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Increments in aerofoil lift coefficient at zero angle of attack and in maximum lift coefficient due to deployment of a double-slotted or triple-slotted trailing-edge flap, with or without a leading-edge high-lift device, at low speeds

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INCREMENTS IN AEROFOIL LIFT COEFFICIENT AT ZERO ANGLE OF ATTACK AND IN MAXIMUM LIFT COEFFICIENT DUE TO DEPLOYMENT OF A DOUBLE-SLOTTED OR TRIPLE-SLOTTED TRAILING-EDGE FLAP, WITH OR WITHOUT A LEADING-EDGE HIGH-LIFT DEVICE, AT LOW SPEEDS

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INCREMENTS IN AEROFOIL LIFT COEFFICIENT AT ZERO ANGLE OF ATTACK AND IN MAXIMUM LIFT COEFFICIENT DUE TO DEPLOYMENT OF A DOUBLE-SLOTTED OR TRIPLE-SLOTTED TRAILING-EDGE FLAP, WITH OR WITHOUT A LEADING-EDGE HIGH-LIFT DEVICE, AT LOW SPEEDS

1. NOTATION AND UNITS

		SI	British
$(a_1)_0$	basic aerofoil lift-curve slope in incompressible flow	rad^{-1}	rad^{-1}
C_{Lm}	maximum lift coefficient of aerofoil with high-lift devices deployed, based on c		
$(C_{LmB})_d$	maximum lift coefficient of basic aerofoil at datum Reynolds number $R_c = 3.5 \times 10^6$		
C_{L0}	lift coefficient at zero angle of attack for aerofoil with high-lift devices deployed, based on c		
ΔC_{Lm}	increment in maximum lift coefficient due to deployment of high-lift devices, based on c , Equation (3.2)		
ΔC_{Lml}	increment in maximum lift coefficient due to deployment of leading-edge high-lift device, based on c , see Section 5		
ΔC_{Lmt}	increment in maximum lift coefficient due to deployment of trailing-edge flap, based on c , Equation (3.4)		
$\Delta C'_{Lmt}$	increment in maximum lift coefficient due to deployment of double-slotted or triple-slotted trailing-edge flap, based on c' , at datum Reynolds number $R_c = 3.5 \times 10^6$, Equation (4.19) or (4.31)		
$(\Delta C'_{Lmt})_0$	component of $\Delta C'_{Lmt}$ independent of flap deflection, Equation (4.16)		
$(\Delta C'_{Lmt})_1$	component of $\Delta C'_{Lmt}$ due to flap deflection, Equation (4.16)		
ΔC_{L0}	increment in lift coefficient at zero angle of attack due to deployment of high-lift devices, based on c , Equation (3.1)		
ΔC_{L0l}	increment in lift coefficient at zero angle of attack due to deployment of leading-edge high-lift device, based on c , see Item No. 94027		

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ΔC_{L0t}	increment in lift coefficient at zero angle of attack due to deployment of trailing-edge flap, based on c , Equation (3.3)		
$\Delta C'_{L0t}$	increment in lift coefficient at zero angle of attack due to deployment of double-slotted or triple-slotted trailing-edge flap, based on c' , Equation (4.4) or (4.20)		
$\Delta C'_{L0t1}, \Delta C'_{L0t2}$	contribution to $\Delta C'_{L0t}$ arising from first, second element of trailing-edge flap, Equation (4.2), (4.3)		
$\Delta C'_{L1}, \Delta C'_{L2}, \Delta C'_{L3}$	increment in lift coefficient associated with deployment of equivalent first, second, third element of slotted trailing-edge flap on aerofoil with lift-curve slope of 2π , based on c' , Figure 4, Figure 5, Figure 6		
c	basic (plain) aerofoil chord (<i>i.e.</i> chord with high-lift devices undeployed), see Sketch 4.1 or 4.2	m	ft
c'	extended aerofoil chord (<i>i.e.</i> chord with high-lift devices deployed), see Sketch 4.1 and Equation (4.9) for double-slotted flap and Sketch 4.2 and Equation (4.23) for triple-slotted flap	m	ft
$c_{et1}, c_{et2}, c_{et3}$	equivalent flap chord of first, second, third element of slotted trailing-edge flap, see Sketch 4.1 and Equations (4.13), (4.15) for double-slotted flap, Sketch 4.2 and Equations (4.26), (4.28), (4.30) for triple-slotted flap	m	ft
c_{t1}, c_{t2}, c_{t3}	chord of first, second, third element of slotted trailing-edge flap, see Sketch 4.1 for double-slotted flap, Sketch 4.2 for triple-slotted flap	m	ft
$\Delta c_{t1}, \Delta c_{t2}, \Delta c_{t3}$	increment in c_{t1}, c_{t2}, c_{t3} , see Sketch 4.1 for double-slotted flap, Sketch 4.2 for triple-slotted flap	m	ft
$c'_{t1}, c'_{t2}, c'_{t3}$	extended chord of first, second, third element of slotted trailing-edge flap, see Sketch 4.1 and Equations (4.8), (4.11) for double-slotted flap, Sketch 4.2 and Equations (4.8), (4.11) and (4.24) for triple-slotted flap	m	ft
Δc_l	chord extension due to deployment of leading-edge device, see Sketch 4.1 or 4.2	m	ft
F_R	factor for effect of Reynolds number on ΔC_{Lml} and ΔC_{Lmt} , see Equation (3.5)		
J_{t1}, J_{t2}, J_{t3}	correlation factor (efficiency factor) for first, second, third element of slotted trailing-edge flap, Figure 1, Figure 2, Figure 3		
K_l	correlation factor for effect of leading-edge device deflection, see Section 5		

