the laminate	m	in
L_1, L_2	first and last ply interfaces, respectively, to be included in stress analysis		
M_0	applied bending moment per unit length (see Sketch 1.2)	N m/m	lbf in/in
m	number of a particular radial mesh point, $1 \leq m \leq n_h$		
N_0	applied direct force per unit length (see Sketch 1.2)	N/m	lbf/in
n	total number of plies in laminate		

n_h	number of radial points through laminate (including those at h_1 and h_2) in stress distribution mesh		
n_θ	number of angular points in stress distribution mesh (including θ_1 and θ_2)		
R	failure factor (see Equation (1.10))		
r	radius of a point in corner (see Sketch 1.2)	m	in
S_0	applied shear force per unit length (see Sketch 1.2)	N/m	lbf/in
t_k	thickness of k th ply	m	in
Z_{rck}, Z_{rtk}	compressive and tensile strengths, respectively, of ply material in radial through-the-thickness direction	N/m ²	lbf/in ²
$Z_{r\theta k}$	shear strength of ply material in through-the-thickness (r, θ) direction	N/m ²	lbf/in ²
ZI_{rck}, ZI_{rtk}	compressive and tensile strengths, respectively, of interface in radial through-the-thickness direction	N/m ²	lbf/in ²
$ZI_{r\theta k}$	shear strength of interface in through-the-thickness (r, θ) direction	N/m ²	lbf/in ²
$\beta_{11}, \beta_{22}, \beta_{12}, \beta_{66}$	terms defined by Equations (1.6)	m ² /N	in ² /lbf
θ	angle of a point around the corner (see Sketch 1.2)	degree	degree
θ_1, θ_2	angles of least and greatest angular points, respectively, in stress distribution mesh	degree	degree
λ, μ	defined by Equations (1.4) and (1.5) respectively		
$\nu_{x\theta}$	Poisson's ratio of corner resulting from force applied in x -direction with induced strain in θ -direction, calculated from laminated plate theory		
$\nu_{xr}, \nu_{\theta r}$	Poisson's ratios of corner resulting from force applied in x - and θ -directions, respectively, with induced strains in r -direction, assuming material homogeneity		
$\nu_{\alpha\beta k}$	Poisson's ratio for k th ply resulting from force applied in α_k -direction with induced strain in β_k -direction		
σ	direct stress component in corner	N/m ²	lbf/in ²
$\tau_{r\theta}$	shear stress component in corner in (r, θ) direction	N/m ²	lbf/in ²
ϕ	angle subtended by corner ($0 < \phi \leq 360^\circ$) (see Sketch 1.2)	degree	degree
ψ_k	angle of principal axis of orthotropy, α_k , of k th layer measured from plate reference axis, $0x$, as shown in Sketch 1.2	degree	degree

$0(x,y,z)$	laminate main reference axes (see Sketch 1.2) (x is also the longitudinal axis of angled section, also see Sketch 1.2)
$0(r,\theta)$	polar coordinates of corner, through-the-thickness (see Sketch 1.2)
$0(\alpha,\beta,z)$	principal axes of orthotropy of a ply (see Sketch 1.2)

1.2.2 Suffixes

k	refers to k th layer, starting with number 1 at inside of corner
max	maximum
0	refers to applied forces and moment at start of corner when $\theta = 0^\circ$
r, θ	relate to r - and θ -directions, respectively
t, c	denote tension and compression failure stresses, respectively
x	relates to x -direction
α, β	relate to α - and β -directions, respectively

1.2.3 Sign convention

The sign convention for the direct forces, bending moments and shear forces as applied to the corner is shown in Sketch 1.2.

Tensile stresses and strengths are positive.
Compressive stresses are negative.
Compressive strengths are positive.

1.3 Theoretical Assumptions and Governing Equations

With some modifications, the governing equations for the two stress components σ_r and $\tau_{r\theta}$ are obtained from Derivation 1. The analysis is based on the assumption of cylindrical anisotropy (see Section 12 of Derivation 1) and in Derivation 1 the problem is formulated in terms of plane stress. However, in this present Data Item it is considered that the curved section would more likely be in a state of plane strain, with the strain in the x -direction taken to be zero, because of the restraint offered by the surrounding structure in the longitudinal direction. The equations of Derivation 1 have therefore been modified for the case of plane strain.

The laminate is assumed to be homogeneous and specially orthotropic with respect to the x - and y -axes (see Sketch 1.2).

