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Supersedes ESDU 79007,
79021, 79023, and 80022

Kinematic analysis of disc cams

**Associated software: ESDUpac A9501
VIEWpac 9501A**

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THE PREPARATION OF THIS DATA ITEM

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The Mechanisms Committee, which first met in 1975, is continuing the work of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

KINEMATIC ANALYSIS OF DISC CAMS

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KINEMATIC ANALYSIS OF DISC CAMS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI. All angles are measured in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

		Units
B	swinging follower arm crank length (see Figure 4 for sign convention)	m
D	distance between cam rotational centre and swinging follower pivot (see Figures 4, 5 and 6 for sign convention)	m
E	translating follower offset (see Figures 1, 2 and 3 for sign convention)	m
H	translating follower lift corresponding to one motion segment (see Figures 1, 2 and 3)	m
k	constant of value + 1 or – 1 (see Section 3.2.2)	–
L	distance between swinging follower pivot and roller centre or centre of follower radius (see Figures 5 and 6)	m
R_b	reference circle radius of cam (see Figures 1 to 6)	m
R_f	radius of domed follower (see Figures 2 and 5)	m
R_g	radius of cutter	m
R_r	radius of roller (see Figures 3 and 6)	m
r_g	distance between cam centre and centre of cutter (see Sketch 7.1)	m
r_p	distance between cam centre and contact point (see Sketch 6.1)	m
u	normalised cam angle of rotation	–
w	normalised follower displacement	–
w', w'', w'''	non-dimensional follower velocity, acceleration and jerk, respectively	–
X_c, Y_c	axes fixed in, and rotating with, cam	–
X_m, Y_m	axes fixed in machine frame	–
x_p, y_p	coordinates of cam profile (see Sketch 6.1) relative to X_c, Y_c axes	m
x_g, y_g	coordinates of cutter centre (see Sketch 7.1)	m

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y	translational displacement of follower	m
$\dot{y}, \ddot{y}, \dddot{y}$	translational follower velocity (m/s), acceleration (m/s ²) and jerk (m/s ³), respectively	–
y_b	height of translating follower reference point above X_m -axis at innermost position	m
y_0	height of translating follower reference point above X_m -axis at beginning of segment	m
y_m	instantaneous height of translating of follower reference point above X_m -axis	m
α	pressure angle (see Sketch 4.1)	rad
α	constant pressure angle for flat-faced translating follower	rad
θ_g	angle between X_c -axis and line connecting cam centre to centre of cutter	rad
θ_p	angle between X_c -axis and line connecting cam centre to contact point	rad
ρ	radius of curvature of pitch curve	m
ρ_p	radius of curvature of cam profile (see Sketch 5.1)	m
Φ	total angle of cam rotation corresponding to one motion segment	rad
ϕ	cam angle of rotation	rad
Ψ	follower swing corresponding to one motion segment (see Figures 4, 5 and 6)	rad
ψ	rotational displacement of swinging follower	rad
$\dot{\psi}, \ddot{\psi}, \dddot{\psi}$	rotational velocity (rad/s), acceleration (rad/s ²) and jerk (rad/s ³), respectively, of swinging follower	–
ψ_b	reference angle for swinging follower at innermost position	rad
ψ_0	reference angle for swinging follower at beginning of segment	rad
ψ_m	instantaneous reference angle for swinging follower	rad
ω_c	angular velocity of cam	rad/s

Subscripts

max	refers to maximum value
min	refers to minimum value

2. INTRODUCTION

When the basic dimensions of a disc cam and its follower have been chosen provisionally it is necessary to check that the resulting cam profile is satisfactory. This involves calculation of the pressure angle, the radius of curvature of the cam profile, the radius of curvature of the pitch curve and, possibly, the coordinates of the cam profile. In addition, when the profile has been finalised and the cam is to be manufactured, it is necessary to generate a series of cutter coordinates for direct transfer to a computer-controlled machine tool.

This Data Item presents methods for the calculation of the above quantities for a disc cam with any one of the commonly-used follower types and a specified follower motion. The Item is also applicable to slot or ridged cams (see Example 1). The calculations are time consuming and are best carried out using the computer program (ESDUpac A9501) supplied with this Data Item. The program calculates all quantities throughout one complete cycle of the cam at user-specified intervals of cam rotation. The program checks that interference (undercutting) between the cam and the follower or between the cam and the cutter is avoided.

The derivation of the equations presented in this Data Item is given in ESDU ME2 (Derivation 1).

2.1 Assumptions

It is assumed that:

- (i) both the cam and the follower are sufficiently stiff to be considered rigid,
 - (ii) imperfections in the cam profile (primarily due to manufacturing) can be ignored,
 - (iii) the cam rotates in the anticlockwise sense at constant angular velocity
- and (iv) the cam and the follower remain in contact throughout the operating cycle.

