

SHEAR FORCE, BENDING MOMENT AND TORQUE OF RIGID AIRCRAFT IN SYMMETRIC STEADY MANOEUVRING FLIGHT

1. INTRODUCTION

A previously issued Data Item, No. 94009 (Reference 1), identified the aerodynamic load and mass data required for the components of a wing-fuselage combination of an aircraft in a steady symmetric manoeuvre in order to undertake a calculation of structural strength. The Item restricted attention to an aircraft of classical configuration for flight at subsonic speeds in which the incidence of the aircraft was assumed constant. It was also assumed in the analysis that both the pitch angle and the incidence of the aircraft were small throughout the manoeuvre so that load estimation could be obtained by the process of linear superposition. The drag and thrust of the aircraft were neglected in the analysis. The distribution of aerodynamic load over the tailplane was ignored and the tailplane contribution to lift and moment were represented by a single force acting at the aerodynamic centre of the tailplane. Each nacelle was assumed to contribute a concentrated lift and moment at and about its own aerodynamic centre and was treated as a concentrated mass with its own mass centre.

The present Item uses the data identified by Item No. 94009 for the case of a steady normal acceleration manoeuvre and provides the evaluation of the distribution of shear force, bending moment and torque along the wing span and along the fuselage required to determine the structural strength.

The wing shear force, bending moment and torque variations along the span are determined for both an unswept high aspect ratio wing and for a swept, medium to high aspect ratio, wing. For the unswept wing a straight flexural axis is defined perpendicular to the aircraft plane of symmetry and the distributions of force and moment are determined in relation to this axis; loads acting on this axis will result in bending deflections only and torques about this axis (in the plane normal to this axis) will result only in twist about this axis. For the swept wing case two types of flexural axis are envisaged. The first identifies a straight flexural axis parallel to the sweep line of the wing and also defines an effective wing root perpendicular to the flexural axis lying in the x - y plane of the aircraft (see Sketch 4.5). The wing is assumed to bend through this axis and twist in the plane normal to this axis. The alternative model identifies a curved flexural axis along the length of the span and in this case the wing is assumed to bend and twist in the plane parallel to the plane of symmetry of the aircraft.

The forces and moments on the fuselage are taken along the centre line of the aircraft in the x - z plane of the aircraft and consequently only shear forces and bending moments are determined.

2. NOTATION

Any coherent system of units may be used.

ac	aerodynamic centre of wing-fuselage combination
arg	argument of a function
BM	bending moment
b	wing span
cm	centre of mass of total aircraft

F	shear force
f	local force in z-direction
$H\{arg\}$	step function (= 0 when $arg < 0$; = 1 when $arg \geq 0$)
i	general integer
K	number of engine nacelles
L	lift
l	distance in x-direction measured as positive from cm position
l_{front}	distance from cm to extreme tip of fuselage nose
l_{nose}	distance from cm to start of fuselage nose region
l_{rear}	distance from cm to extreme rear of fuselage
$l_W, l_W(y)$	distance from cm to aerodynamic centre of complete wing and of wing section at station y , respectively
$(l_m)_W, l_{mW}(y)$	distance from cm to mass centre of complete wing and of wing section at station y , respectively
ℓ	local lift per unit span and per unit fuselage length
ℓ_q	rate of change of local lift per unit span with q , $d\ell/dq$
ℓ_0	local lift per unit span at zero incidence
ℓ_α	local lift curve slope per unit span, $d\ell/d\alpha$
M	pitching moment about ac
m	local pitching moment per unit span about ac
m_q	rate of change of local pitching moment per unit span with q , dm/dq
m_0	local pitching moment coefficient about local aerodynamic centre when $q = 0$
n	load factor, L/W
P	pitching moment about y-axis
p	pitching moment of wing section about y-axis
q	steady pitch rate
T	engine thrust
T	torque
t	time

V	true airspeed of aircraft
W	weight
$w(x)_{Fus}$	weight per unit length of fuselage at fuselage station x
$w(y)_W$	weight per unit span of wing at span station y
x, x'	general fuselage stations measured from cm position
x, y, z	axis directions
y, y'	general span stations measured from fuselage centre line
y_r	distance in y direction of wing root from fuselage centre line
y_x	flexural axis
α	incidence
γ	flight path angle
$\dot{\gamma}$	rate of change of γ with, $d\gamma/dt(\equiv q)$
θ	pitch angle
Λ	wing sweep angle