The assumption of material homogeneity is not strictly valid for the type of laminate considered in the present Data Item because the magnitude and distribution of the radial stresses depends very largely on the through-the-thickness distribution of the tangential end loads reacting the bending moment. This distribution will depend on the number of layers in the laminate and the orientation of their axes of orthotropy, and will in general differ from that induced in the assumed homogeneous section. This difference and hence the accuracy of the analysis will therefore depend on the type of lay-up, with a tendency towards greater accuracy with a large number of layers having their various orientations spread uniformly through the thickness. However, bearing in mind that the values of the through-the-thickness allowable stresses are not usually known with great accuracy, the analysis will provide a solution with adequate accuracy for many types of lay-up likely to be used in practice. The program tests the validity of the assumption of material homogeneity for the laminate specified. The test is based on an approximate value of the ratio of the differential end loads due to bending, for the actual laminate compared with the homogenised section. This test gives an indication of the possible error of the calculated value of the maximum radial stress. If the test indicates that the maximum radial stresses, and therefore the value of R_{max} , are likely to be underestimated a warning is printed out at the start of the results, under the heading "Homogeneity Test Warning", see Section 2.4.3, which gives a value that can be used to assess the applicability of the program.

The requirement that the material should be specially orthotropic means that terms in the stress-strain matrix equation that would couple together shear and direct effects are taken to be zero, see ESDU Data Item No. 82013*. It is anticipated that in such cases the fibre alignments would be in the x-direction and in the y-direction together with fibres at $\psi_k = \pm 45^\circ$.

For each of the three loading cases considered, namely the application of N_0 , S_0 and M_0 , the plane strain solution satisfies the partial differential equation for the stress function, together with the stress free boundary conditions at $r = a$ and $r = b$ and the load condition on the loaded face at $\theta = 0^\circ$. However, it should be noted that in Derivation 1 the solution for the radial stresses σ_r due to the end load N_0 is incorrect. This is because this solution is not able to satisfy, completely, the boundary conditions on the loaded face at $\theta = 0^\circ$. It is found that when the circumferential stresses, σ_θ , are integrated across the thickness at $\theta = 0^\circ$, the distribution is such as to give rise to a resultant load N_0 (as required), but that a bending moment equal to $-N_0(a + b)/2$ is also present. Thus the requirement for zero end moment, in this case, is violated. In order to correct for this, the radial stress distribution due to an opposing end moment equal to $N_0(a + b)/2$ is superimposed on to the radial stresses given by Derivation 1 to give Equations (1.1) and (1.7) below. The expressions for the shear stresses, $\tau_{r\theta}$, given in Derivation 1 are correct since they are unaffected by the additional end moment.

* ESDU Data Item No. 82013 "Laminate stacking sequences for special orthotropy. (Application to fibre reinforced composites.)".

1.3.1 Equations for the stresses

1.3.1.1 Radial stresses

The following equation is used to calculate the radial stresses due to M_0 , N_0 and S_0 .

$$\sigma_r = \frac{M_0 + N_0(a+b)/2}{b^2 g} \left[1 - \frac{(1-c^{\lambda+1})}{1-c^{2\lambda}} \left(\frac{r}{b} \right)^{\lambda-1} - \frac{(1-c^{\lambda-1})c^{\lambda+1}}{1-c^{2\lambda}} \left(\frac{b}{r} \right)^{\lambda+1} \right] + (N_0 \cos \theta + S_0 \sin \theta) \left(\frac{1}{g_1 r} \right) \left[\left(\frac{r}{b} \right)^\mu + c^\mu \left(\frac{b}{r} \right)^\mu - 1 - c^\mu \right], \quad (1.1)$$

$$g = \frac{(1-c^2)}{2} - \frac{\lambda(1-c^{\lambda+1})^2}{(\lambda+1)(1-c^{2\lambda})} + \frac{\lambda c^2(1-c^{\lambda-1})^2}{(\lambda-1)(1-c^{2\lambda})}, \quad (1.2)$$

$$g_1 = \frac{2(1-c^\mu)}{\mu} + (1+c^\mu) \ln c, \quad (1.3)$$

$$\lambda = \left(\frac{\beta_{11}}{\beta_{22}} \right)^{1/2}, \quad (1.4)$$

$$\mu = \left[1 + \frac{(\beta_{11} + 2\beta_{12} + \beta_{66})}{\beta_{22}} \right]^{1/2}, \quad (1.5)$$

$$\left. \begin{aligned} \beta_{11} &= \frac{1}{E_r} - \frac{v_{xr}^2}{E_x}, \\ \beta_{22} &= \frac{1}{E_\theta} - \frac{v_{x\theta}^2}{E_x}, \\ \beta_{12} &= -\frac{v_{\theta r}}{E_\theta} - \frac{v_{xr} v_{x\theta}}{E_x} \\ \text{and} \\ \beta_{66} &= \frac{1}{G_{r\theta}}. \end{aligned} \right\} \quad (1.6)$$

The two moduli E_x and E_θ and the Poisson's ratio $v_{x\theta}$ are calculated in the computer program using laminated plate theory. The remaining properties, E_r , $G_{r\theta}$, v_{xr} and $v_{\theta r}$, are input data supplied by the user. Data Item 89013* can be used to obtain a value for $G_{r\theta}$, where it will be found that $G_{r\theta} = H_{22}/h$.