2.2 Scope

2.2.1 Follower type

This Data Item is applicable to the design of disc cams with any one of the six main types of follower illustrated in Sketch 2.1. As this Item deals only with kinematic quantities the method by which closure between the cam and follower is maintained does not affect the results. Thus the Item can be used for the design of cam systems with spring force closure or with body force closure, for example cognate cams and slot/grooved cams as shown in Example 1 and in ESDU 82024 (Reference 5).

2.2.2 Follower motion

The methods presented in this Item can be used to analyse cams with a wide range of profiles. In cam design the relationship between the motion of the cam and the motion of the follower, during a dwell-rise-dwell (DRD), a dwell-return-dwell (DRD) motion, or a dwell-rise-return-dwell (DRRD) motion, is usually governed by a mathematical equation known as a cam law (see ESDU 82006, Reference 3). Although the equations (presented in Sections 4, 5, 6 and 7) are general, in that any mathematical cam law can be specified, the computer program (ESDUpac A9501) contains only six commonly-used cam laws (identified in Section 12.2.2(a)).

If the required cam law for a particular motion segment is not included in the current version of ESDUpac A9501, the motion of a follower, throughout a cam cycle, can be provided to the program via a file. This motion data file must contain values of follower displacement, follower velocity, follower acceleration and follower jerk at a specified interval of cam rotation. The required format of the file is described in Section 12.2.2(b) and the reasons for specifying motion in this way are explained in Section 8.1.

