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**Dimensions, deflections and stresses for
Hertzian contacts under combined
normal and tangential loading
(Guide to use of computer program
A9434)**

Associated software: ESDUpac A9434

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The Tribology Steering Group, which first met in September 1973, is continuing the work that was previously the responsibility of the former Bearing and Lubrication Committee, which first met in July 1964.

DIMENSIONS, DEFLECTIONS AND STRESSES FOR HERTZIAN CONTACTS UNDER COMBINED NORMAL AND TANGENTIAL LOADING. (GUIDE TO USE OF COMPUTER PROGRAM A9434)

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DIMENSIONS, DEFLECTIONS AND STRESSES FOR HERTZIAN CONTACTS UNDER COMBINED NORMAL AND TANGENTIAL LOADING. (GUIDE TO USE OF COMPUTER PROGRAM A9434)

1. INTRODUCTION

ESDUpac A9434 provides a FORTRAN 77 computer program for calculating the stresses in the concentrated point contacts that occur, for example, in rolling element bearings, gears and cams. Both normal and combined normal and tangential loading are allowed.

The program determines the contact dimensions, orientation and gross deflection (normal closure) from specified radii and other properties of the two contacting bodies. If, however, the contact dimensions are already known from a separate analysis these can be input directly into the program.

The program locates the critical stress points with respect to three criteria which consider yield failure, brittle failure and fatigue failure. The program can also produce stress distribution tables suitable for plotting. The stress distributions can be constrained to a cross-section through the contact region or to a line. Stress distributions around the perimeter of the contact area can also be produced.

The program implements the calculation methods of ESDU 78035 entitled "Contact phenomena. I: stresses, deflections and contact dimensions for normally-loaded unlubricated elastic components", ESDU 84017 entitled "Contact phenomena. II: stress fields and failure criteria in concentrated elastic contacts under combined normal and tangential loading" and ESDU 85007 entitled "Contact phenomena. III: calculation of individual stress components in concentrated elastic contacts under combined normal and tangential loading". The usual limitations and assumptions of Hertzian theory apply. In particular, the contacting surfaces are assumed to be topographically smooth and unlubricated but the results can be applied with sufficient accuracy to the majority of practical elastohydrodynamically lubricated contacts. In order to use the program effectively, ESDU 78035, ESDU 84017 and ESDU 85007 must be referred to for the theoretical limitations of the method and for discussion of practical considerations.

2. NOTATION AND UNITS

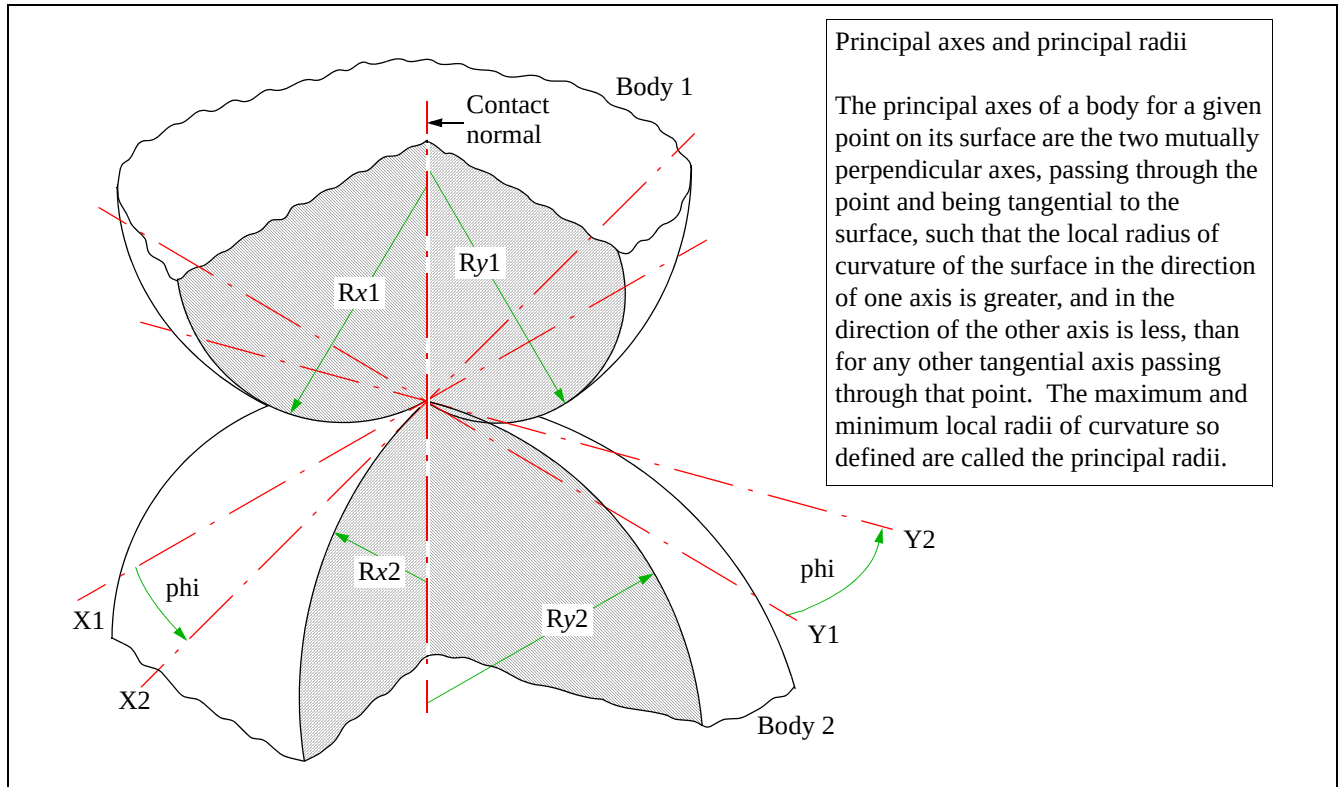
The symbols (variable names) used in the associated data files for ESDUpac A9434 (see Section 4) differ from those used in ESDU 78035, ESDU 84017 and ESDU 85007. Both sets of symbols are listed in this Notation. Note also that the International System of units is used.