For the special case when $\lambda = 1$, for example for the isotropic plate (where $\mu = 2$), the above equation for σ_r cannot be used because the expression for g becomes indeterminate. Instead, the following equation

* Data Item No. 89013 "Transverse (through-the-thickness) shear stiffnesses of fibre reinforced composite laminated plates (computer program)".

is used.

$$\sigma_r = \frac{M_0 + N_0(a + b)/2}{b^2 g_2} [c^2 \{1 - (b/r)^2\} \ln c - (1 - c^2) \ln(b/r)] \\ + (N_0 \cos \theta + S_0 \sin \theta)(1/g_1 r) [(r/b)^\mu + c^\mu (b/r)^\mu - 1 - c^\mu] \quad (1.7)$$

where
$$g_2 = \left(\frac{1 - c^2}{2} \right) - (c \ln c)^2 . \quad (1.8)$$

1.3.1.2 Shear stresses

The shear stresses due to M_0 are zero and the following equation is used to calculate the shear stresses due to N_0 and S_0 . The equation applies to both anisotropic and isotropic cases.

$$\tau_{r\theta} = (N_0 \sin \theta - S_0 \cos \theta)(1/g_1 r) [(r/b)^\mu + c^\mu (b/r)^\mu - 1 - c^\mu] . \quad (1.9)$$

Expressions for g_1 and μ are given by Equations (1.3) and (1.5) respectively.

1.3.2 Failure analysis

The criterion for transverse failure is governed by the following quadratic stress equation proposed by Chang and Springer (Derivation 2).

$$\left(\frac{\tau_{r\theta}}{Z_{r\theta}} \right)^2 + \left[\left(\frac{\sigma_r}{Z_{rt}} \right)^2 \text{ or } \left(\frac{\sigma_r}{Z_{rc}} \right)^2 \right] = R^2 . \quad (1.10)$$

Note that the usual reserve factor is $1/R$.

If R^2 is found to be greater than or equal to unity, then failure is predicted. One or other of the terms inside the square brackets is used, depending on whether or not the radial stress is positive or negative. In the program the ply material strengths $Z_{r\theta}$, Z_{rt} and Z_{rc} are generally used in this calculation. However, at ply interfaces this calculation is also made using the interface strengths $ZI_{r\theta}$, ZI_{rt} and ZI_{rc} and the more critical value of ply or interface failure is given in the results.

1.3.3 Modes of failure

It is often the case that the through-the-thickness tensile strength of composite laminates is low compared with the compressive and shear strengths. In such instances the radial tensile stresses are likely to cause damage. This can take the form of an opening delamination mode between the plies or a matrix cracking mode within the plies. Once this damage has started, then it is possible that the delamination and cracks will spread under repeated loading, which could lead to overall premature failure of the structure.

Similar arguments apply when cracks are caused by high radial compressive and shear stresses. In these cases there will not be a tendency to form an initial opening failure mode. However, this opening mode may well develop if the loading is reversed following initial cracking.

1.4 Some Numerical Results

In this section some numerical results from the present analysis are compared with results obtained from other sources that have used the finite element method, and from experiments. It was necessary to ensure that reasonable comparisons were obtained because of the simplifying assumption of homogeneity which has been made in the present theoretical solution.

1.4.1 Stress distribution and delamination prediction

Derivation 3 gives a theoretical distribution for the stress, σ_r , for a $[90_3/0/90_3/0/90/0]_s$ 20 ply carbon fibre corner specimen. The stresses from the present solution are found to be only slightly below those obtained using the finite element method.

In Derivation 2, the stress fields in laminated corners are calculated using the finite element method, and delamination is predicted using the quadratic failure equation, Equation (1.10) given in Section 1.3.2 above. A large number of theoretical results that show the effects of geometry and ply orientation on the strengths of the corners are presented in graphical form. Detailed strength comparisons have been made for a 90° bend with $[90/0/90]$ lay-up, subjected to a shear force and bending moment. The correlation between the two sets of results is very good with the present analysis giving values that are from 4 per cent to 8 per cent below those obtained from the finite element analysis.

1.4.2 Comparison with experiment

Derivation 3 also gives some experimental results for delamination tests carried out on the 20 ply specimen discussed in Section 1.4.1 above. Tests on 16 identical specimens (see Table 4 of Derivation 3) gave a mean delamination failure load of 44 lbf/in, with a range of 36 to 66 lbf/in. A value of 38.2 lbf/in is obtained from the present analysis, which may be compared with the mean experimental failure load.

1.4.3 General comments

This Data Item provides a program based on a mathematical model that can be used to describe the behaviour in the corner radius of a laminated composite plate. The model, which has been constructed using the theory and assumptions described in Section 1.3, contains a very large number of independent variables. Test data that can be used to verify its accuracy are only available for particular sets of the independent variables. Sufficient data with which to verify the model over its complete range are highly unlikely to become available in the foreseeable future. The model is based upon well known principles of elastic behaviour which have been employed successfully in many models of structural elements and components. Within the validity of the assumptions made it is therefore reasonable to assume that the agreement demonstrated in comparisons with the available sets of test data gives an indication of the accuracy of the model. However, because of the lack of comprehensive test data consideration should be given to testing the structure.