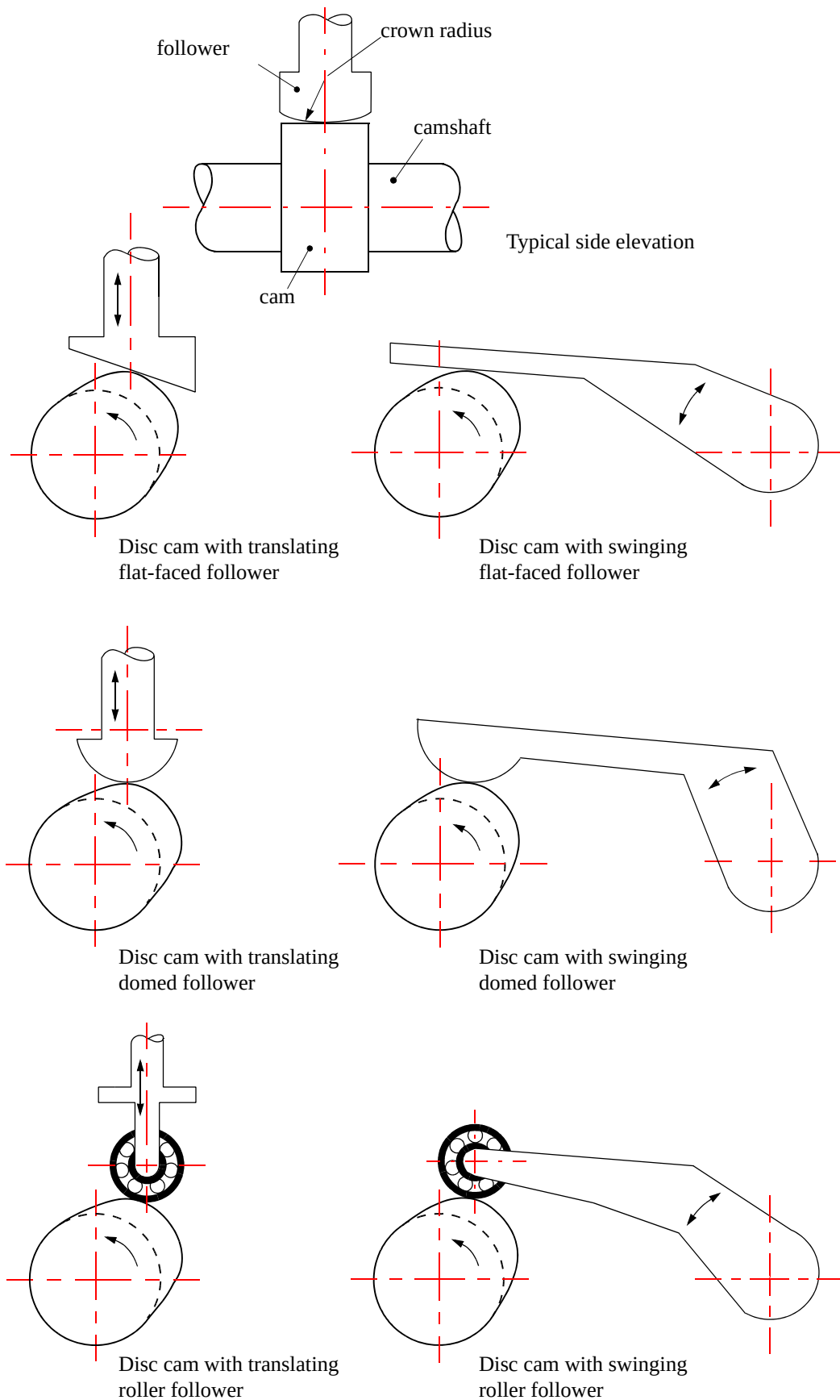
2.3 Use of This Data Item

The equations used in this Data Item are presented in Sections 4, 5, 6 and 7 but it is not necessary to have a deep understanding of the derivation of these equations to carry out the required calculations using the accompanying computer program (ESDUpac A9501) as described in Section 12. A guide to the choice of input values for this program and a description of the program output are presented in Section 8. Design guidance is presented in Section 9. The examples in Section 11 also provide valuable information on the effective use of the program. The calculation of pressure angle, radius of curvature and profile coordinates is useful:

- (a) during a feasibility study (to assess the potential of a particular cam mechanism type for the provision of a specified motion path)
- or (b) as part of the detail design (to ensure that a specified design has acceptable kinematic performance).

The calculation of profile cutting coordinates is essential prior to manufacturing the cam. Modern computer controlled machines can make direct use of a file of cutting coordinates to drive the cutting tool along the required path.

When an acceptable kinematic design has been achieved it is recommended that ESDU 94017 (Reference 15) be used to aid the choice of materials.



Sketch 2.1 Disc cams with six commonly-used followers

3. SIGN CONVENTIONS

3.1 General Comments

To use this Item it is necessary to adopt sign conventions for certain aspects of the cam and follower geometry and for the follower motion. These conventions are indicated in the following statements and illustrated in Figures 1, 2, 3, 4, 5 and 6.

- (1) All angles are measured as positive in the anticlockwise sense. A negative angle returned during the calculations indicates that the quantity has been measured in the clockwise sense.
- (2) The cam is always considered as rotating in an anticlockwise sense. The angle, ϕ , between the fixed machine axes (X_m , Y_m) and the rotating axes (X_c , Y_c) is always positive as shown in Figures 1 to 6.
- (3) The total follower lift during a rise or a return segment is always considered a positive scalar quantity.
- (4) For cams with translating followers the offset of the follower line-of-action, E , is positive if it passes to the right of the cam centre and negative if it passes to the left (see Figures 1 to 3). The height of the follower above the X_m -axis, y_m is always considered as positive (see Table 3.1).

TABLE 3.1 Follower reference height

Follower type	Follower reference point
Translating flat-faced	point at which follower face intersects follower line-of-action (see Figure 1)
Translating domed	centre of domed follower radius (see Figure 2)
Translating roller	centre of roller (see Figure 3)

- (5) For cams with swinging followers the distance between the cam centre and the follower pivot, D , is positive if the pivot is to the right of the cam centre and negative if it is to the left (see Figures 4 to 6). The angle (anticlockwise positive) between the X_m -axis and the follower reference line (see Table 3.2) is always positive.

TABLE 3.2 Follower reference line

Follower type	Follower reference line
Swinging flat-faced	line forming the contact face of the follower (see Figure 4)
Swinging domed	line joining follower pivot to centre of domed follower radius (see Figure 5)
Swinging roller	line joining follower pivot to centre of roller (see Figure 6)

An additional quantity, the length of the follower arm crank, B , is required for a cam with a swinging flat-faced follower. This has a positive sign if it lies above the X_m -axis and a negative sign if it lies below that axis (see Figure 4).

3.2 Follower Motion

The follower motion, throughout one complete rotation of the cam, must be decided before the calculations described in this Data Item can be performed. Normally the motion is specified in terms of a sequence of distinct motion segments as shown in Sketch 3.1. This information is summarised in the form of a timing chart, an example of which is given by Sketch 8.1.

For each motion segment it is necessary to specify the total cam angle of rotation, Φ , the total follower lift, H or Ψ , and the non-dimensional equation, $w = f(u)$ (known as a cam law), that governs the relationship between cam angle and follower displacement. The equations of widely-used cam laws for DRD motion segments are specified in ESDU 82006 (Reference 3).

Note that:

- (a) u is defined by the expression $u = (\phi - \phi_0) / \Phi$,
 - (b) $w = f(u)$, $w' = f(u)'$ and $w'' = f(u)''$ for each DRD or DRRD motion segment, where u varies between 0 and 1,
 - (c) w and w' are positive for a rise segment and negative during a return motion
- and (d) w'' changes sign during every motion segment (positive during the first part of a rise segment and negative during the first part of a return segment).