K_T	correlation factor for effect of basic aerofoil geometry, Figure 7		
K_{t1}, K_{t2}, K_{t3}	correlation factor for effect of deflection for first, second, third element of slotted trailing-edge flap, Figure 8, Figure 9		
M	free-stream Mach number		
R_c	Reynolds number based on free-stream conditions and aerofoil chord c		
t	maximum thickness of aerofoil	m	ft
x	chordwise distance aft from basic aerofoil leading edge	m	ft
x_{ts}	chordwise location of flap-shroud trailing edge, see Sketch 4.1 or 4.2	m	ft
x_{um}	chordwise location of maximum upper-surface ordinate	m	ft
$z_{u1.25}$	upper-surface ordinate at $x = 0.0125c$ for basic aerofoil	m	ft
δ_l, δ_l°	deflection of leading-edge device, see Sketch 4.1 or 4.2	rad, deg	rad, deg
$\delta_{t1}, \delta_{t2}, \delta_{t3}$ $\delta_{t1}^\circ, \delta_{t2}^\circ, \delta_{t3}^\circ$	deflection of first, second, third element of slotted trailing-edge flap, positive trailing edge down, see Sketch 4.1 for double-slotted flap, Sketch 4.2 for triple-slotted flap	rad deg	rad deg
ρ_l	leading-edge radius of basic aerofoil, see Sketch 4.1 or 4.2	m	ft

Subscripts

$()_{expt}$	denotes experimental value
$()_{pred}$	denotes predicted value

2. INTRODUCTION

2.1 Scope of the Item

This Item provides semi-empirical methods for estimating the incremental effects on aerofoil lift at zero angle of attack and on maximum lift due to the deployment of a double-slotted or triple-slotted trailing-edge flap, with or without the deployment of a leading-edge device, at low speeds.

Section 3 summarises the equations relating the contributions to the total lift increments at zero angle of attack and at maximum lift arising from the deployment of leading-edge high-lift devices and trailing-edge flaps. Section 4 presents methods whereby the contributions from double-slotted and triple-slotted trailing-edge flaps are obtained. The contribution to the lift increment at zero angle of attack arising from a variety of leading-edge high-lift devices is obtainable from Item No. 94027 (Derivation 22). However, with regard to the lift increment at maximum lift, the situation is hampered by the lack of test data for other than slats in combination with slotted flaps. Nevertheless, Section 5 provides a guide to the use of the data in Item No. 94027 in the present context for the full range of leading-edge devices covered in that Item.

Section 6 addresses applicability and accuracy, Section 7 gives the Derivation and References and Section 8 presents three detailed examples illustrating the use of the methods.

2.2 Application of Data to Calculation of Total Lift Coefficient Values C_{L0} and C_{Lm}

In order to use the data obtained from the present Item in the wider context in which the total lift coefficient at zero angle of attack, C_{L0} , and at maximum lift, C_{Lm} , are required for an aerofoil with high-lift devices deployed, it is necessary to refer to Item No. 94026 (Reference 26). That Item acts as an introduction to, and a link between, each of the Items in the complete series dealing with the incremental effects of high-lift device deployment on aerofoil lift at zero angle of attack and on maximum lift. It describes how the incremental effects are summed and added to the contributions from the basic (*i.e.* plain) aerofoil to give the total values C_{L0} and C_{Lm} .