Subscripts

F	fuselage nose region
Fus	complete fuselage
i	general integer
m	relates to mass quantity
Nac	nacelle
q	relates to differential with respect to q
T	tailplane and fin assembly
W	wing
WF	wing-fuselage combination
0	relates to $q = 0$
$1, 2, 3$	used to identify shear force, bending moment and torque through and about flexural axis for various cases
α	relates to differential with respect to α

3. BACKGROUND DESCRIPTION

3.1 Forces and Moments on a Complete Aircraft

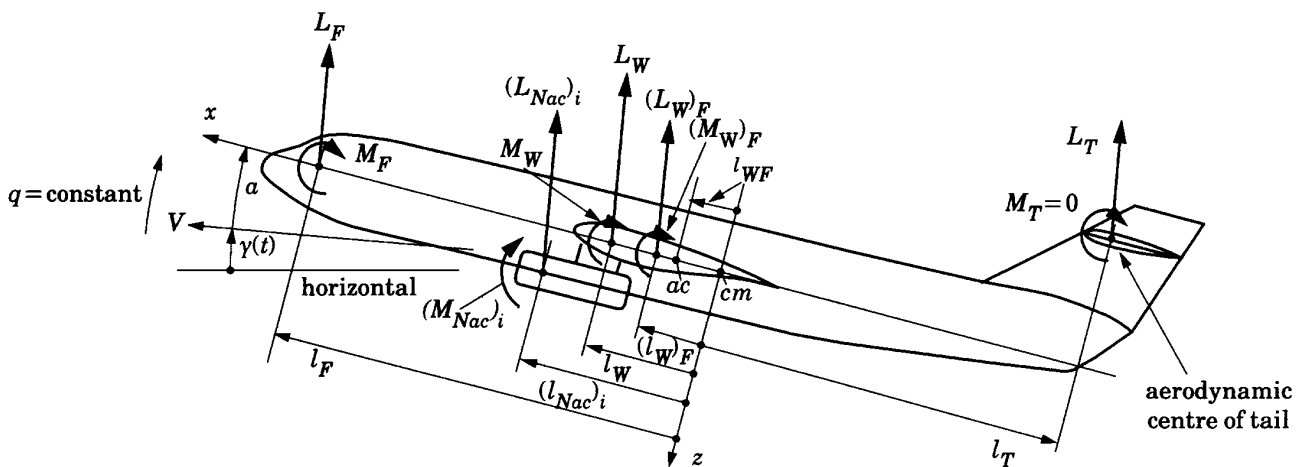
An aircraft is assumed to be executing a normal acceleration manoeuvre, that is a motion along the arc of a circle at constant forward speed, V , constant incidence, α , and constant pitch rate, q . The flight path angle, $\gamma(t)$, is equal to $\theta(t) - \alpha$, where θ is the pitch angle. The pitch rate is equal to the rate of change of flight path angle and so $q = \dot{\gamma}(t) = \text{constant}$. The orthogonal axis system chosen for the aircraft is a body-fixed system consisting of the x -axis and z -axis in the aircraft plane of symmetry with the x -axis directed towards the nose, the z -axis directed downwards and the third axis y to starboard.

Sketch 3.1 shows the aerodynamic forces on the components of an aircraft, where the components consist of two halves of a net (*i.e.* an exposed) wing, a carry-over section which is a body section between each half of the net wing, a fuselage, a tailplane and a single nacelle. The effects of the nacelles are illustrated by treating the effects of a single nacelle. The lift, moment and aerodynamic centre position of each aircraft component in relation to the centre of mass of the complete aircraft, cm , is denoted by

L_W, M_W, l_W	for both halves of the net (<i>i.e.</i> exposed) wing,
$(L_W)_F, (M_W)_F, (l_W)_F$	for the carry-over section on the fuselage,
L_F, M_F, l_F	for the fuselage (assumed to act on the fuselage nose),
L_T, M_T, l_T	for the tailplane
and $(L_{Nac})_i, (M_{Nac})_i, (l_{Nac})_i$	for the i^{th} engine nacelle,

