

EFFECTS OF SMALL CHANGES ON RATE OF CLIMB

1. NOTATION AND UNITS^{*†}

		SI	British
C_D	drag coefficient ($= D/\rho V^2 S/2$)		
C_L	lift coefficient ($= L/\rho V^2 S/2$)		
D	airframe drag	N	lbf
D_M	intake momentum drag (per aircraft)	N	lbf
F	net jet thrust along flight path (= component of F_G along flight path minus D_M , see Data Item No. 69006 (Reference 2) for definitions)	N	lbf
F_G	engine gross thrust (per aircraft)	N	lbf
f	conversion factor in expression for effect of small change in power		
g_0	standard sea-level value [†] of acceleration due to gravity	m/s ²	ft/s ²
H	geopotential height [†]	m	ft
K_1, K_2	coefficients in drag equation $C_D = K_1 + K_2 C_L^2$		
L	airframe lift	N	lbf
m	mass of aircraft	kg	slug
n	load factor, (aircraft acceleration in lift direction)/ g_0		
P	total engine power supplied to propeller(s)	kW	hp
S	wing area	m ²	ft ²
V	true airspeed (air-path velocity or resultant velocity relative to air)	m/s	ft/s
V_e	equivalent airspeed ($= V\sigma^{1/2}$)	m/s	ft/s
v_c	rate of climb	m/s, m/min	ft/s, ft/min
W	aircraft weight ($= mg_0$)	N	lbf
η	propeller efficiency		

* The equations and the expressions for the effects of small changes in this Data Item are written for coherent units. Section 1.1 gives conversion factors for use with various mixed units.

† See Data Item No. 77022 (Reference 3) for definitions.

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ρ	density of air	kg/m ³	slug/ft ³
ρ_0	standard sea-level value* of ρ	kg/m ³	slug/ft ³
σ	relative density		

* See Data Item No. 77011 (Reference 3) for definitions.

1.1 Conversion Factors For Mixed Units

Quantity	SI units	British units
P	1 kW $\equiv$ 1000 mN/s	1 hp $\equiv$ 550 ft lbf/s
V	1 knot $\equiv$ 0.5144 m/s 1 km/h $\equiv$ 0.2778 m/s	1 knot $\equiv$ 1.688 ft/s 1 km/h $\equiv$ 0.911 ft/s
v_c	1 m/s $\equiv$ 60 m/min	1 ft/s $\equiv$ 60 ft/min

2. INTRODUCTION

This Data Item gives simple formulae for estimating the effect on rate of climb of a small change in any one of the independent variables F , K_1 , K_2 , W , ηP , V and σ . Typical uses of the formulae would be as follows,

adjustment of measured results to contractual guarantee operating conditions,

assessment of changes necessary to achieve a required performance level.

The method ignores the effects of any consequent changes in the other independent variables.

3. RATE OF CLIMB

The rate of climb (for constant wind velocity) may be expressed as (Reference 5)

$$v_c = \frac{(\eta P + FV - DV)}{W} \times \frac{1}{\left[1 + \frac{V}{g_0} \frac{dV}{dH}\right]} \quad (3.1)$$

The quantity in square brackets in Equation (3.1) is referred to as the acceleration factor and its magnitude depends on the manner in which the aircraft climbs or descends. Values of acceleration factor for several common climb techniques are given in Data Item No. 81046 (Reference 4).

For conditions in which it is permissible to use a parabolic expression to represent aircraft drag coefficient,

$$C_D = K_1 + K_2 C_L^2 \quad (3.2)$$

and where the angle of climb is small enough to assume that $L = W$ and, therefore, that $C_L = W/\frac{1}{2}\rho V^2 S$, drag may be expressed as

$$D = \frac{\rho V^2 S K_1}{2} + \frac{2 K_2 W^2}{\rho V^2 S}. \quad (3.3)$$

The rate of climb then becomes

$$v_c = \left(\frac{\eta P}{W} + \frac{FV}{W} - \frac{\rho V^3 S K_1}{2W} - \frac{2 K_2 W}{\rho V S} \right) \frac{1}{\left[1 + \frac{V}{g_0} \frac{dV}{dH} \right]}. \quad (3.4)$$

If the climb is flown at constant true airspeed (acceleration factor = 1.0) Equation (3.4) becomes

$$v_c = \frac{\eta P}{W} + \frac{FV}{W} - \frac{\rho V^3 S K_1}{2W} - \frac{2 K_2 W}{\rho V S}. \quad (3.5)$$

4. EFFECTS OF SMALL CHANGES

4.1 Effects of Changes in ηP , F , W , K_1 and K_2

The effects of small changes in the independent variables ηP , F , W , K_1 and K_2 at constant true airspeed may be obtained by partial differentiation of Equation (3.5). The resulting expressions are presented in the following table.

Variable	Partial derivative	SI units	British units
ηP	$\frac{\partial v_c}{\partial (\eta P)} = \frac{f}{W}$	$f = 1000$	$f = 550$
F	$\frac{\partial v_c}{\partial F} = \frac{V}{W}$		
W	$\frac{\partial v_c}{\partial W} = - \left(v_c + \frac{4 K_2 W}{\rho V S} \right) \frac{1}{W}$		
K_1	$\frac{\partial v_c}{\partial K_1} = - \frac{\rho V^3 S}{2W}$		
K_2	$\frac{\partial v_c}{\partial K_2} = \frac{2W}{\rho V S}$		

These expressions for the partial derivatives are for v_c in m/s (SI units) and ft/s (British units). For m/min and ft/min the results should be multiplied by 60.