

## FATIGUE LIFE ESTIMATION UNDER VARIABLE AMPLITUDE LOADING USING CUMULATIVE DAMAGE CALCULATIONS

### 1. INTRODUCTION

This Item describes methods of estimating the fatigue life of a component subjected to a variable amplitude loading. It may be applied to any load history. All of the methods described use the concept of fatigue damage accumulation and this is presented together with the use of the linear cumulative damage theory in Sections 3 and 4. The application of this linear damage summation to nominal stress, local stress, and nominal and local strain data is then given; Section 5 describes the stress approach and Section 6 the strain approach. Once the stress or strain history has been established, cycle counting can be performed and the damage sum calculated. Two algorithms for defining and counting cycles are described in Section 7. The Item also describes the effects of variables such as residual stress, fretting, notch size, bending and surface treatment on the endurance of a component under variable amplitude loading and how the effect of these variables on the local stress should be taken into account. The entire calculation procedure and the differences between the stress history and strain-life methods are described, and Section 8 provides some guidance on the data required to carry out cumulative damage calculations. Fully worked examples are given. This Data Item does not include any factors of safety. It should be noted that appropriate factors should be applied as required.

Definitions of certain terms used in this Item are given in Data Item No. 70016\*.

### 2. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

|              |                                                            |                  |                     |
|--------------|------------------------------------------------------------|------------------|---------------------|
| $b$          | fatigue strength exponent                                  |                  |                     |
| $c$          | fatigue ductility exponent                                 |                  |                     |
| $D$          | summation of damage increments                             |                  |                     |
| $D_f$        | value of $D$ at failure                                    |                  |                     |
| $e$          | nominal strain                                             |                  |                     |
| $E$          | modulus of elasticity                                      | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_{act\ r}$ | actual stress at notch root at stress reversal point $r$   | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_n$        | material reference stress (see Data Item No. 76014*)       | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_R$        | initial residual stress at surface of component notch root | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_t$        | tensile strength of material                               | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $K'$         | cyclic strength coefficient                                | N/m <sup>2</sup> | lbf/in <sup>2</sup> |

*For footnote see end of Notation section*

\* Data Item No. 70016, "Terms and notation for fatigue endurance data".

|                    |                                                                                         |                  |                     |
|--------------------|-----------------------------------------------------------------------------------------|------------------|---------------------|
| $K_f$              | fatigue notch factor                                                                    |                  |                     |
| $K_t$              | elastic stress concentration factor                                                     |                  |                     |
| $K_\epsilon$       | local strain concentration factor                                                       |                  |                     |
| $K_\sigma$         | local stress concentration factor                                                       |                  |                     |
| $m$                | material characteristic (see Data Item No. 76014)                                       |                  |                     |
| $m'$               | index characterising shape of $R$ – $M$ diagram<br>(see Data Item No. 76014)            |                  |                     |
| $n$                | number of cycles                                                                        |                  |                     |
| $N$                | endurance                                                                               |                  |                     |
| $N_b$              | endurance, in cycles, under constant amplitude loading at same<br>stress cycle as block |                  |                     |
| $n'$               | cyclic strain hardening coefficient                                                     |                  |                     |
| $n_b$              | number of stress cycles in programme block                                              |                  |                     |
| $p$                | programme number (also used as suffix)                                                  |                  |                     |
| $p_f$              | number of programmes to failure                                                         |                  |                     |
| $q$                | number of stress reversal points per programme                                          |                  |                     |
| $r$                | sequential stress reversal point number (also used as suffix)                           |                  |                     |
| $S$                | stress                                                                                  | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $S_a$              | alternating stress                                                                      | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $S_{a0}$           | alternating stress at zero mean stress                                                  | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $S_{am}$           | fatigue strength                                                                        | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $S_{el\ r}$        | theoretical elastic stress at stress reversal point $r$                                 | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $S_m$              | mean stress                                                                             | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $S_{nom\ r}$       | nominal applied stress at stress reversal point $r$                                     | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $\Delta e$         | nominal strain range                                                                    |                  |                     |
| $\Delta S$         | nominal stress range                                                                    | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $\Delta S_{el\ r}$ | theoretical elastic stress range between stress reversal points $r$<br>and $(r - 1)$    | N/m <sup>2</sup> | lbf/in <sup>2</sup> |