3. LIFT COEFFICIENT INCREMENTS ΔC_{L0} AND ΔC_{Lm}

The increments in the lift coefficient at zero angle of attack, ΔC_{L0} , and at maximum lift, ΔC_{Lm} , due to the deployment of a leading-edge high-lift device and a trailing-edge flap on an aerofoil are given by the sum of the individual increments, *i.e.*

$$\Delta C_{L0} = \Delta C_{L0l} + \Delta C_{L0t} \quad (3.1)$$

$$\text{and} \quad \Delta C_{Lm} = \Delta C_{Lml} + \Delta C_{Lmt} \quad (3.2)$$

The increments in Equations (3.1) and (3.2) are based on the chord, c , of the basic aerofoil.

The values of ΔC_{L0l} and ΔC_{Lml} for various leading-edge devices are obtainable from Item No. 94027. However, see Section 5 with regard to ΔC_{Lml} for use with the present Item.

For correlation purposes it is more convenient to present the right-hand sides of Equations (3.1) and (3.2) in terms of increments based on the extended chord c' . Also, whereas values of ΔC_{L0l} and ΔC_{L0t} can be taken to be independent of Reynolds number, the values of ΔC_{Lml} and ΔC_{Lmt} are influenced by Reynolds number. Thus, for a general trailing-edge flap, Item No. 94028 (Derivation 23) shows that

$$\Delta C_{L0t} = \frac{c'}{c} \Delta C'_{L0t} \quad (3.3)$$

and
$$\Delta C_{Lmt} = F_R \frac{c'}{c} \Delta C'_{Lmt} \quad (3.4)$$

where
$$F_R = 0.153 \log_{10} R_c \quad (3.5)$$

The value of $\Delta C'_{Lmt}$ in Equation (3.4) is therefore appropriate to the datum Reynolds number $R_c = 3.5 \times 10^6$, for which $F_R = 1$, (see Section 6.1).

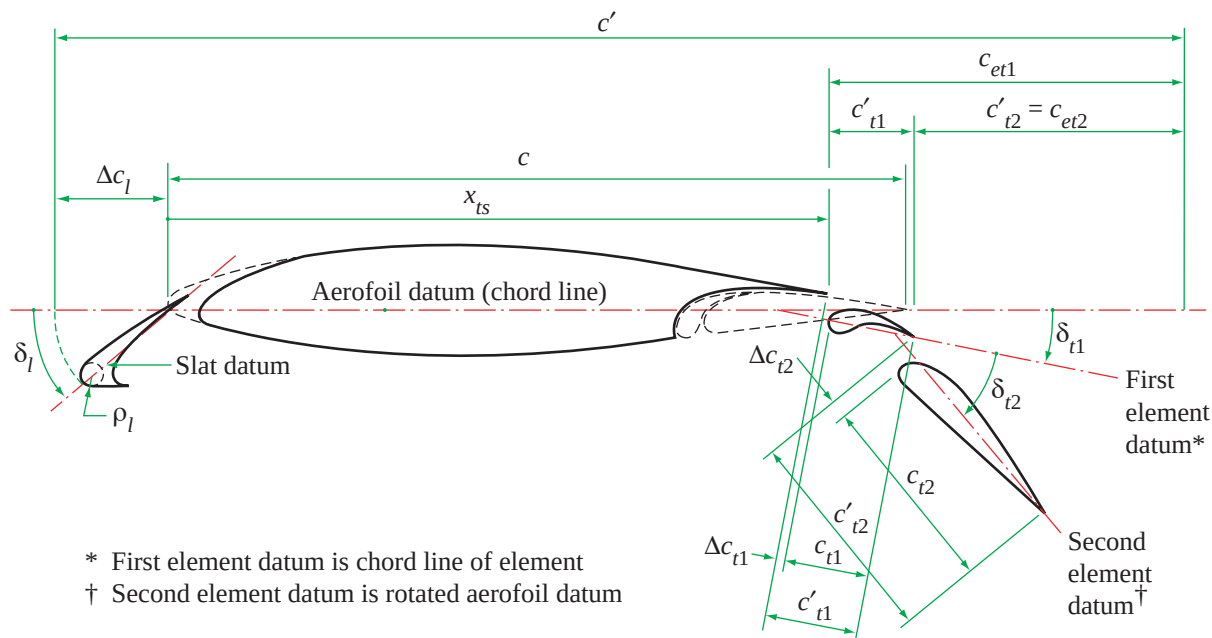
The values of $\Delta C'_{L0t}$ and $\Delta C'_{Lmt}$ in Equations (3.3) and (3.4) are determined for double-slotted and triple-slotted trailing-edge flaps by the methods of Sections 4.1 and 4.2 respectively.