3.2.1 Translating follower

For a rise motion, the displacement of a translating follower driven by a disc cam is given by

$$y = wH \quad (3.1)$$

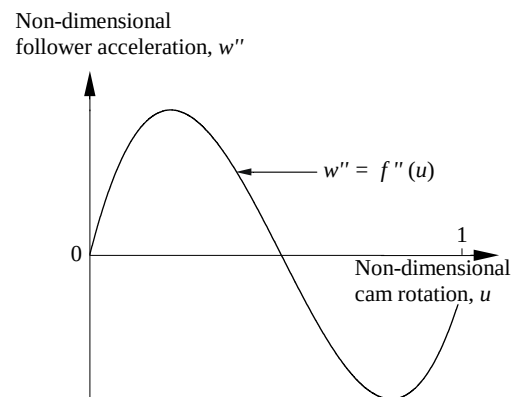
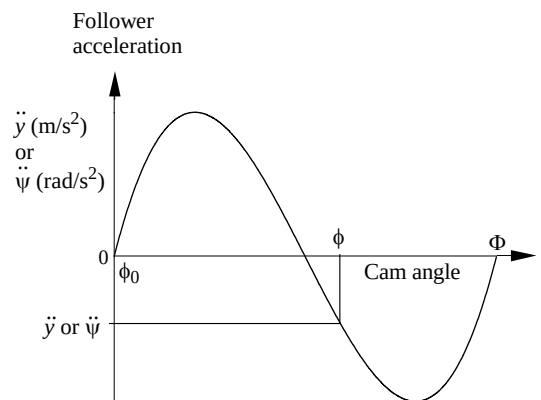
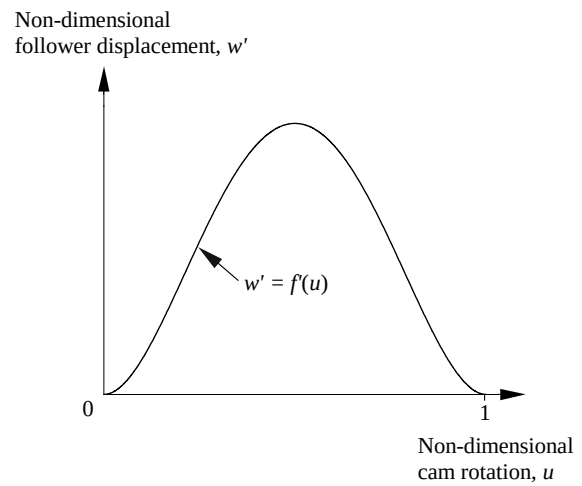
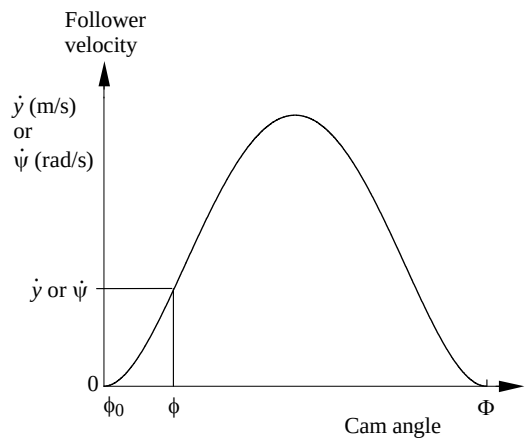
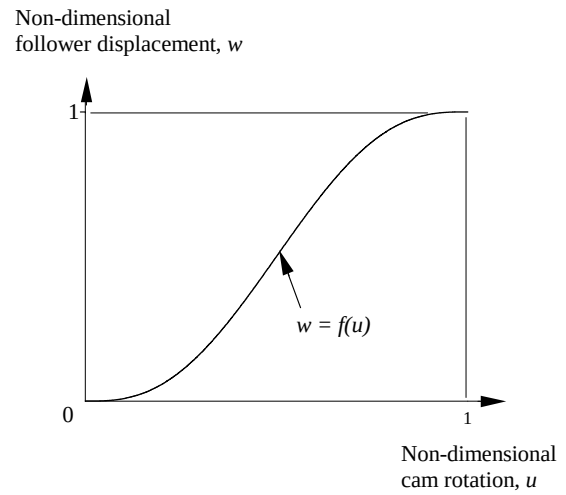
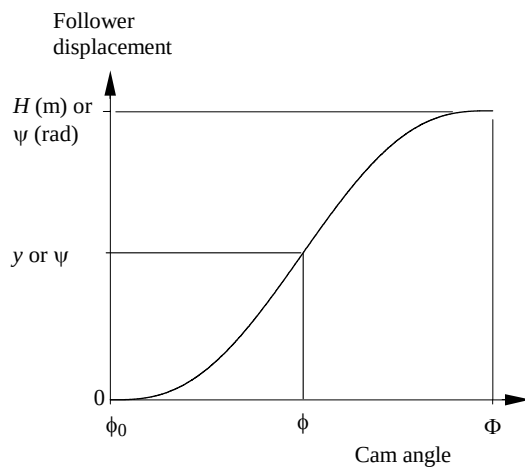
and the distance between the X_m -axis and the reference point on the follower (see Figures 1, 2 and 3)

$$y_m = y_0 + wH \quad (3.2)$$

where y_0 is defined as the follower displacement at the start of the current motion segment.