Symbol used by ESDUpac A9434	Symbol used by ESDU 78035, ESDU 84017 and ESDU 85007	Description	Units
a	a	major semi-axis of contact ellipse, measured in x direction (see Sketch 2.4)	m
b	b	minor semi-axis of contact ellipse, measured in y direction (see Sketch 2.4)	m

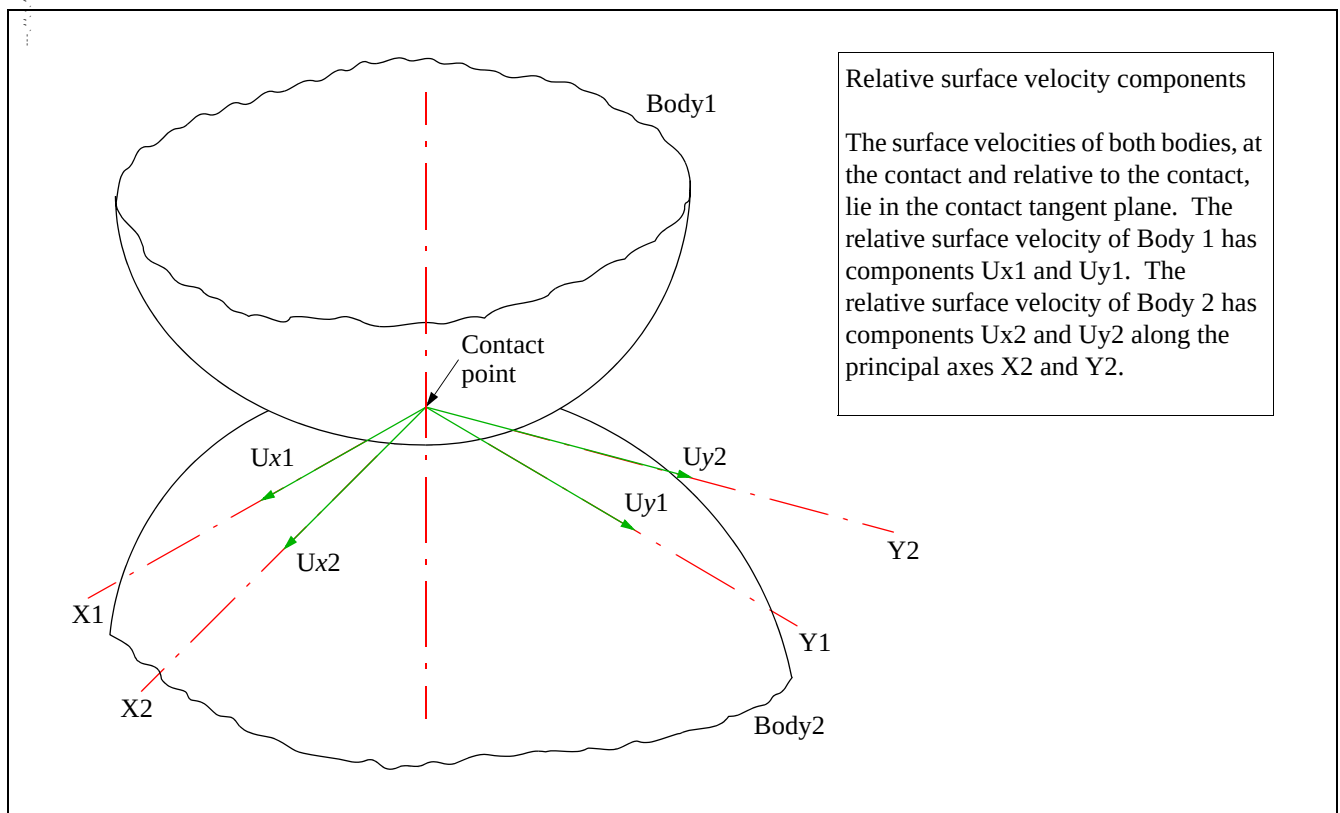
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beta	β	axis ratio of contact ellipse, b/a	–
chi	–	direction of contact motion used in calculations	degree
chi1, chi2	–	direction of contact motion on Bodies 1 and 2 (see Sketch 2.5)	degree
d	–	horizontal offset of a trajectory for orthogonal shear stress change (see Sketch 2.8)	m
delta	δ	normal approach of contacting bodies defined as decrease in separation between two points on axis of symmetry, one in each body, remote from compressed region	m
dQo	Δq_{orth}	range in orthogonal shear for a given element of material as it passes under the contact	N/m ²
dQoMax	$(\Delta q_{orth})_{max}$	maximum range in orthogonal shear stress	N/m ²
E1, E2	E	elastic modulus for Body 1 and 2	N/m ²
fx, fy, fz	f_x, f_y, f_z	direct stress components of stress tensor acting in directions parallel to, and over planes perpendicular to, x-, y- and z- axes respectively (positive direct stress in tensile)	N/m ²
gama	γ	direction of friction used in calculations	degree
gama1, gama2	–	direction of friction on Bodies 1 and 2 (see Sketch 2.6)	degree
nGrid	–	number of steps for stress distributions (see Section 4.5)	–
P	P	normal load applied to contact	N
Po	P_0	maximum Hertzian pressure	N/m ²
phi	ϕ	angle of skew of principal axes measured positive anticlockwise from x1-axis of Body 1 to the x2-axis of Body 2 (see Sketch 2.1)	degree
psi	–	orientation angle of contact ellipse measured positive from x1-axis of Body 1 to y-axis of the contact ellipse (see Sketch 2.3)	degree
Q	Q	resulting tangential load	N
Qe	q_e	equivalent shear stress defined as square-root of second invariant of stress deviator tensor	N/m ²
QeMax	$(q_e)_{max}$	maximum equivalent shear stress	N/m ²
Qo	q_{orth}	orthogonal shear stress defined as stress acting on planes parallel and perpendicular to surface of contact	N/m ²