4. LIFT COEFFICIENT INCREMENTS $\Delta C'_{L0t}$ AND $\Delta C'_{Lmt}$ FOR DOUBLE-SLOTTED AND TRIPLE-SLOTTED FLAPS

First approximations to the lift coefficient increments due to the deployment of trailing-edge flaps can be obtained from the theory for an equivalent thin hinged plate with empirical correlation factors to account for the geometry of practical airfoils and flaps. In the theory the lift increment due to a multi-element flap is treated as the sum of the contributions arising from an equivalent flap representing each flap element. To make some allowance for the effects of chord extension in the theory the flap chord ratios and the lift coefficient increments are based on the aerofoil extended chord. This approach was used in Derivation 16 and was the basis for the methods developed for plain trailing-edge flaps in Item No. 94028 which was used as a model for the methods given in the present Item for double-slotted and triple-slotted trailing-edge flaps, see Sections 4.1 and 4.2. However, some adjustments were required to adapt to the considerable departure from the thin hinged plate basis of the model when applied to a slotted flap, possibly involving large extensions in chord.

The presentation of the methods for triple-slotted flaps in Section 4.2 is given in summary only, since the methods of development are identical to those given in detail for double-slotted flaps in Section 4.1.

4.1 Double-slotted Flaps



Sketch 4.1 Double-slotted trailing-edge flap with typical leading-edge high-lift device (slat)

4.1.1 Increment in lift coefficient at zero angle of attack

The increment in lift coefficient at zero angle of attack due to the deployment of a double-slotted trailing-edge flap is given by

$$\Delta C'_{L0t} = \Delta C'_{L0t1} + \Delta C'_{L0t2}, \quad (4.1)$$

where $\Delta C'_{L0t1}$ and $\Delta C'_{L0t2}$ are the contributions arising from the first and second flap elements respectively. Item No. 94030 (Derivation 24) shows that

$$\Delta C'_{L0t1} = J_{t1} \Delta C'_{L1} (a_1)_0 / 2\pi, \quad (4.2)$$

and similarly, $\Delta C'_{L0t2}$ is given by

$$\Delta C'_{L0t2} = J_{t2} \Delta C'_{L2} (a_1)_0 / 2\pi. \quad (4.3)$$

Therefore, substitution of Equations (4.2) and (4.3) into Equation (4.1) gives

$$\Delta C'_{L0t} = [J_{t1} \Delta C'_{L1} + J_{t2} \Delta C'_{L2}] (a_1)_0 / 2\pi. \quad (4.4)$$

In this equation J_{t1} and J_{t2} are empirical correlation (or efficiency) factors which are functions of δ_{t1} and are given by

$$J_{t1} = 1.17[\sin(3.83\delta_{t1}^\circ)]^{1/2} \quad \text{for } 0 \leq \delta_{t1}^\circ \leq 23.5^\circ \quad (4.5)$$

$$\text{and} \quad J_{t1} = 1.17 \quad \text{for } \delta_{t1}^\circ > 23.5^\circ, \quad (4.6)$$

which are plotted in Figure 1, and

$$J_{t2} = 2.2 - 0.04|\delta_{t1}^\circ| \quad \text{for } -10^\circ \leq \delta_{t1}^\circ \leq 30^\circ \quad (4.7)$$

$$\text{and} \quad J_{t2} = 1.0 \quad \text{for } \delta_{t1}^\circ > 30^\circ, \quad (4.8)$$

which are plotted in Figure 2. Configurations with negative values of δ_{t1}° require special treatment, see Section 6.1.

The deflection of the first flap element, δ_{t1} , is seen from Sketch 4.1 to be the angle between the aerofoil datum (i.e. the aerofoil chord line) and the first element datum. The first element datum (as with the slat) is the chord line of the element, defined as the line passing through the centre of the leading-edge radius and the trailing edge. The deflection of the second flap element, δ_{t2} , is the angle between the first and second element datum lines, see Sketch 4.1. In contrast to the first flap element, the second flap element datum is the rotated aerofoil datum.