|                     |                                   |                  |                     |
|---------------------|-----------------------------------|------------------|---------------------|
| $\Delta\varepsilon$ | local strain range                |                  |                     |
| $\Delta\sigma$      | local stress range                | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $\varepsilon$       | local strain                      |                  |                     |
| $\varepsilon_{el}$  | elastic component of local strain |                  |                     |
| $\varepsilon'_f$    | fatigue ductility coefficient     |                  |                     |
| $\varepsilon_p$     | plastic component of local strain |                  |                     |
| $\sigma$            | local stress                      | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $\sigma'_f$         | fatigue strength coefficient      | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $\sigma_{max}$      | maximum local stress              | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $\sigma_0$          | local mean stress                 | N/m <sup>2</sup> | lbf/in <sup>2</sup> |

## Suffixes

|            |                                                                           |
|------------|---------------------------------------------------------------------------|
| <i>act</i> | actual value at notch root                                                |
| <i>C</i>   | compressive value                                                         |
| <i>d</i>   | relates to feature under design                                           |
| <i>i</i>   | relates to <i>i</i> th stress or damage increment of <i>k</i> occurrences |
| <i>k</i>   | total number of damage increments                                         |
| <i>r</i>   | relates to a sequential stress reversal point number                      |
| <i>s</i>   | relates to test specimen                                                  |
| <i>T</i>   | tensile value                                                             |

\* Data Item No. 76014, "Estimation of endurance and construction of constant amplitude *S-N* curves from related data corrected for notch and mean stress effects".

Suffixes 1, 2, 3, *et cetera*, are used to number sequentially the stress reversal points of each programme and the number of programmes.

## Sign Convention

Tensile stresses are positive.

Throughout this Data Item the terms "nominal stress" and "nominal strain" indicate the free field stress or strain.

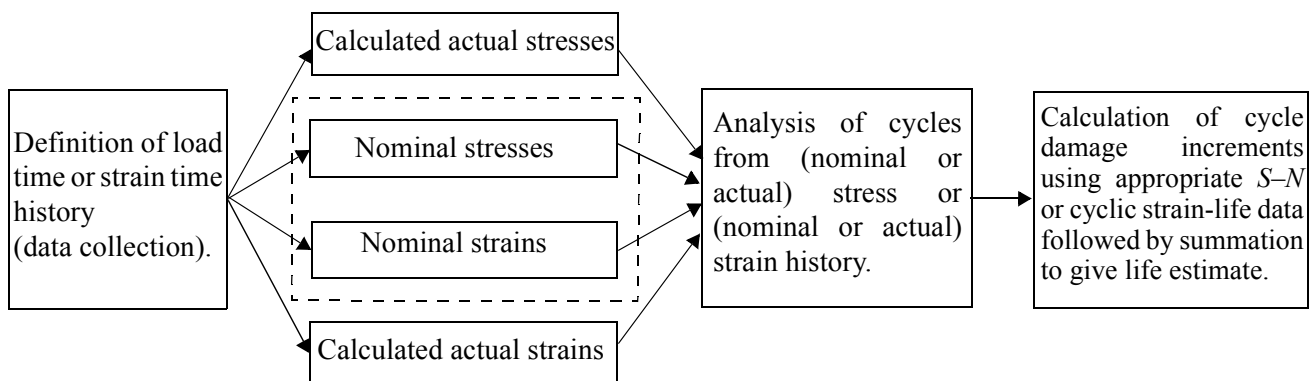
### 3. FATIGUE DAMAGE UNDER VARIABLE AMPLITUDE LOADING

The basic precept of damage accumulation is that application of a cyclic stress will produce fatigue damage. In a component subject to fatigue loading, the severity of the damage will be a function of the number and magnitude of the stress cycles applied. When this accumulated damage reaches a critical value, fatigue failure of the component is deemed to have occurred. Although this concept is simple, in practice difficulties are encountered because it is difficult to assess the damage caused by operation at any given stress level,  $S_i$ , for a specified number of cycles,  $n_i$ . Furthermore, the order and relative magnitude of the applied stress cycles may strongly influence the damage accumulation process.