Q_{xy}, Q_{yz}, Q_{zx}	q_{xy}, q_{yz}, q_{zx}	shear stress components of stress tensor where first subscript gives direction of normal to plane in which stress acts, and second subscript gives direction of stress	N/m^2
r_{μ}	μ	coefficient of kinetic friction	–
R_{x1}, R_{y1}	R_{11}, R_{12}	principal radii at contact point for Body 1 (see Sketch 2.1)	m
R_{x2}, R_{y2}	R_{21}, R_{22}	principal radii at contact point for Body 2 (see Sketch 2.1)	m
sigma	σ	Poisson's ratio used in calculations	–
sigma1, sigma2	–	Poisson's ratio for Bodies 1 and 2	–
theta	θ	angle of radius vector to point on perimeter of contact area measured positive anticlockwise from x-axis (see Sketch 2.7)	degree
T_{max}	T_{max}	maximum tensile stress on perimeter of contact ellipse	N/m^2
U_{x1}, U_{y1}	–	surface velocity components relative to contact point for Body 1 (see Sketch 2.2)	m/s
U_{x2}, U_{y2}	–	surface velocity components relative to contact point for Body 2 (see Sketch 2.2)	m/s
$X1, Y1$	–	principal axes of Body 1 (see Sketch 2.1)	–
$X2, Y2$	–	principal axes of Body 2 (see Sketch 2.1)	–
x, y	x, y	axes of contact ellipse (see Sketch 2.3)	m
z	z	distance measured normal to surface and directed towards interior of body	m
$x_{Bot}, y_{Bot}, z_{Bot}$	–	lower extents for stress distributions (see Section 4.5)	m
$x_{Top}, y_{Top}, z_{Top}$	–	upper extents for stress distributions (see Section 4.5)	m

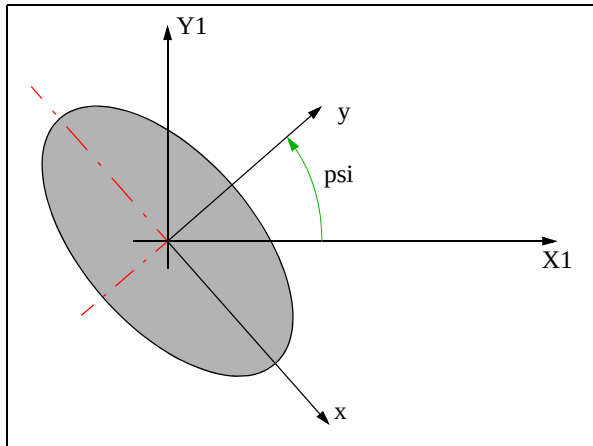


Sketch 2.1 Contacting bodies

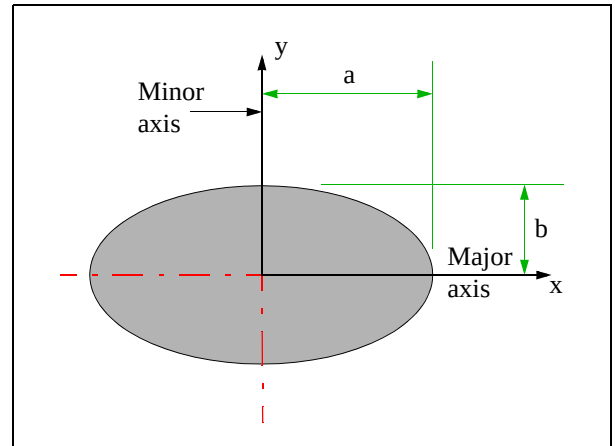


Sketch 2.2 Relative surface velocities

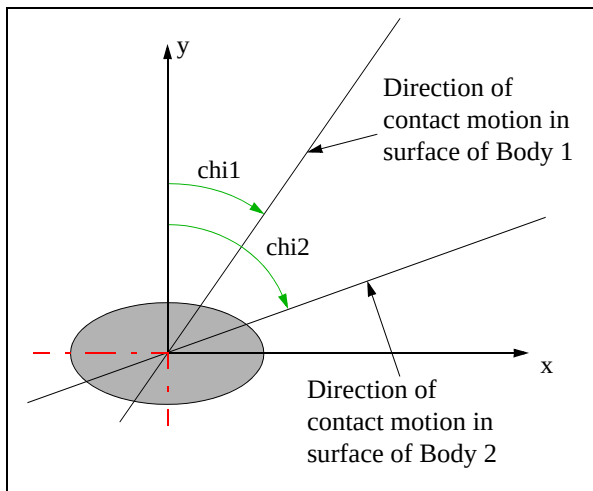
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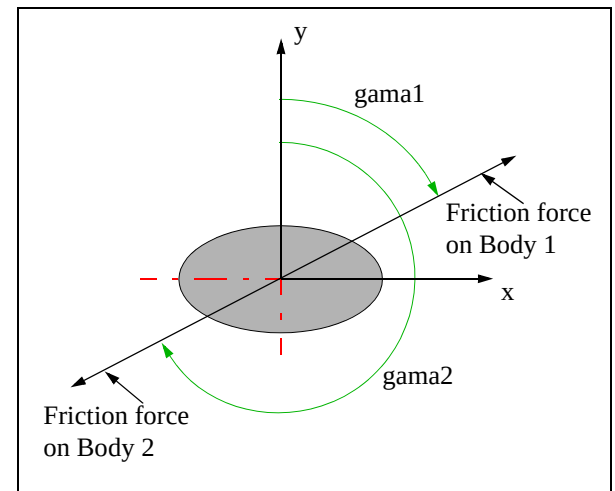
Sketch 2.3 Orientation of contact ellipse



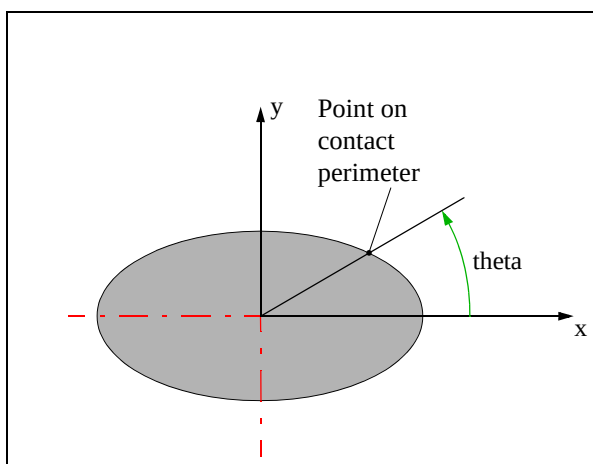
Sketch 2.4 Dimensions of contact ellipse