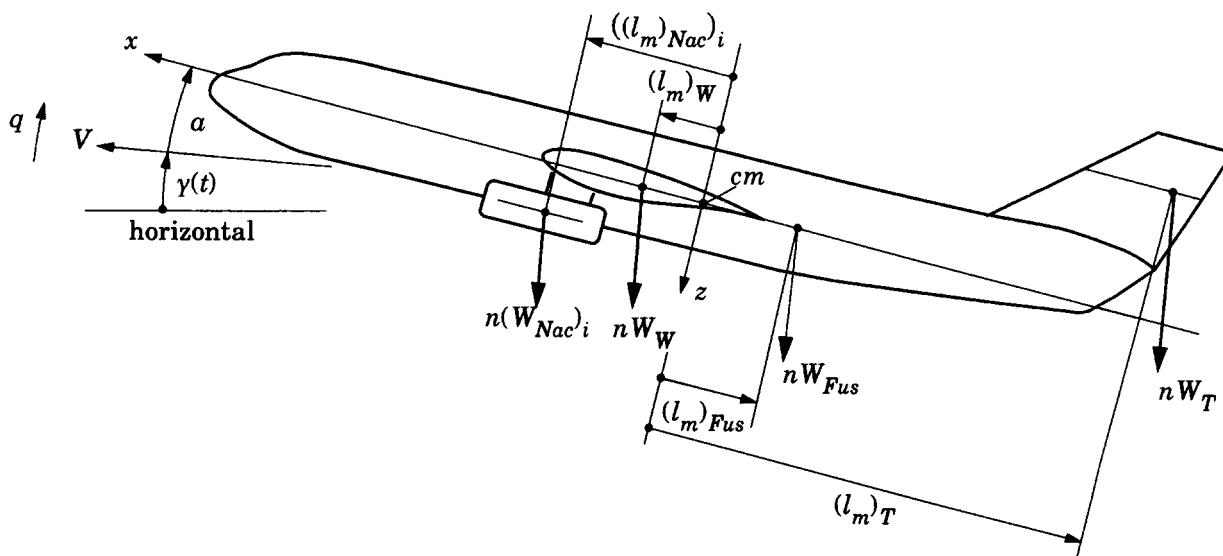
where the moment of the tailplane about the aerodynamic centre of the tailplane, M_T , is assumed zero.

It is assumed that the aerodynamic centres and mass centres of each aircraft component (other than the tail) and the centre of mass of the complete aircraft lie in the x - y plane of the aircraft. Thus terms in the moment equations that would have been introduced when such conditions do not apply are zero. These assumptions simplify the analysis procedure.



Sketch 3.1 Lift forces on components of an aircraft

Similarly, Sketch 3.2 shows the combined weight and D'Alembert reaction force (see Item No. 94009, Reference 1), acting on each of the components of an aircraft, where the tailplane now includes the effects of the fin assembly. It may be noted that the line of action of these inertial forces is in a direction perpendicular to the aircraft flight direction.



Sketch 3.2 Aircraft inertial force breakdown

The combined weight and D'Alembert's reaction force of each aircraft component and the position of the centre of mass of each component in relation to the aircraft centre of mass, cm , is denoted by

$nW_W, (l_m)_W$	for both halves of the net (i.e. exposed) wing,
$nW_{Fus}, (l_m)_{Fus}$	for the complete fuselage (including the carry-over section),
$nW_T, (l_m)_T$	for the tailplane and fin assembly
and $n(W_{Nac})_i, ((l_m)_{Nac})_i$	for the i^{th} engine nacelle,

where n is the load factor, given by $n = L/W = 1 + q/Vg$.