It is conventional to designate the rise motion segment, which begins from the least cam radius, as the first motion segment. Then y_0 is equal to y_b and the follower displacement at inner dwell is given by

$$y_b \begin{cases} = R_b \cos \alpha & \text{for a flat-faced follower} & (3.3a) \\ = [(R_b + R_f)^2 - E^2]^{1/2} & \text{for a domed follower} & (3.3b) \\ = [(R_b + R_r)^2 - E^2]^{1/2} & \text{for a roller follower.} & (3.3c) \end{cases}$$



Sketch 3.1 Rise motion segment

For a rise motion, the follower velocity, acceleration and jerk, respectively, are given by

$$\dot{y} = \frac{w' \omega_c H}{\Phi}, \quad (3.4)$$

$$\ddot{y} = \frac{w'' \omega_c^2 H}{\Phi^2} \quad (3.5)$$

and
$$\ddot{\ddot{y}} = \frac{w''' \omega_c^3 H}{\Phi^3}. \quad (3.6)$$

3.2.2 Swinging follower

For a rise motion, the displacement of a swinging follower driven by a disc cam is given by

$$\psi = w\Psi \quad (3.7)$$

and the angle between the X_m -axis and the follower arm (see Figures 4, 5 and 6) is given by

$$\psi_m = \psi_0 - kw\Psi, \quad (3.8)$$

where ψ_0 is defined as the follower displacement at the start of the current motion segment and where k is a constant of value unity that adopts the sign of D to take account of configurations in which the follower pivot lies to the left of the cam centre ($D - ve$) or to the right of the cam centre ($D + ve$).

It is conventional to designate the rise motion segment, which begins from the least cam radius, as the first motion segment. Then ψ_0 is equal to ψ_b and the follower displacement is given by

$$\left. \begin{aligned} \psi_b &= \pi - \sin^{-1} \left[\frac{R_b - B}{D} \right] && \text{for a flat-faced follower } (D + ve) && (3.9a) \\ \psi_b &= \sin^{-1} \left[\frac{B - R_b}{D} \right] && \text{for a flat-faced follower } (D - ve) && (3.9b) \\ \psi_b &= \cos^{-1} \left[\frac{- \left(L^2 + D^2 - (R_b + R_f)^2 \right)}{2LD} \right] && \text{for a domed follower} && (3.9c) \\ \psi_b &= \cos^{-1} \left[\frac{- \left(L^2 + D^2 - (R_b + R_r)^2 \right)}{2LD} \right] && \text{for a roller follower.} && (3.9d) \end{aligned} \right\}$$

For a rise motion, the follower velocity, acceleration and jerk, respectively, are given by

$$\dot{\psi} = - \frac{w' \omega_c \Psi k}{\Phi}, \quad (3.10)$$

$$\ddot{\psi} = - \frac{w'' \omega_c^2 \Psi k}{\Phi^2} \quad (3.11)$$

and
$$\ddot{\ddot{\psi}} = - \frac{w''' \omega_c^3 \Psi k}{\Phi^3}. \quad (3.12)$$