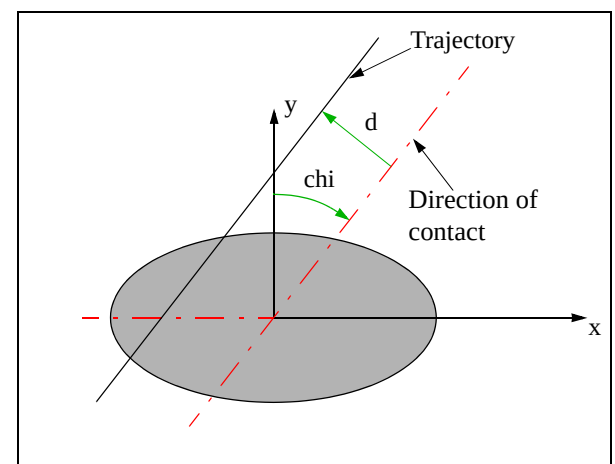
Sketch 2.5 Directions of contact motion



Sketch 2.6 Directions of friction



Sketch 2.7 Definition of theta



Sketch 2.8 Definition of d

3. RUNNING THE PROGRAM

The supplied executable version of the ESDUpac program is suitable for Microsoft Windows operating systems.

To run the executable version of the program from the keyboard, type the program name[†], A9434V10, at the operating system prompt and press return. The program will prompt for the name of the input data file and the name of the output data file. Alternatively, the name of the input data file and the name of the output data file can be specified on the command line using the operating system redirection symbols as in,

```
A9434V10 < EXP1.IN > EXP1.OUT
```

If a problem occurs in reading the data from the input file or if the data are invalid in any way, error messages are sent to the output file. These error messages describe the error and indicate the number of the line at which the error occurred in the input data file.

A number of example input data files are supplied with the program (files with the extension “.IN”). The corresponding output data files (with the extension “.OUT”) can be examined using a text editor.

[†] The program name used here is that of Version 1.0 (V10) which may not be the latest.

4. THE INPUT DATA FILE

Input data files are prepared using a text editor and, in most cases, new input files will be produced by editing existing input files. An example input file is shown here.

```
ESDU A9434V10
* A normally loaded elliptical contact of known dimensions.
a      semi-major axis of contact ellipse      = 0.4 E-3  m
b      semi-minor axis of contact ellipse      = 0.2 E-3  m
P      normal load on contact                  = 280    N
```

The following points should be noted:

- (i) All lines beginning with an asterisk, '*', are comment lines. Such comment lines may be freely inserted for documentation purposes.
- (ii) The first non-comment line in the input file specifies the name of the ESDUpac program that is to be used for the analysis. This line must be present in all input data files.
- (iii) Each input parameter has an identifying symbol. The explanatory text following the parameter symbol may be omitted in which case the '=' sign is not strictly necessary.
- (iv) Units have been stated in the file purely for documentation purposes. All text following the actual numerical value specified is ignored by the program.

In the above example input file the dimensions of the contact ellipse have been specified directly. In most situations, however, these dimensions will not be known. Also, no tangential loading has been specified. The following example input file includes a more realistic problem specification in which the geometry of the contacting bodies has been specified so that ESDUpac A9434 can determine the contact dimensions. The surface velocities of the contacting bodies have also been specified so that ESDUpac A9434 can determine the direction of friction and the direction of contact motion.

```
ESDU A9434V10
* Contacting cylinders with skewed axes
* ~~~~~
* Based on Example 3 of 89045 with surface velocities specified