### 4. METHODS OF ENDURANCE ESTIMATION

Many theories have been proposed for endurance estimation under variable amplitude loading, the majority of which use constant amplitude fatigue data. This Item uses a linear method for estimating the damage increments resulting both from the application of individual cycles of stress and from stress sequences. The basis of the method is described in this section.

Sections 5 and 6 describe how local stresses or strains may be derived from the applied load history. Once the nominal or local stress or strain history of the component has been determined it is transformed, using a suitable cycle counting method, to define the spectrum of stress or strain amplitudes and associated numbers of counts. The damage sum can then proceed as described in Section 8, using the corresponding stress of a suitable constant amplitude  $S-N$  curve or suitable strain-life curve. Thus, the necessary order of calculation for a cumulative damage endurance estimate is summarised by the flow chart shown in Sketch 4.1.



**Sketch 4.1 Flowchart for damage calculations**

#### 4.1 The Linear Cumulative Damage (Palmgren-Miner) Rule

The simplest cumulative damage calculations such as those proposed by Miner (see Derivation 3) rest on the hypothesis that each cycle of any loading sequence damages the material and that the damage is the same as the damage per cycle under constant amplitude loading to failure under the same cycle. The cumulative damage ratio,  $D$ , is defined as the summation of the damage increments for the various stress cycles, represented by the following equation.

$$\sum_{i=1}^{i=k} \frac{n}{N} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots + \frac{n_k}{N_k} = D. \quad (4.1)$$

The value of  $D$  at failure is given the symbol  $D_f$ , and failure in any of the cumulative damage theories is assumed to occur when the summation of damage increments equals 1.

There are several sources of error in cumulative damage estimation processes, including the scatter inherent in constant amplitude  $S-N$  data. Even for plain specimens under laboratory conditions, scatter in available curves drawn through  $S-N$  data will give rise to variations in the value of  $D_f$  because, for a given value of  $S_i$ , there will be a variation in the fraction  $n_i/N_i$ . In practice, such variations are normally accounted for by the use of factors (see Derivation 10). Furthermore, the assumption that identical processes occur both in the variable and constant amplitude loading situations is known to be incorrect; this is the case for plain specimens, and for components containing stress concentrators.

A major source of error in the simple hypothesis occurs for components containing a concentrator if the positive (tensile) peaks and/or negative (compressive) troughs in the nominal loading history are sufficiently large to cause localised yielding at the concentrator. Such yielding may occur under both constant and variable amplitude loading. In the former case the stress-strain condition at the notch root is affected largely by yielding during the first load cycle but in the latter case the stress-strain condition is affected by the previous loading history in conjunction with the load being applied. Therefore there may be differences between the fatigue damage inflicted by nominally identical load cycles contained within different sequences of loading.

For the above reasons the value of the cumulative damage ratio at failure,  $D_f$ , may not be equal to unity. If  $D_f$  is greater than unity, tensile loads during the variable amplitude stress-time history may have been high enough to cause compressive residual stresses which retard crack initiation and propagation, thus increasing life. Conversely,  $D_f$  values may also be considerably less than unity, for example, where there are tensile residual stresses. The effect on the fatigue life of components containing such residual stresses may be estimated by the local stress or strain method.

It should be noted that use of the Palmgren-Miner rule may not produce accurate predictions for reasons including those given above, and that there are other cumulative damage models available. However, none of these models is universally applicable and many are subject to the same problems of estimating the damage caused by particular cycles. While the Palmgren-Miner rule may be inadequate in some respects, it has the virtue of being simple in principle; it has been widely applied in endurance estimation and as a result the reasons for its inadequacies are recognised and can be allowed for in design.