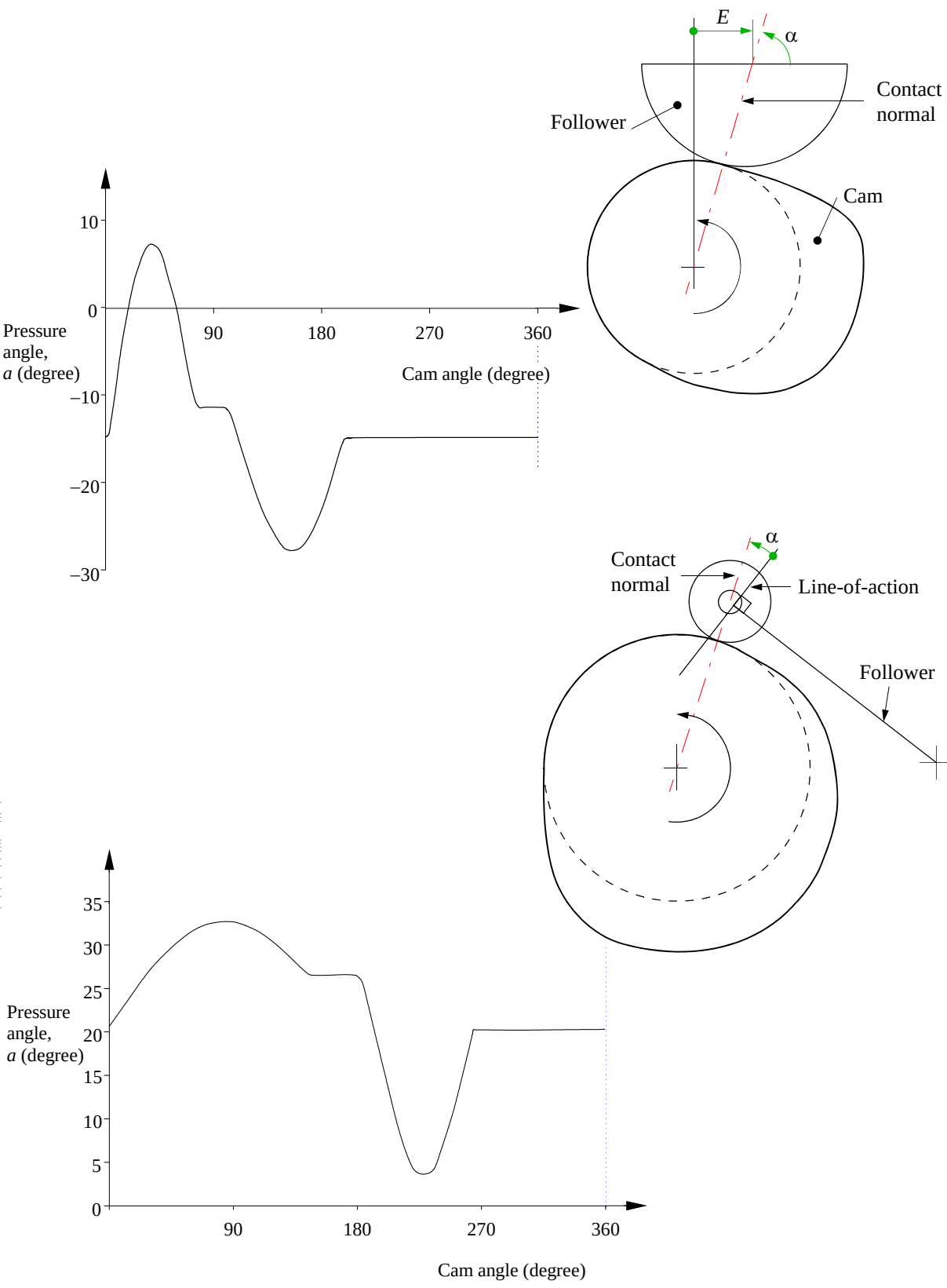
4. PRESSURE ANGLE CALCULATIONS

The pressure angle is the acute angle between the follower line-of-action and the contact normal (see Sketch 4.1). It can be used to judge the quality of load transmission between the cam and the follower as described in Section 9.2.1.

Table 4.1 gives expressions for the pressure angle for each of the cam/follower configurations covered by this Data Item. These are derived in ESDU ME2 (Derivation 1).

TABLE 4.1 Pressure angle expression

Follower type	Pressure angle (degrees)
translating flat-faced	$\alpha = \text{constant}$
translating domed	$\alpha = \tan^{-1} \left[\frac{\frac{\dot{y}}{\omega_c} - E}{y_m} \right]$
translating roller	$\alpha = \tan^{-1} \left[\frac{\frac{\dot{y}}{\omega_c} - E}{y_m} \right]$
swinging flat-faced	$\alpha = \tan^{-1} \left[\frac{-kB \left[1 - \frac{\dot{\psi}}{\omega_c} \right]}{D \cos \psi_m} \right]$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$
swinging domed	$\alpha = \tan^{-1} \left[\frac{L \left[1 - \frac{\dot{\psi}}{\omega_c} \right] + D \cos \psi_m}{D \sin \psi_m} \right]$
swinging roller	$\alpha = \tan^{-1} \left[\frac{L \left[1 - \frac{\dot{\psi}}{\omega_c} \right] + D \cos \psi_m}{D \sin \psi_m} \right]$



Sketch 4.1 Typical examples of pressure angle

5. RADIUS OF CURVATURE CALCULATIONS

The instantaneous value of the radius of curvature of the cam profile (see Sketch 5.1) must be calculated. This is necessary to ensure that the cam profile is not undercut (see Section 9.2.2).