Rx1 = 0.02    m
Rx2 = 0.02    m
Ry1 = 00
Ry2 = 00

phi = 45      degree

E1 = 200E9    N/m^2
E2 = 200E9    N/m^2

P   = 280     N

Ux1 = -9.05   m/s
Uy1 = 0.0
Ux2 = 0.0
Uy2 = 12.8

rMu = 0.1

$CASE
* Higher friction case

rMu = 0.2
```

Note that an additional case, that for a friction coefficient of 0.2, has been appended to the initial data. Whenever stresses are to be determined over a range of conditions or dimensions, further cases can be appended to the initial data with the key-word '\$CASE'. Any number of additional cases can be appended and the key-word '\$CASE' must be used for each one. A single parameter, or several parameters, may be specified for each additional case and the values of the specified parameters overwrite the values existing for the previous case.

Input parameters can be specified in any order. Input parameters are specified with the symbol of the parameter followed by the value (with optional comment text). Note that not all input parameters need be specified for any one analysis. Indeed, mutually exclusive means of specifying certain aspects of the contact are provided. An input parameter may be left unspecified in one of three ways.

- (i) Put a question mark, '?', in place of the parameter value.
- (ii) Comment-out the parameter by placing an asterisk, '*', at the start of the line.
- (iii) Omit the parameter specification entirely.

The input parameters for this ESDUpac program fall naturally into several groups as described in the following sections.

4.1 Contact Dimensions

Sufficient information must be provided to enable the dimensions of the contact ellipse to be established. These dimensions can be specified directly or the geometry of the contacting bodies can be specified in which case ESDUpac A9434 determines the contact dimensions.

To specify the contact dimensions directly simply specify the values of the semi-major and semi-minor axes as illustrated here.

a	semi-major axis of contact ellipse	=	0.647 E-3	m
b	semi-minor axis of contact ellipse	=	0.203 E-3	m

To specify the contact geometry, the principal radii of the contacting bodies, the angle between the principal axes and the elastic moduli for both bodies must be specified as illustrated by the input file fragment shown here.

Rx1	principal radii of curvature of body 1	=	0.02	m
Ry1	" " " " " "	=	00	
Rx2	principal radii of curvature of body 2	=	0.02	m
Ry2	" " " " " "	=	00	
phi	angle of skew of principal axes	=	45	degree
E1	elastic modulus of material of body 1	=	200 E9	N/m^2
E2	elastic modulus of material of body 2	=	200 E9	N/m^2

Note that radii of curvature can be positive (for convex surfaces), negative (for concave surfaces) or infinite (for "flat" surfaces). Infinite radii are specified by two zeros as shown in the above example. Note also that by convention $-90^\circ < \phi < 90^\circ$.

4.2 Loading Conditions

It is mandatory that the normal load on the contact be specified.

P	normal load on contact	=	280	N
---	------------------------	---	-----	---

ESDUpac A9434, however, allows combined normal and tangential loading to be applied to the contact. Tangential loading can be specified in one of two ways. If already known, the direction of sliding relative to the contact ellipse can be specified directly; otherwise the surface velocities of the contacting bodies can be specified. In both cases the coefficient of kinetic friction must also be specified.

To specify the direction of sliding directly simply specify the angle of sliding and the coefficient of kinetic friction as illustrated here.

P	normal load on contact	=	280	N
gama1	direction of friction on body 1	=	-60	degree
rMu	coefficient of kinetic friction	=	0.15	

The direction of friction on Body 2 is, of course, opposite to that on Body 1. Instead of specifying gama1 = -60° the direction of friction could equally be specified as gama2 = 120°. Note that by convention -180° < gama1 < 180° and -180° < gama2 < 180°.

To specify the surface velocities, the surface velocity components of both bodies relative to the contact must be specified as illustrated by the input file fragment shown here.

P	normal load on contact	=	280	N
Ux1	surface velocity component of body 1	=	-9.05	m/s
Uy1	" " " " " "	=	0.0	m/s
Ux2	surface velocity component of body 2	=	0.0	m/s
Uy2	" " " " " "	=	12.8	m/s
rMu	coefficient of kinetic friction	=	0.15	

Note that if the surface velocities are to be specified then the geometry of the contacting bodies must also have been specified. It is not possible to specify the dimensions of the contact ellipse directly and to then specify surface velocities.

If the variation of the orthogonal shear stress, which is associated with sub-surface initiated fatigue (see Derivation 2), is to be investigated then the direction of motion of the contact relative to the surfaces of the bodies must be known. This direction is not necessarily coincident with the direction of friction but if the surface velocity components have been specified, as illustrated above, ESDUpac A9434 will determine the contact motion direction. If the surface velocity components have not been specified but it is still required to investigate the variation of orthogonal shear stress, the direction of contact motion must be specified directly as illustrated by the following input file fragment.

P	normal load on contact	=	280	N
chi1	direction of contact motion on body 1	=	22.5	degree

There is no simple relationship between the directions of contact motion on the two bodies as there is with the direction of friction and so if the stresses in Body 2 are to be investigated, the value of chi2 should be specified. By convention -90° < chi1 < 90° and -90° < chi2 < 90°.

4.3 Other Parameters

By default, ESDUpac A9434 determines the stresses for Body 1. When the material properties of both bodies are identical the stress field in one body can be obtained by an appropriate reflection of the field in the other. However, there might be occasions where the material properties of the two bodies are very different or where it is simply more convenient to visualise the stresses in Body 2.

To obtain the contact stresses in Body 2 instead of Body 1, include the following specification in the input file.

body	=	2
------	---	---

Note that the direction of friction and the direction of contact motion (see "Loading conditions" above) can be specified relative to Body 1 (gama1 and chi1) and/or relative to Body 2 (gama2 and chi2). There is, of course, a relationship between the directions and ESDUpac A9434 checks for inconsistent specifications.

For many contact situations, especially where any tangential loading is small, stress fields tend not to be sensitive to Poisson's ratio. Indeed, the orthogonal shear stress is independent of Poisson's ratio for all contacts. In any case, most materials have a value close to 0.3 and this is the value assumed by ESDUpac A9434 unless an alternative value has been specified. The following example specification sets the Poisson's ratio for body 1 to an alternative value.

sigma1	Poisson's ratio for body 1	=	0.4
--------	----------------------------	---	-----

4.4 Constraints

By default, ESDUpac A9434 performs a global search to locate the critical stress points wherever they might occur. However, constraints can be placed on the search carried out to locate these critical stress points. Constraints are applied by specifying one or more of the ordinates x , y and z .

To constrain the search for critical stress points to a plane, specify a value for one of the ordinates x , y or z . The specified ordinate will then be held constant during the search. For example, if it is required to locate the critical stress points at a depth of 0.0001 m below the surface, the following specification would be added to the input file.

z	depth below surface of body	=	0.0001	m
---	-----------------------------	---	--------	---

To constrain the search for critical stress points to a line, specify a value for two of the ordinates x , y and z . Again, the specified ordinates will be held constant during the search. For example, if it is required to locate critical stress points on the surface of the contacting bodies along the minor axis of the contact ellipse, the following specifications would be included in the input file.

x	distance along major axis of contact ellipse	=	0.0001	m
z	depth below surface of body	=	0.0	m

If all three ordinates are specified, ESDUpac A9434 determines the stress at the specified single point. The following example input file fragment specifies a point located at a depth of 0.0001 m directly below the centre of the contact.

x	distance along major axis of contact ellipse	=	0.0	m
y	distance along minor axis of contact ellipse	=	0.0	m
z	depth below surface of body	=	0.0001	m

4.5 Stress Distributions

ESDUpac A9434 will, if requested, produce a stress distributions file containing tables of stress values that can be plotted. If a stress distributions file is required simply specify the name of the file in the input file. For example, to obtain a stress distributions file named "STRESS.DAT", the following specification would be added to the input file (the quotes around the name are optional).

datOut	name of stress distributions file	= 'STRESS.DAT'
--------	-----------------------------------	----------------

The type of stress distributions contained in the file depends on the constraints that have been placed on the calculations. Briefly, if no constraints have been placed on the calculations (*i.e.* none of the ordinates x , y or z has been assigned values in the input file), the stress distributions file will contain values of surface stress around the perimeter of the contact and values of the maximum stresses at various depths. If a single constraint has been placed on the calculations (*i.e.* one of the ordinates x , y or z has been assigned a value in the input file), the stress distributions file will contain stress values over a cross-section. If values have been assigned to two of the ordinates x , y and z , the stress distributions file will contain stress values along a line. No stress distributions are produced if all three ordinates, x , y and z have been assigned values. The contents of the stress distributions file are described in more detail in Section 6 and in Tables 5.1, 6.1 and 6.2.

The extent of the stress distributions generated can be controlled by the parameters $xBot$, $xTop$, $yBot$, $yTop$, $zBot$ and $zTop$ although normally these parameters will not need to be specified. By default all stress distributions (except those around the perimeter of the contact area) are confined to a cuboidal volume extending 1.5 times the contact ellipse axis dimensions in the horizontal (x and y) directions and twice the contact ellipse semi-minor axis dimension into the depth of the body. This volume is sufficiently large to encompass all positions of interest in the majority of situations. If required, however, the extent and position of this volume can be changed as can the number of stress values that are calculated for the stress distributions. The following example specifications, added to the input file, redefine the horizontal (x and y) extent of the volume.

$xBot$	lower limit on x for stress distributions	=	0.001	m
$xTop$	upper limit on x for stress distributions	=	0.0015	m
$yBot$	lower limit on y for stress distributions	=	0.0	m
$yTop$	upper limit on y for stress distributions	=	0.0002	m

If required, the upper and lower limits in the z direction, $zBot$ and $zTop$, could be similarly redefined. Redefining the extent of the volume in this manner might be used to examine in more detail selected regions after first obtaining stress distributions for the original default volume. In this way the dimensions of the contact area will have been established before the region of interest is specified.

By default, all intervals over which the stress distributions are obtained are divided into 30 equal sized steps. Thus, for a distribution over a cross-section, stress values would be calculated at 31 by 31 points. To increase the number of steps to 50, say, include the following specification in the input file.

nGrid	number of steps for stress distributions	=	50
-------	--	---	----

Note that increasing the number of steps, $nGrid$, will result in increased program running time and a larger stress distributions file (see Tables 5.1, 6.1 and 6.2). The default value of 30 is likely to be sufficient when it is required to plot the variation of stress with a single ordinate only. The value might need to be increased when it is intended to plot contours, for example, of a particular stress component over a cross-section. Note that the size of the stress distributions file, and the running time of the program, can be reduced by selecting only those stress components which are of interest. For example, to select just the direct stress component, f_z , and the equivalent stress, Q_e , the following specification would be added to the input file.

distr	=	f_z Q_e
-------	---	-------------

Without such a specification in the input file, ESDUpac A9434 includes all stress components in any stress distributions file produced.

4.6 Summary of Input Parameters

The following table, arranged in the format of an input data file, includes comments on the purpose of all input parameters.