The incremental lift coefficients $\Delta C'_{L1}$ and $\Delta C'_{L2}$ are associated with the deployment of the equivalent first and second flap elements respectively on an aerofoil having a lift-curve slope of 2π . They are based on Item Nos Flaps 01.01.08 and Flaps 01.01.09 (Derivations 13 and 12 respectively) and are given in Figures 4 and 5 respectively as functions of δ_{t1}° and c_{et1}/c' for $\Delta C'_{L1}$ and δ_{t2}° and c_{et2}/c' for $\Delta C'_{L2}$.

The parameters $\Delta C'_{L1}$ and $\Delta C'_{L2}$ which are based on test data, replace the value derived from thin plate theory used for plain flaps in Item No. 94028. This was necessary because slot effects are not represented in the simple theory.

The aerofoil extended chord, c' , shown in Sketch 4.1 is

$$c' = \Delta c_l + x_{ts} + c'_{t1} + c'_{t2}. \quad (4.9)$$

The parameter Δc_l the chord extension due to the deployment of the leading-edge device, obtained for a variety of such devices from Item No. 94027. The quantity x_{ts} is the chordwise location of the trailing edge of the flap shroud and c'_{t1} and c'_{t2} are the extended chord lengths of the first and second flap elements, given by

$$c'_{t1} = c_{t1} + \Delta c_{t1} \quad (4.10)$$

$$\text{and} \quad c'_{t2} = c_{t2} + \Delta c_{t2}. \quad (4.11)$$

In Sketch 4.1 Δc_{t1} and Δc_{t2} are drawn as having magnitudes sufficiently small to neglect. For those cases in which Δc_{t1} and Δc_{t2} are not small, graphical estimation would be necessary.

From Sketch 4.1 it is seen that, as used in Derivation 16, the chord lengths of the equivalent flaps representing the first and second flap elements, and required in Figures 4 and 5, are

$$c_{et1} = c'_{t1} + c'_{t2} \quad (4.12)$$

$$= c_{t1} + \Delta c_{t1} + c_{t2} + \Delta c_{t2}, \quad (4.13)$$

from Equations (4.10) and (4.11), and

$$c_{et2} = c'_{t2} \quad (4.14)$$

$$= c_{t2} + \Delta c_{t2}. \quad (4.15)$$

The parameter $(a_l)_0$ in Equation (4.4) is the basic aerofoil lift-curve slope in incompressible flow from Item No. Wings 01.01.05 (Derivation 15).

The value of $\Delta C'_{L0t}$ is then used in Equation (3.3) to determine ΔC_{L0t} .

4.1.2 Increment in maximum lift coefficient

In the development of the method for predicting the increment in maximum lift coefficient it was found necessary to adapt further the thin hinged plate theoretical model to cater for slotted flaps, especially those involving large chord extensions. With multi-slotted flaps it is possible to obtain significant increments in maximum lift (with a low drag penalty) by deployment as effectively a single-element flap without deflection. The increment in maximum lift coefficient is then very largely due to chord extension and is related to the basic aerofoil maximum lift coefficient. The increment in maximum lift coefficient can then be considered to consist of two components, one, $(\Delta C'_{Lmt})_0$, independent of flap deflection and the other, $(\Delta C'_{Lmt})_1$, due to the deflection.

Thus

$$\Delta C'_{Lmt} = (\Delta C'_{Lmt})_0 + (\Delta C'_{Lmt})_1, \quad (4.16)$$

where

$$(\Delta C'_{Lmt})_0 = (1 - c/c')(C_{LmB})_d \quad (4.17)$$

and

$$(\Delta C'_{Lmt})_1 = K_T K_{t1} J_{t1} \Delta C'_{L1} + K_T K_{t2} J_{t2} \Delta C'_{L2} - (1 - c/c') \sin \delta_{t1} (C_{LmB})_d, \quad (4.18)$$

so that

$$\Delta C'_{Lmt} = (1 - c/c')(1 - \sin \delta_{t1})(C_{LmB})_d + K_T K_{t1} J_{t1} \Delta C'_{L1} + K_T K_{t2} J_{t2} \Delta C'_{L2}. \quad (4.19)$$

In Equation (4.19) $(C_{LmB})_d$ is the maximum lift coefficient for the basic aerofoil at the datum Reynolds number $R_c = 3.5 \times 10^6$, obtained from Item No. 84026 (Derivation 21).