## 5. THE CALCULATION OF STRESSES FOR THE STRESS LIFE METHOD

For a given loading sequence the nominal stresses associated with each successive point of stress reversal can be converted into a series of local stresses in the component at particular locations such as at a notch root. If yielding does not occur the local stresses are simply the product of the nominal stresses and the elastic stress concentration factor, so that the nominal and local stresses are linearly related. However, in the majority of practical cases where fatigue is an issue, the stress level reached somewhere in the sequence of loads will be sufficiently high to cause localised yielding. When this occurs the local stress is determined by the form of the material cyclic stress-strain curve and is therefore not linearly related to the nominal stress. In general, this yielding modifies subsequent stress values. Thus, even for simple cases, where the nominal stress multiplied by the stress concentration factor gives a stress that is within the elastic range, the nominal and local stress may no longer be proportional.

### 5.1 Notch Root Stresses

All components contain some type of geometrical or microstructural discontinuity. These discontinuities, or stress concentrators, usually result in maximum local stresses,  $\sigma_{max}$ , at the discontinuity which are

greater than the nominal stress in the component,  $S$ . In ideally elastic components the ratio of these stresses is designated  $K_t$ , the theoretical stress concentration factor.

$$K_t = \frac{\sigma_{max}}{S} \quad (5.1)$$

This theoretical stress concentration factor is solely dependent on geometry and mode of loading. Stress concentration curves for standard configurations may be obtained from the appropriate Data Items.

In the stress life approach, the effect of the notch may be accounted for by the fatigue notch factor,  $K_f$ . This value relates the unnotched fatigue strength to the notched fatigue strength subject to the notch root stress level at a given life. Where  $K_f \leq K_t$  for any given endurance,

$$K_f = \frac{S_{am}^{(unnotched)}}{S_{am}^{(notched)}} \quad (5.2)$$

$K_f$  is usually taken at the endurance limit value.

Unlike the theoretical stress concentration factor,  $K_t$ , the fatigue notch factor,  $K_f$ , is a function of the material properties as well as geometry and loading. For example, for a notched specimen under variable amplitude loading,  $K_f$  may be less than  $K_t$  in the presence of tensile overloads which induce a compressive residual stress at the notch root. It should be noted that the use of  $K_f$  may prove non-conservative in such instances.

Two methods of determining the actual notch root stresses are fully described in Data Item No. 76014. The first employs an elastic/perfectly plastic stress-strain curve. This method may not be satisfactory if both tensile and compressive yielding occur. The second method offers an improvement in the calculation of notch root stress and allows for the effects of tensile and compressive yielding in the same cycle. This second method uses the Neuber equation for plastic stress concentration factors in conjunction with a description of the material stress-strain curve, based on a material reference stress,  $f_n$ , and a characteristic index,  $m$ . Methods for obtaining  $f_n$  and  $m$  from standard material properties are given in Appendix A of Item No. 76014. Figure A1 of Data Item No. 76014 presents curves that relate the theoretical elastic stress at the notch root to the actual stress for various values of  $m$ .

Analysis of the local stresses at load reversal points using Figure A1 of Item No. 76014 is carried out strictly in order of load application, as illustrated in the worked example in Section 10.1. The procedure may be restated as follows.

- (i) Any residual stress prevailing at the surface of the component at the stress concentration will affect the stress reached at the first load reversal point. The theoretical elastic stress at this point is given by

$$S_{el1} = f_R + K_t S_{nom1} \quad (5.3)$$

From this equation  $S_{el1}/f_{nT}$  (or  $S_{el1}/f_{nC}$  if  $S_{el1}$  is compressive) is calculated. Figure A1 of Data Item No. 76014 is then used, with the appropriate value of  $m$ , to determine  $f_{act1}/f_{nT}$  (or  $f_{act1}/f_{nC}$  if the stresses are compressive) and hence  $f_{act1}$  is determined.

- (ii) For the second and all subsequent load reversals  $\Delta S_{el\ r}$  is calculated as

$$\Delta S_{el\ r} = K_t S_{nom(r-1)} - K_t S_{nom\ r} \quad (5.4)$$

so that

$$S_{el\ r} = f_{act(r-1)} - \Delta S_{el\ r} \quad (5.5)$$

The value of  $S_{el\ r}$  is then used to find  $f_{act\ r}$  in the same way as described in (i).