```

ESDU A9434V10
* Contact dimensions:
* ~~~~~
a      major semi-axis of contact ellipse      = 0.647 E-3 m
*      This dimension would be specified if the contact dimensions had
*      already been determined. Leave unspecified if principal radii
*      of contacting bodies are to be specified.
b      minor semi-axis of contact ellipse      = 0.203 E-3 m
*      This dimension would be specified if the contact dimensions had
*      already been determined. Leave unspecified if principal radii
*      of contacting bodies are to be specified.
Rx1    principal radius at contact point for Body 1 = 0.02      m
*      If the contact dimensions are unknown specify, instead, the
*      principal radii of both bodies. Radii are positive for convex
*      surfaces and negative for concave surfaces. For infinite radii
*      ("flat" surfaces) specify 00.
Ry1    principal radius at contact point for Body 1 = 00
*      If the contact dimensions are unknown specify, instead, the
*      principal radii of both bodies. Radii are positive for convex
*      surfaces and negative for concave surfaces. For infinite radii
*      ("flat" surfaces) specify 00.
Rx2    principal radius at contact point for Body 2 = 0.02      m
*      If the contact dimensions are unknown specify, instead, the
*      principal radii of both bodies. Radii are positive for convex
*      surfaces and negative for concave surfaces. For infinite radii
*      ("flat" surfaces) specify 00.
Ry2    principal radius at contact point for Body 2 = 00
*      If the contact dimensions are unknown specify, instead, the
*      principal radii of both bodies. Radii are positive for convex
*      surfaces and negative for concave surfaces. For infinite radii
*      ("flat" surfaces) specify 00.
phi    angle of skew of principal axes          = 45      degree
*      This angle must be specified (in degree) if the principal radii
*      of curvature have been specified. The angle must be between
*      -90 and +90 degree.
E1     elastic modulus for material of Body 1    = 200 E9      N/m^2
*      The elastic modulii for the materials of both bodies must be
*      specified if the principal radii of curvature have been specified
E2     elastic modulus for material of Body 2    = 200 E9      N/m^2
*      The elastic modulii for the materials of both bodies must be
*      specified if the principal radii of curvature have been specified
* Loading conditions:
* ~~~~~
P      normal load on contact                    = 280      N
*      The normal load applied to the contact must be specified.
gama1  angle of friction on Body 1              = -60      degree
*      This angle would be specified if tangential loading is present
*      and the direction of sliding has been determined. Leave un-
*      specified if only normal loading is present or if the surface
*      velocities of the contacting bodies are to be specified. The
*      angle must be between -180 and +180 degree.

```

(continued)

```

gama2  angle of friction on Body 2          = 120          degree
*      This angle would be specified if tangential loading is present
*      and the direction of sliding has been determined. Leave un-
*      specified if only normal loading is present or if the surface
*      velocities of the contacting bodies are to be specified. The
*      angle must be between -180 and +180 degree.

chi1    angle of contact motion for Body 1    = 22.5          degree
*      This angle would be specified if variation of the orthogonal
*      shear stress is to be investigated and if the direction of
*      contact motion has been determined. Leave unspecified if the
*      variation in orthogonal shear stress is not required or if the
*      surface velocities of the contacting bodies are to be specified.
*      The angle must be between -90 and +90 degree.

chi2    angle of contact motion for Body 2    = 22.5          degree
*      This angle would be specified if variation of the orthogonal
*      shear stress is to be investigated and if the direction of
*      contact motion has been determined. Leave unspecified if the
*      variation in orthogonal shear stress is not required or if the
*      surface velocities of the contacting bodies are to be specified.
*      The angle must be between -90 and +90 degree.

Ux1     surface velocity component of Body 1  = -9.05          m/s
*      If the direction of friction and/or the direction of contact
*      motion is unknown specify, instead, the surface velocity
*      components for both bodies relative to the contact point.