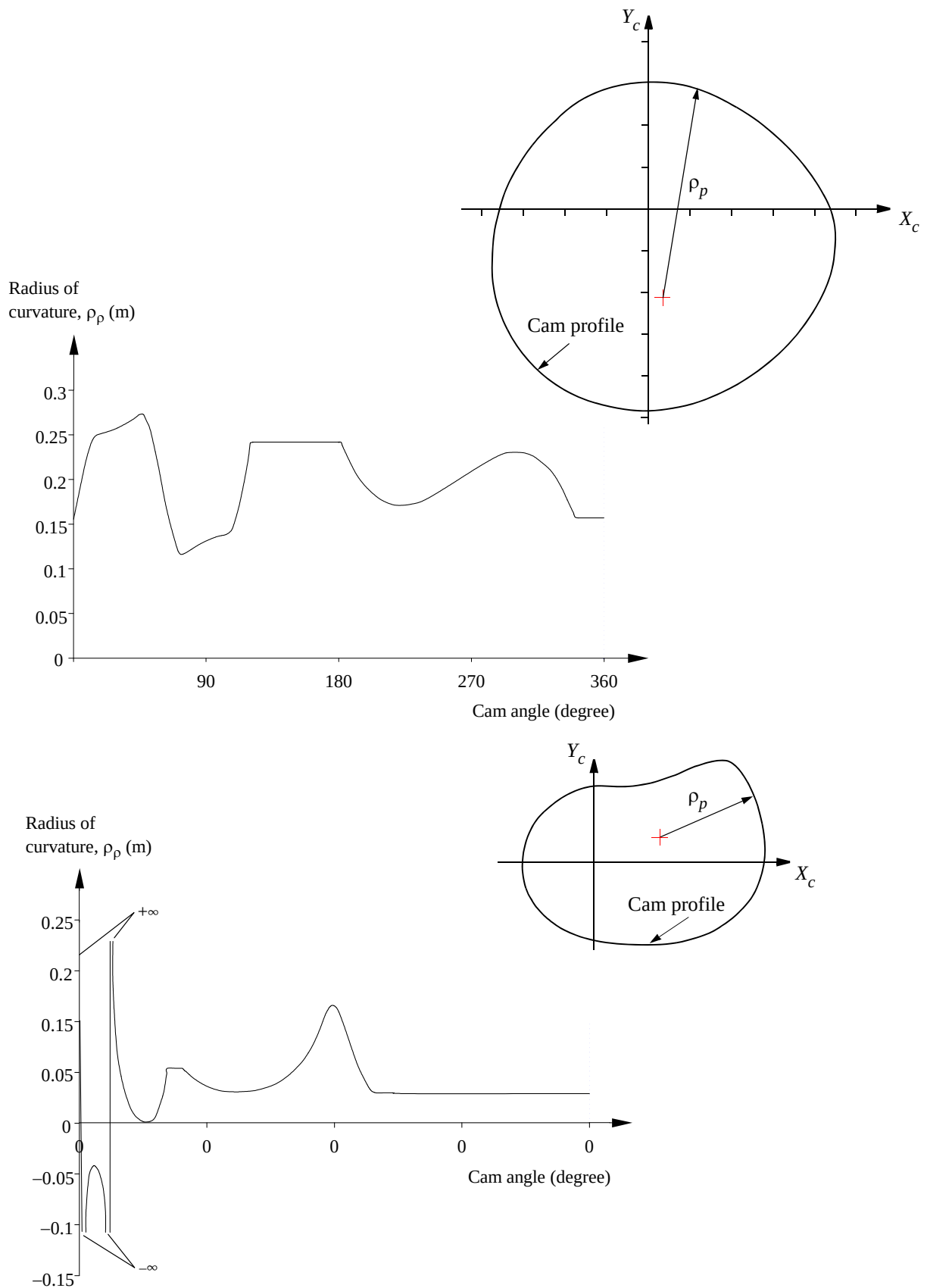
Table 5.1 gives expressions for the radius of curvature of the cam profile for each of the cam/follower configurations covered by this Data Item. These are derived in ESDU ME2 (Derivation 1). Tests for undercutting (with domed or roller followers) require the radius of curvature of the pitch curve and this is given by