Following this procedure two special cases may arise.

- For blocks of identical loadings it is only necessary to determine  $f_{act\ r}$  for the first four reversals since all subsequent reversals in the block will be identical to the second pair of these, see the example in Section 10.1.
- Where the loading sequence has been idealised to a repeated programme of loads the stress at the first load reversal of the second programme is calculated from the values obtained at the end of the first programme so that

$$\Delta S_{el1(p=2)} = K_t S_{nom\ q(p=1)} - K_t S_{nom\ 1(p=2)} \quad (5.6)$$

$$S_{el1(p=2)} = f_{act\ q(p=1)} - \Delta S_{el1(p=2)} \quad (5.7)$$

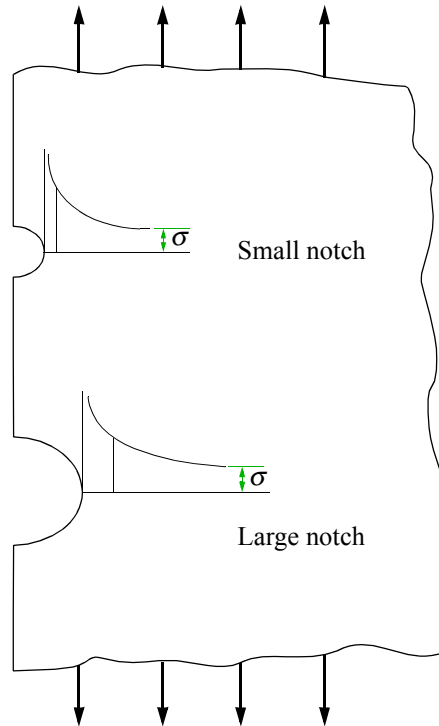
and the chain calculation continues (see the example in Section 10.1). Prediction of  $f_{act\ 1}$  for the second programme from  $f_{act\ q}$  of the first programme thus will give a different value of  $f_{act\ 1}$  from that found for the first programme. Because of this it is necessary to follow the calculations of  $f_{act\ r}$  through the second programme. It is not normally necessary to continue this procedure into the third programme since variations in  $f_{act\ q}$  after the second programme are usually negligible. The requirement to extend the procedure to the third programme may be judged by the difference between the values of  $f_{act\ q}$  for the first and second programmes.

### 5.1.1 Correction for notch size effects

Notch sensitivity is not only a function of shape but also of notch size. The size effect of the notch root radius may be attributed to a volume effect. For a given notch geometry the shape of the stress gradient is constant when normalised with respect to the dimension of the notch root radius, the specimen size and loading. As the absolute size of the notch increases, the volume of highly stressed material near the notch increases. This is illustrated in Sketch 5.1.

In relatively small notches, the probability is that the very small volume of material under high stress will not contain a flaw and the fatigue crack that determines endurance may be initiated from a nearby flaw at a slightly lower stress value than in the case of a scaled-up component. Since the flaw will be subject to a lower stress the endurance will be increased over that for larger notches that are geometrically similar. Size effect is also observed during the crack growth phase of the fatigue process.

The degree of this effect may be estimated using the method given by Equation (4.23) in Data Item No. 76014.



**Sketch 5.1 Volume of critically stressed material at small and large notches**

## 5.2 Joints and Fretting

Data on the endurance of structural joints may be located via the ESDU Index. Data are available on four types of joints, namely bonded joints, riveted joints, bolted joints and joint systems incorporating patented fastener systems. The fatigue endurances of various joint configurations under the FALSTAFF standard loading sequence are given in these latter Data Items.

The fatigue strength of a bonded joint depends markedly on the local stress distribution. A major factor affecting the fatigue strength of bolted joints is fretting, as well as the local stress distribution determined by the position of the bolts and the clamping force applied thorough them. Similarly, in a riveted joint, the failure is dependent on the rivet fit and the clamping force between the sheets, which influence the local stress distribution; fretting is also an important consideration.