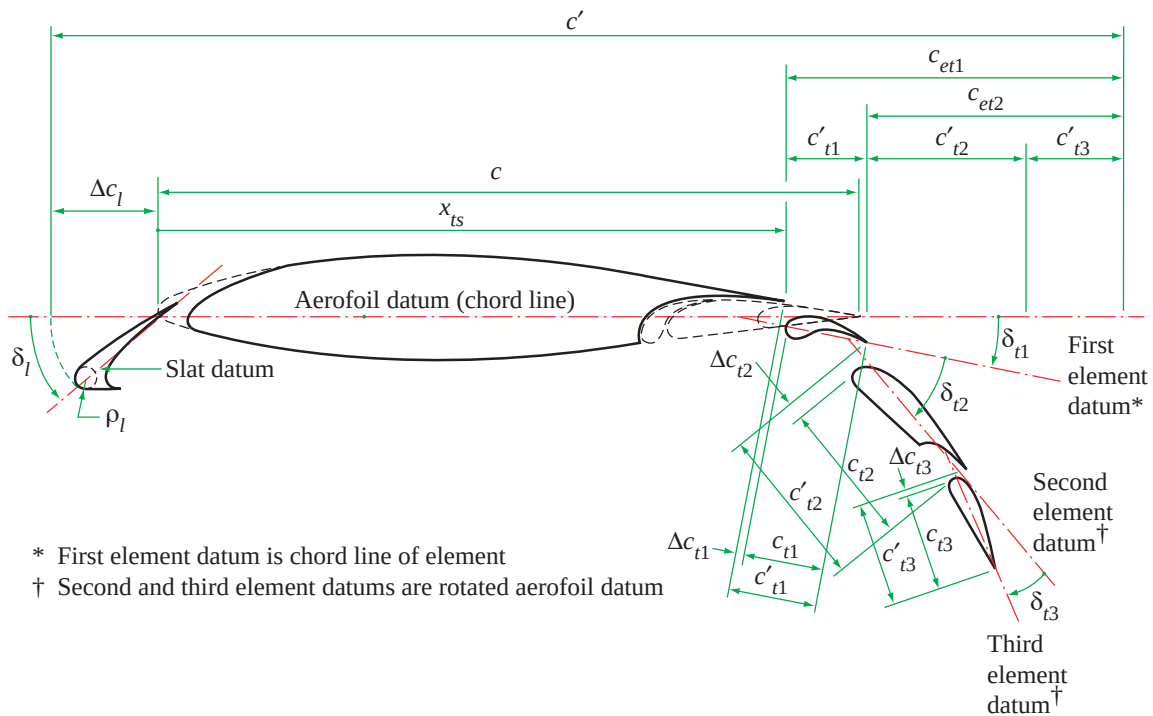
The extended chord, c' , is given by Equations (4.9) to (4.11), J_{t1} and J_{t2} are obtained either from Figures 1 and 2 or from Equations (4.5) to (4.8) and $\Delta C'_{L1}$ and $\Delta C'_{L2}$ are obtained from Figures 4 and 5.

The correlation factor K_T , given in Figure 7, allows for the effect of basic aerofoil geometry via $z_{u1.25}/c$, the dimensionless upper-surface ordinate at $0.0125c$, and x_{um}/c , the dimensionless chordwise location of the maximum upper-surface ordinate. Values of $z_{u1.25}/c$ for a range of NACA sections are given in Item No. 66034 (Reference 25). The correlation factors K_{t1} and K_{t2} are given as functions of δ_{t1}° in Figures 8 and 9 respectively.

Finally, with the value of $\Delta C'_{Lmt}$ obtained from Equation (4.19), ΔC_{Lmt} is evaluated from Equation (3.4) with F_R given by Equation (3.5).

4.2 Triple-slotted Flaps

The development of the methods for triple-slotted flaps follows very closely that for double-slotted flaps given in Section 4.1 and so the presentation will be given in an abbreviated form.



Sketch 4.2 Triple-slotted trailing-edge flap with typical leading-edge high-lift device (slat)

4.2.1 Increment in lift coefficient at zero angle of attack

The increment in lift coefficient at zero angle of attack due to the deployment of a triple-slotted trailing-edge flap is given by

$$\Delta C'_{L0t} = [J_{t1} \Delta C'_{L1} + J_{t2} \Delta C'_{L2} + J_{t3} \Delta C'_{L3}] (a_1)_0 / 2\pi \quad (4.20)$$

In this Equation J_{t1} and J_{t2} are obtained either from Figures 1 and 2 or from Equations (4.5) to (4.8) and J_{t3} is obtained either from Figure 3 or the following equations.