Uy1     surface velocity component of Body 1  = 0.0           m/s
*      If the direction of friction and/or the direction of contact
*      motion is unknown specify, instead, the surface velocity
*      components for both bodies relative to the contact point.

Ux2     surface velocity component of Body 2  = 0.0           m/s
*      If the direction of friction and/or the direction of contact
*      motion is unknown specify, instead, the surface velocity
*      components for both bodies relative to the contact point.

Uy2     surface velocity component of Body 2  = 12.8          m/s
*      If the direction of friction and/or the direction of contact
*      motion is unknown specify, instead, the surface velocity
*      components for both bodies relative to the contact point.

rMu     coefficient of kinetic friction        = 0.15
*      This coefficient must be specified if either the direction of
*      friction or the surface velocity components have been specified.

* Other parameters:
* ~~~~~

body     body for which stresses will be calculated = 2
*      Specify 2 if stresses for Body 2 are required. Specify 1, or
*      leave unspecified if stresses for Body 1 are required.

sigma1   Poisson's ratio for material of Body 1 = 0.4
*      Specify a value for Poisson's ratio to override the default value
*      of 0.3.

sigma2   Poisson's ratio for material of Body 2 = 0.2
*      Specify a value for Poisson's ratio to override the default value
*      of 0.3.

* Constraints:
* ~~~~~

x        position along contact ellipse major axis = 0.0001    m
*      Specify a value for this parameter if x is to be held constant
*      while searching for critical stress points. If the stress at a
*      single point is required then all three ordinates x, y and z
*      would be specified. Leave unspecified if no constraint on x
*      is required.

```

(continued)

```

y      position along contact ellipse minor axis = 0.0001      m
*      Specify a value for this parameter if y is to be held constant
*      while searching for critical stress points. If the stress at a
*      single point is required then all three ordinates x, y and z
*      would be specified. Leave unspecified if no constraint on y
*      is required.

Z      depth below the surface of the body      = 0.0001      m
*      Specify a value for this parameter if z is to be held constant
*      while searching for critical stress points. If the stress at a
*      single point is required then all three ordinates x, y and z
*      would be specified. Leave unspecified if no constraint on z
*      is required.

* Stress distributions file:
* ~~~~~

datOut name of stress distributions file      = 'STRESS.DAT'
*      If stress distributions for plotting are required specify the
*      name of the stress distributions file. Leave unspecified if a
*      stress distributions file is not required.

xBot   lower limit on x for stress distributions = 0.001      m
*      Specify this parameter if stress distributions are required to
*      be confined to a particular region. Leave unspecified if the
*      default stress distributions extent (see Section 4.5) is
*      satisfactory.

xTop   upper limit on x for stress distributions = 0.002      m
*      Specify this parameter if stress distributions are required to
*      be confined to a particular region. Leave unspecified if the
*      default stress distributions extent (see Section 4.5) is
*      satisfactory.

yBot   lower limit on y for stress distributions = 0.001      m
*      Specify this parameter if stress distributions are required to
*      be confined to a particular region. Leave unspecified if the
*      default stress distributions extent (see Section 4.5) is
*      satisfactory.

yTop   upper limit on y for stress distributions = 0.002      m
*      Specify this parameter if stress distributions are required to
*      be confined to a particular region. Leave unspecified if the
*      default stress distributions extent (see Section 4.5) is
*      satisfactory.

zBot   lower limit on z for stress distributions = 0.001      m
*      Specify this parameter if stress distributions are required to
*      be confined to a particular region. Leave unspecified if the
*      default stress distributions extent (see Section 4.5) is
*      satisfactory.

zTop   upper limit on z for stress distributions = 0.002      m
*      Specify this parameter if stress distributions are required to
*      be confined to a particular region. Leave unspecified if the
*      default stress distributions extent (see Section 4.5) is
*      satisfactory.

nGrid  number of steps for stress distributions = 50
*      Specify this parameter if the default of 30 steps for stress
*      distributions is unsatisfactory.

distr  list of stress components to output      = fx Qe
*      Specify this parameter if it is required to reduce the size of
*      the stress distributions file by only including selected stress
*      components. Leave unspecified if all components are of interest.

```

(continued)

The allowable values for all input parameters and, where applicable, the default values are summarised in Table 4.1.

TABLE 4.1 ALLOWABLE VALUES AND DEFAULTS		
Symbol	Allowable values	Default (if any)
a b	1.0×10^{-6} m to 10.0 m 1.0×10^{-6} m to 10.0 m	
Rx1 Ry1 Rx2 Ry2	$-\infty$ to -0.001 m or 0.001 m to $+\infty$ $-\infty$ to -0.001 m or 0.001 m to $+\infty$ $-\infty$ to -0.001 m or 0.001 m to $+\infty$ $-\infty$ to -0.001 m or 0.001 m to $+\infty$	
phi	-90 degree to 90 degree	
E1 E2	1.0×10^9 N/m ² to 1.0×10^{12} N/m ² 1.0×10^9 N/m ² to 1.0×10^{12} N/m ²	
P rMu	0.0 to 1.0×10^{12} N 0.0 to 1.0	0.0
Ux1 Uy1 Ux2 Uy2	-100.0 m/s to 100.0 m/s -100.0 m/s to 100.0 m/s -100.0 m/s to 100.0 m/s -100.0 m/s to 100.0 m/s	0.0 0.0 0.0 0.0
gama1 gama2	-180 degree to 180 degree -180 degree to 180 degree	0.0 0.0
chi1 chi2	-90 degree to 90 degree -90 degree to 90 degree	0.0 0.0
sigma1 sigma2	0.0 to 0.9 0.0 to 0.9	0.3 0.3
body	1 or 2	1
x y z	-10.0 m to 10.0 m -10.0 m to 10.0 m -10.0 m to 10.0 m	
xBot xTop yBot ytop zBot zTop	-10.0 m to 10.0 m -10.0 m to 10.0 m -1.0 m to 1.0 m -1.0 m to 10.0 m -1.0 m to 1.0 m 0.0 m to 1.0 m	-1.5 a 1.5 a 1.5 b 1.5 b 0.0 2.0 b
nGrid	10 to 100	30
datOut	valid file name or path (75 characters maximum)	blank (i.e. no distributions)
distr	a list of any of fx, fy, fz, Qxy, Qyz, Qzx, Qe, Qo and Tmax	fx, fy, fz, Qxy, Qyz, Qzx, Qe, Qo and Tmax (i.e. all stresses)