Structural joints in particular may experience some form of fretting which reduces the fatigue life. The stress values at the region of fretting are a controlling factor in the fatigue life of the joint and it is necessary to take as full account as possible of the actual stress cycle in the manner described above. This should also take full account of the actual applied loading, for example any applied static loads should also be considered. Furthermore, the mode of failure of a structural joint is dependent on joint design, assembly variables and applied stress level.

Although some relationships have been proposed for the reduction in fatigue life due to fretting fatigue, none of these has been sufficiently used to have been shown to be generally applicable. Therefore it will be necessary to use *S-N* data obtained from appropriate specimens when taking account of fretting fatigue.

for components subjected to variable amplitude loading. It may also be necessary to obtain endurance data from appropriate specimens tested under a variable amplitude loading spectrum representative of the expected service conditions.

Data Item No. 90031\* considers the factors affecting fatigue life under fretting conditions in more detail.

Fretting has an especially important influence on the fatigue strength of lugs. The fretting of lugs will depend, for example, on the fit and material of the pin, and also on whether or not a bush is used, all of which influence the endurance of the joint under fatigue loading. Pin and lug material combinations also have an important influence on the endurance. These effects are in addition to size and other geometry effects, and to any residual stress effects induced by the fit of the pin and any bush. Data Item No. A.05.02† describes how these factors may be accounted for if suitable experimental data for a particular component are unavailable. As with the other types of joints described above, care must be taken that the constant amplitude  $S-N$  data used in the damage calculation are consistent with the component design under consideration. Data on the endurance of specific lug configurations may be located via the ESDU Index.

As with bolted joints, it is usually more appropriate to use the stress-life approach for lugs because it is difficult to predict the precise failure location and measure the appropriate strain. Furthermore, it is difficult to quantify the effects of fretting, which may be strongly influenced by factors such as pin fit and pin bending.

### 5.3 Other Factors Affecting the Calculation of Stresses

Other factors that affect the fatigue endurance of components include surface treatments (such as anodising and shot peening) and other effects such as pre-service cold working around the hole. Cold working and those surface treatments that introduce a compressive residual stress may increase the endurance of the component to some degree. As with fretting fatigue, the effect on the endurance of factors such as surface treatments cannot easily be quantified. It is advisable, therefore, to use  $S-N$  data obtained from appropriate specimens tested under representative loadings where possible. Some data for the effect of different surface treatments on endurance are given in Data Items, which may be located via the ESDU Index.

Where a compressive stress of known (or calculable) value is applied and causes a residual stress in the component (having caused localised plastic deformation), the local stress at the notch root may be estimated using the methods described above.

## 6. THE STRAIN-LIFE METHOD AND THE USE OF STRAIN-LIFE DATA

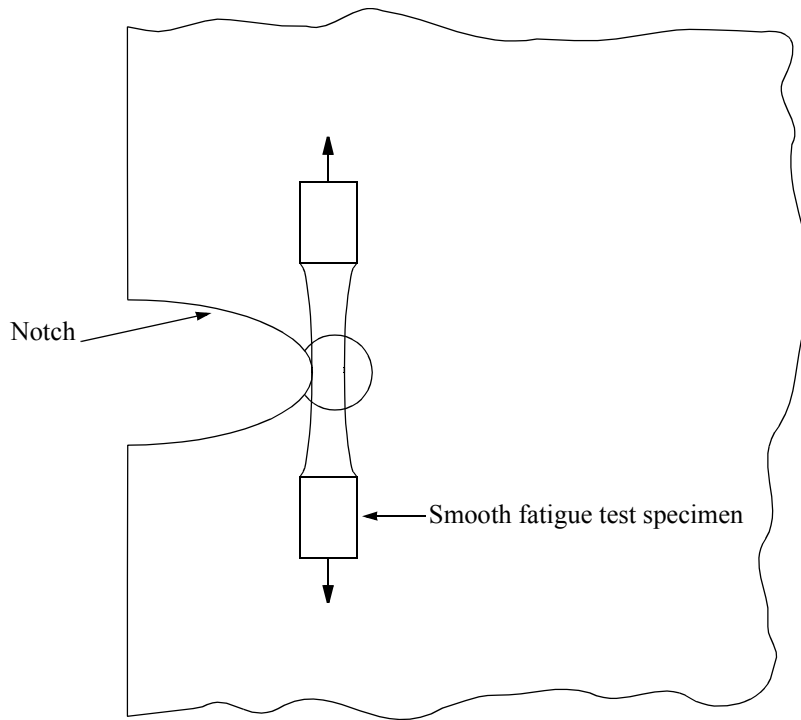
The strain-life method is a means of accounting for notch root plasticity. From a knowledge of the notch root strain history and smooth specimen strain-life data or fatigue properties, fatigue life estimates may be made for notched components. One of the advantages of this method is that it takes into account changes in local mean and residual stresses.