$$\rho = \rho_p + R_f \text{ for a domed follower}$$

or
$$\rho = \rho_p + R_r \text{ for a roller follower.}$$

TABLE 5.1 Radius of curvature of cam profile

Follower type	Radius of curvature of cam profile (m)
translating flat-faced	$\rho_p = \left(y_m + \frac{\ddot{y}}{\omega_c^2} \right) \cos \alpha - \left(E - 2 \frac{\ddot{y}}{\omega_c^2} \right) \sin \alpha$
translating domed	$\rho_p = \frac{y_m^2}{\frac{\dot{y}}{\omega_c} \cos^2 \alpha \sin \alpha - \frac{\ddot{y}}{\omega_c^2} \cos^3 \alpha + y_m \cos \alpha} - R_f$
translating roller	$\rho_p = \frac{y_m^2}{\frac{\dot{y}}{\omega_c} \cos^2 \alpha \sin \alpha - \frac{\ddot{y}}{\omega_c^2} \cos^3 \alpha + y_m \cos \alpha} - R_r$
swinging flat-faced	$\rho_p = B + k \left[\frac{D \sin \psi_m}{\left(1 - \frac{\dot{\psi}}{\omega_c} \right)} - \frac{\frac{\dot{\psi}}{\omega_c} D \sin \psi_m}{\left(1 - \frac{\dot{\psi}}{\omega_c} \right)^2} + \frac{\frac{\ddot{\psi}}{\omega_c^2} D \cos \psi_m}{\left(1 - \frac{\dot{\psi}}{\omega_c} \right)^3} \right]$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$
swinging domed	$\rho_p = \frac{-kD^2 \left(1 - \frac{\dot{\psi}}{\omega_c} \right) \frac{\sin^2 \psi_m}{\cos^2 \alpha}}{\frac{\dot{\psi}}{\omega_c} L \left(1 - \frac{\dot{\psi}}{\omega_c} \right)^2 \sin \alpha + D \frac{\ddot{\psi}}{\omega_c^2} \cos(\alpha + \psi_m) - D \left(1 - \frac{\dot{\psi}}{\omega_c} \right) \frac{\sin \psi_m}{\cos \alpha}} - R_f$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$
swinging roller	$\rho_p = \frac{-kD^2 \left(1 - \frac{\dot{\psi}}{\omega_c} \right) \frac{\sin^2 \psi_m}{\cos^2 \alpha}}{\frac{\dot{\psi}}{\omega_c} L \left(1 - \frac{\dot{\psi}}{\omega_c} \right)^2 \sin \alpha + D \frac{\ddot{\psi}}{\omega_c^2} \cos(\alpha + \psi_m) - D \left(1 - \frac{\dot{\psi}}{\omega_c} \right) \frac{\sin \psi_m}{\cos \alpha}} - R_r$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$



Sketch 5.1 Typical examples of radius of curvature

6. CAM PROFILE COORDINATE CALCULATIONS

The cam profile can be described by a series of coordinates (cartesian or polar) with the origin at the cam rotational centre (see Sketch 6.1). The coordinates can be used to generate a scale drawing of the cam (see Section 9). Table 6.1 gives expressions for the cartesian coordinates of the cam profile for each of the cam/follower configurations covered by this Data Item. These are derived in ESDU ME2 (Derivation 1).

In each case the polar coordinates of the cam profile are given by

$$r_p = (x_p^2 + y_p^2)^{1/2}$$

and
$$\theta_p = \tan^{-1}\left(\frac{y_p}{x_p}\right).$$

TABLE 6.1 Cartesian coordinates of cam profile

Follower type	Profile coordinates (m)
translating flat-faced	$x_p = y_m \sin \phi - \left[y_m \sin \alpha - \frac{\dot{y}}{\omega_c} \cos \alpha \right] \cos (\alpha - \phi)$ $y_p = y_m \cos \phi + \left[y_m \sin \alpha - \frac{\dot{y}}{\omega_c} \cos \alpha \right] \sin (\alpha - \phi)$
translating domed	$x_p = E \cos \phi + y_m \sin \phi - R_f \sin (\phi - \alpha)$ $y_p = -E \sin \phi + y_m \cos \phi - R_f \cos (\phi - \alpha)$
translating roller	$x_p = E \cos \phi + y_m \sin \phi - R_r \sin (\phi - \alpha)$ $y_p = -E \sin \phi + y_m \cos \phi - R_r \cos (\phi - \alpha)$
swinging flat-faced	$x_p = D \cos \phi + kB \sin (\psi_m - \phi) - \frac{D \cos \psi_m \cos (\psi_m - \phi)}{\left(1 - \frac{\dot{\psi}}{\omega_c}\right)}$ $y_p = -D \sin \phi - kB \cos (\psi_m - \phi) - \frac{D \cos \psi_m \sin (\psi_m - \phi)}{\left(1 - \frac{\dot{\psi}}{\omega_c}\right)}$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$
swinging domed	$x_p = D \cos \phi + L \cos (\psi_m - \phi) + kR_f \sin (\phi - \psi_m - \alpha)$ $y_p = -D \sin \phi + L \sin (\psi_m - \phi) + kR_f \cos (\phi - \psi_m - \alpha)$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$
swinging roller	$x_p = D \cos \phi + L \cos (\psi_m - \phi) + kR_r \sin (\phi - \psi_m - \alpha)$ $y_p = -D \sin \phi + L \sin (\psi_m - \phi) + kR_r \cos (\phi - \psi_m - \alpha)$ where $k = 1$ if $D > 0$ and $k = -1$ if $D < 0$