5. THE OUTPUT DATA FILE

The output data file includes the results of the analysis together with a copy of the corresponding input data (see Section 4). The file can be examined using an ASCII text editor. An example output data file is shown here.

```
-----
ESDU International plc.

PROGRAM A9434V10

ESDUpac Number:   A9434
ESDUpac Title:    Dimensions, deflections and stresses for
                  Hertzian contacts under combined normal
                  and tangential loading.

Data Item Number: 94034
Data Item Title:  Dimensions, deflections and stresses for
                  Hertzian contacts under combined normal
                  and tangential loading.
                  (Guide to use of computer program A9434).

ESDUpac Version:  1.0, December 1994.
(See Data Item for full input/output specification and interpretation)
-----
Name of input data file
C:\ESDU\A9434\SPEC2.IN

INPUT DATA
~~~~~

* Contacting cylinders with skewed axes
* ~~~~~
* Based on Example 3 of Data Item 89045 with surface velocities specified

Rx1  = 0.02    m
Rx2  = 0.02    m
Ry1  = 00
Ry2  = 00

phi  = 45      degree

E1   = 200E9   N/m^2
E2   = 200E9   N/m^2

P    = 280     N

Ux1  = -9.05   m/s
Uy1  = 0.0
Ux2  = 0.0
Uy2  = 12.8

rMu  = 0.1

OUTPUT DATA
~~~~~

CONTACT DIMENSIONS
~~~~~
a      major semi-axis of contact ellipse      = 0.647 E-03 m
b      minor semi-axis of contact ellipse      = 0.203 E-03 m
beta   axis ratio, b/a, of contact ellipse     = 0.314
psi    orientation angle of contact ellipse     = 22.5      degree
        (angle from X1 axis to minor axis of contact ellipse)
delta  normal approach of contacting bodies    = 4.83 E-06 m
```

(continued)

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LOADING CONDITIONS

Combined normal and tangential loading.

P	normal load on contact	=	0.280 E+03	N
Q	frictional shear load on contact	=	28.0	N
rMu	coefficient of kinetic friction	=	0.100	
gama	direction of friction	=	-67.5	degree
	(angle between frictional force and minor axis of contact area)			
chi	direction of contact motion	=	22.5	degree
	(angle between contact trajectory and minor axis of contact area)			
Po	maximum Hertzian pressure	=	1.02 E+09	N/m^2

STRESSES FOR BODY 1

sigma	assumed value of Poisson's ratio	=	0.300
-------	----------------------------------	---	-------

Global maxima

QeMax	maximum value of equivalent stress	=	0.354 E+09	N/m^2
	located at	x =	-28.5 E-06	m
		y =	13.2 E-06	m
		z =	0.135 E-03	m
Tmax	maximum value of tensile stress	=	0.242 E+09	N/m^2
	located at	x =	0.647 E-03	m
		y =	7.37 E-06	m
		z =	0.00	m
dQoMax	maximum change in orthogonal shear	=	0.495 E+09	N/m^2
	located at a depth of	z =	95.0 E-06	m

INPUT DATA FOR CASE 2

* Higher friction case

rMu = 0.2

OUTPUT DATA FOR CASE 2

CONTACT DIMENSIONS

Contact dimensions unchanged.

LOADING CONDITIONS

Combined normal and tangential loading.

P	normal load on contact	=	0.280 E+03	N
Q	frictional shear load on contact	=	56.0	N
rMu	coefficient of kinetic friction	=	0.200	
gama	direction of friction	=	-67.5	degree
	(angle between frictional force and minor axis of contact area)			
chi	direction of contact motion	=	22.5	degree
	(angle between contact trajectory and minor axis of contact area)			
Po	maximum Hertzian pressure	=	1.02 E+09	N/m^2

STRESSES FOR BODY 1

sigma	assumed value of Poisson's ratio	=	0.300
-------	----------------------------------	---	-------

Global maxima

QeMax	maximum value of equivalent stress	=	0.363 E+09	N/m^2
	located at	x =	-51.8 E-06	m
		y =	25.6 E-06	m
		z =	0.129 E-03	m

(continued)

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Tmax	maximum value of tensile stress	=	0.355	E+09	N/m ²
	located at	x =	0.646	E-03	m
		y =	11.3	E-06	m
		z =	0.00		m
dQoMax	maximum change in orthogonal shear	=	0.495	E+09	N/m ²
	located at a depth of	z =	95.1	E-06	m

***Normal end					

The following points should be noted:

- (i) The output data file includes an exact copy of the input data.
- (ii) The output data are arranged into three major sections, namely

CONTACT DIMENSIONS,
LOADING CONDITIONS,
STRESSES.

Before determining the contact stresses, ESDUpac A9434 first establishes the contact dimensions and outputs a summary of these dimensions and other associated parameters such as the orientation of the contact ellipse relative to the body axes and the normal approach of the bodies. The loading conditions are then established and, again, a summary is output.

Note that if ESDUpac A9434 has been unable to complete the performance analysis (or some part of the analysis) for some reason then an appropriate error message is included in the output.

In examining the stresses in the contacting bodies, various failure criteria can be applied. The criteria calculated by this ESDUpac program are summarised in Table 5.1 and their use is illustrated by the worked examples in Section 7.

A discussion of the criteria can be found in Derivation 2.

TABLE 5.1 FAILURE CRITERIA

Ductile materials failing by plastic yielding

Criterion: Q_e or Q_{eMax}

Critical value: $0.577 |Y| = |k|$

or $0.192 H$

where Y is the yield stress in uniaxial tension or compression, k is the yield stress in simple shear and H is the Brinell or Vickers hardness.

Brittle materials failing by cracking

Criterion: T_{max}

Critical value: Depends on the size and orientation of pre-existing flaws or scratches. Experience with the same material is the best guide.

Materials failing by sub-surface initiated fatigue

Criterion: dQ_o or dQ_{oMax}

Critical value: Establish using experience with the same material combination.