It has been recognised that the fatigue failure process involves three phases: fatigue damage leading to crack initiation, followed by a stable propagation phase and finally, when the crack reaches a critical size, unstable rapid crack growth to fracture. The local stress-strain approach may be used to predict the crack initiation phase and takes into account localised plastic deformation in the component under consideration. Although it is usual for engineering components to be designed such that the nominal stresses remain elastic, stress concentrators often cause plastic strains to develop in the region of notch roots which may form crack initiation sites. In such a situation the load spectrum is not wholly indicative of the strain history at the

\* Data Item No. 90031, "Fretting fatigue".

† Data Item No. A.05.02, "Estimation of endurance of pin joints".

notch, rendering the stress-life approach less adequate to predict the endurance and the strain-life approach more suitable.



**Sketch 6.1 Equally stressed volumes of material**

Sketch 6.1 illustrates the concept of a small unnotched fatigue specimen used to represent a simple notched component. The basis of the local stress-strain approach is the assumption that the local fatigue response of the material at the site of crack initiation is analogous to the response of a small smooth specimen subjected to the same cyclic strains and stresses (see Derivation 7).

The use of cyclic stress-strain data in the estimation of fatigue endurance is described below, and the following information is necessary to carry out the actual endurance calculation:

1. material properties obtained from smooth specimens tested under constant amplitude strain-controlled conditions, that is, the cyclic stress-strain response and strain-life data;
  2. the stress-strain history at the critical location, for example, at a notch;
  3. techniques for cycle counting;
  4. methods to take account of mean stress effects;
  5. a suitable body of data for a variety of features, including notches, joints, surface treatment effects, *et cetera*;
- and 6. a damage summation technique, for example, the Palmgren-Miner linear damage rule.

Neuber's rule is often used in conjunction with the cyclic stress-strain properties and fatigue stress concentration factor to calculate the local stresses and strains from the external loading and geometry. Neuber's rule (see Derivation 4) is an approximation and may be written as

$$K_t = (K_\epsilon K_\sigma)^{1/2}. \quad (6.1)$$

This may be rearranged, using  $K_\sigma = \sigma/S$  and  $K_\epsilon = \epsilon/e$ , to give the following.

$$K_t^2 = \frac{\sigma \epsilon}{S e}. \quad (6.2)$$

This relationship may be modified for application to fatigue loading by substituting the nominal stress range,  $\Delta S$ , nominal strain range,  $\Delta e$ , local stress range,  $\Delta \sigma$ , and local strain range,  $\Delta \epsilon$ , for nominal stress, nominal strain, local stress and local strain, respectively. The relationship thus becomes

$$K_t(\Delta S \Delta e)^{1/2} = (\Delta \sigma \Delta \epsilon)^{1/2}. \quad (6.3)$$

If the loads are such that the nominal stresses in the component are elastic,  $\Delta S/\Delta e = E$  and Equation (6.3) may be written as

$$\frac{(K_t \Delta S)^2}{E} = \Delta \sigma \Delta \epsilon. \quad (6.4)$$

All of the terms on the left-hand side of this equation are known from the geometry, the loading and the material properties of the component. To resolve the right-hand side of the equation, curves may be prepared (see Reference 16) from the stable cyclic stress-strain curve to give  $\sigma$  versus the product  $\sigma \epsilon$ . This technique provides one method of computing local stresses and strains, although care must be taken with interpolation of values. Test methods are available for obtaining cyclic stress-strain data (see Derivations 5 and 6).