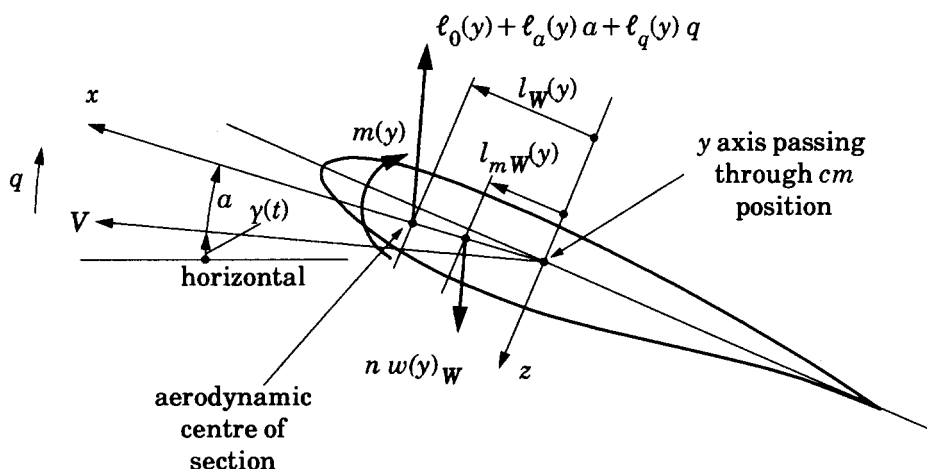
3.2 Forces and Pitching Moments on a Wing Section

The forces and pitching moments acting on a wing section of unit span at a spanwise position of y from the body centre-line are shown in Sketch 3.3. In the analysis it has been assumed, as previously, that the aerodynamic centres and mass centres of each wing section and the centre of mass of the complete aircraft lie in the x - y plane of the aircraft.

The local, unit span, wing aerodynamic loading, $\ell(y)$, consists of the zero incidence constant lift, $\ell_0(y)$, a lift due to incidence and a lift due to pitch rate, given by

$$\ell(y) = \ell_0(y) + \ell_\alpha(y)\alpha + \ell_q(y)q, \quad (3.1)$$

which acts in an upwards direction perpendicular to the flight direction and which passes through the position of the local aerodynamic centre of the section.



Sketch 3.3 Forces and pitching moment acting on a unit span wing section

The position of this local centre, $l_W(y)$, is taken from the spanwise variation for the complete wing. Its position is measured in a direction parallel to the x -axis from the centre of mass of the complete aircraft, cm , as illustrated in Sketch 3.3. The pitching moment on a unit span wing section about the aerodynamic centre, $m(y)$, is equal to

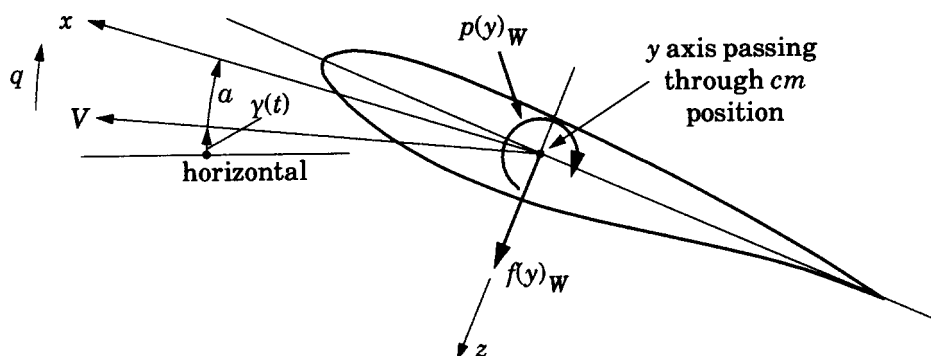
$$m(y) = m_0(y) + m_q(y) q. \quad (3.2)$$

The combined wing section weight and D'Alembert's reaction force, $n w(y)$, is also shown in Sketch 3.3 and acts at the local centre of mass position of the section at a distance $l_{mW}(y)$ from the point at which the y -axis passes through the local section.

The forces and pitching moment on the wing section may be transformed to an equivalent single force in the z -direction, $f(y)_W$, and pitching moment, $p(y)_W$, acting through and about the point at which the y -axis intersects the local section, as shown in Sketch 3.4. They are given by, when α is small, i.e. $\cos \alpha \approx 1$,

$$f(y)_W = n w(y)_W - l(y) \quad (3.3)$$

and
$$p(y)_W = m(y) + l(y) l_W(y) - n w(y)_W l_{mW}(y). \quad (3.4)$$



Sketch 3.4 Force and pitching moment acting on a unit span wing section