Ideally the cyclic stress-strain curve should be established experimentally. However, there are several other methods that may be used to estimate the cyclic stress-strain curve. These include use of an empirical form of the cyclic stress-strain curve, which is an expression of the total strain in terms of a sum of elastic and plastic components (see Derivation 2):

$$\epsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K'} \right)^{1/n'}. \quad (6.5)$$

The equation of the hysteresis loop may then be derived from the equation of the cyclic  $\sigma$ - $\epsilon$  curve, Equation (6.5), using Masing's hypothesis (Derivation 1), which allows the hysteresis loop to be obtained by doubling the cyclic stress-strain curve. The general hysteresis curve equation thus derived is

$$\frac{\Delta \epsilon}{2} = \frac{\Delta \sigma}{2E} + \left( \frac{\Delta \sigma}{2K'} \right)^{1/n'}. \quad (6.6)$$

For fatigue life estimation, strain-life data are also required. It was found by Coffin (Reference 13) and Manson (Reference 12) that plastic strain-life data may be estimated by a power law function:

$$\frac{\Delta \epsilon_p}{2} = \epsilon'_f (2N)^c. \quad (6.7)$$

Using this relationship and a form of Basquin's equation (Reference 11) an empirical expression for the

total strain amplitude may be developed as

$$\frac{\Delta \epsilon}{2} = \frac{\Delta \epsilon_{el}}{2} + \frac{\Delta \epsilon_p}{2} = \frac{\sigma'_f}{E} (2N)^b + \epsilon'_f (2N)^c. \quad (6.8)$$

The constants in the relationship may be obtained from cyclic tests. However, it should be noted that the cyclic response may differ considerably from the monotonic response, and approximate values of these constants should be used with caution.

The strain-life equation may be further modified to account for mean stress effects. One such modification was proposed to include the mean stress in the elastic term of the strain-life equation (see Reference 14):

$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_f - \sigma_0}{E} (2N)^b + \epsilon'_f (2N)^c. \quad (6.9)$$

Other modifications to the strain-life equation have been proposed. All of these equations are empirically based. Therefore, care must be taken to use them with data appropriate to the problem under consideration, as with the other techniques described in this Item.

## 7. DEFINITION OF CYCLES AND CYCLE COUNTING METHODS

Once all of the stresses have been determined from the applied variable amplitude loading, taking proper account of the effects of notches, joints, residual stresses and the order of loading, the next step is to divide the resulting stress-time history into individual stress cycles so that the damage increments,  $n/N$  can be determined. The two most commonly used methods, the range pairs and the rainflow techniques, are described below. Several of the principal alternative methods available are summarised in Appendix A.

It should be noted that the range pairs and rainflow methods define the same cycles. The cycles defined by the range pairs method and the rainflow algorithm are also identical to those formed using the stress-strain hysteresis loop method. Section 7.3 describes how these two cycle counting methods relate to the dynamic stress-strain history.

### 7.1 Range Pairs Cycle Counting Method

In this technique, also known as the range mean pairs method, the procedure is to select and remove, from an ordered list of actual load (or stress or strain) reversal values (that is, load maxima and minima), the adjacent pair having the smallest difference. This is repeated until all possible pairs are removed. Each pair is then considered to constitute the peak and trough of one load cycle for which a mean and alternating load can be determined.

In its elementary form this process is inefficient because only one range mean pair is extracted for each pass through the loading history. However, this basic test may be extended so that the number of passes through the loading history can be reduced.

It can be deduced that this definition of a minimum difference identifies cycles that constitute parts of other, larger cycles and that a test based on this may be used to extract more than one cycle per pass. This test may be illustrated as shown in Sketch 7.1 and may be defined as follows. If there are four successive turning points labelled  $r_1, r_2, r_3$  and  $r_4$  in order, then  $r_2 - r_3$  are a range pair if:

$$|r_2 - r_1| \geq |r_3 - r_2| \leq |r_4 - r_3|. \quad (7.1)$$