



Effect of wing height on lift and aerodynamic centre for a slender wing-body combination

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Mr R. Sanderson	— Daimler-Benz Aerospace Airbus GmbH, Bremen, Germany
Mr A.E. Sewell*	— McDonnell Douglas Corp., Long Beach, Calif., USA
Mr M.R. Smith	— British Aerospace Airbus Ltd, Bristol.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by under contract to ESDU by

Mr A. Stanbrook.

The person with overall responsibility for the work in this subject area is Mr P.D. Chappell, Head of Aircraft Aerodynamics Group.

EFFECT OF WING HEIGHT ON LIFT AND AERODYNAMIC CENTRE FOR A SLENDER WING-BODY COMBINATION

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EFFECT OF WING HEIGHT ON LIFT AND AERODYNAMIC CENTRE FOR A SLENDER WING-BODY COMBINATION

1. NOTATION AND UNITS

		SI	British
A	aspect ratio of net wing, $4(s' - r)^2/S$		
a	parameter defined in Equation (A1.9)		
C_L	lift coefficient based on net wing area		
$C_L(x)$	contribution to C_L from region forward of station x		
C_m	pitching moment coefficient about wing-root leading edge based on body maximum cross-sectional area and diameter, positive nose up		
C_N	normal force coefficient based on body maximum cross-sectional area		
C_{Vf}	forebody volume coefficient, $(\text{volume})/\pi r^2 l_f$		
C_r	root chord, <i>i.e.</i> chord at wing-body junction	m	ft
F_{WBO}	K_{WBO}/K_{WB}		
f_{WBO}	k_{WBO}/k_{WB}		
$G(\phi, \psi)$	function in expression for lift in Appendix A		
g	parameter defined in Equation (A1.13)		
h	height of wing above body centre line at station x , see Sketch 1.1	m	ft
$I(\phi, \psi)$	integral in Equation (A1.9)		
$J(\phi, \psi)$	function in expression for lift in Appendix A		
K_B	ratio of L_B due to angle of attack, α , to corresponding lift of net wing, L_W		
$K_{WB}(x)$	ratio of $L_{WB}(x)$ due to angle of attack, α , at $\delta = 0$, to corresponding lift of net wing, $L_W(x)$		
K_{WB}	value of $K_{WB}(x)$ at $x = x_1$		

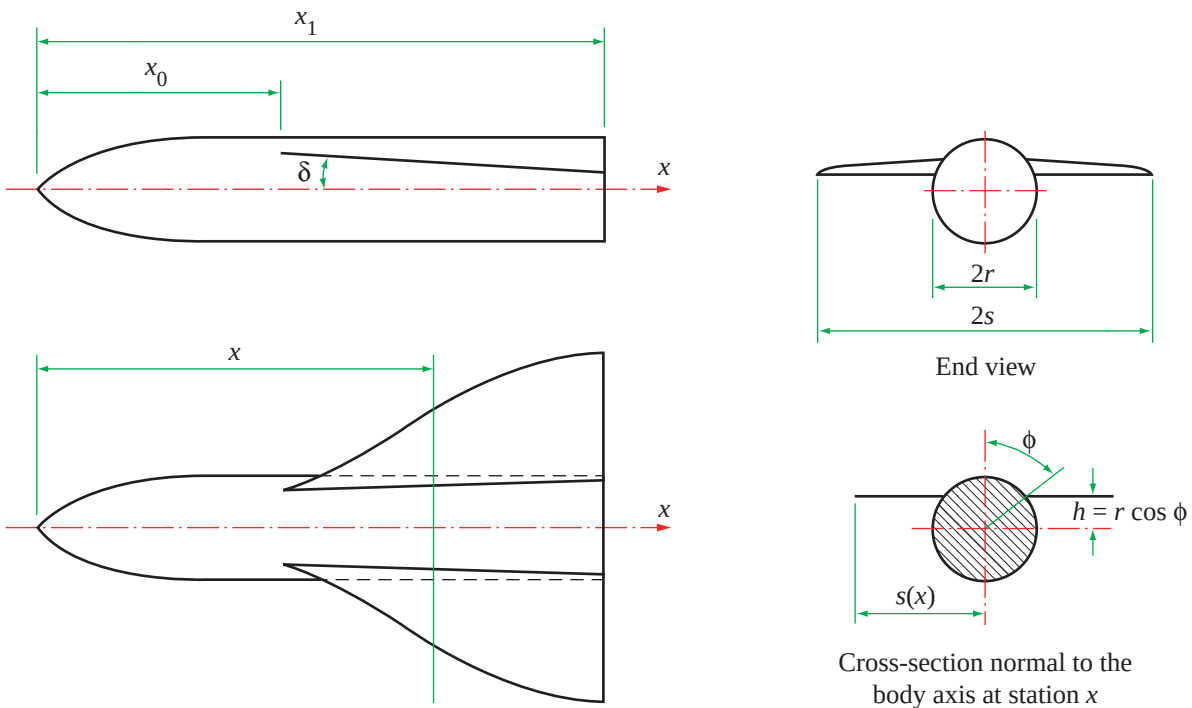
$K_{WBO}(x)$	ratio of $L_{WBO}(x)$ due to angle of attack, α , at $\delta = 0$, to corresponding lift of net wing, $L_W(x)$		
K_{WBO}	value of $K_{WBO}(x)$ at $x = x_1$		
k_{WB}	ratio of L_{WB} due to wing-body angle, δ , at $\alpha = 0$, to lift of net wing, L_W		
k_{WBO}	ratio of L_{WBO} due to wing-body angle, δ , at $\alpha = 0$, to lift of net wing, L_W		
L	lift, $L(x_1)$	N	lbf
$L(x)$	contribution to L from region forward of station x	N	lbf
l_f	forebody length	m	ft
M_∞	free-stream Mach number		
M	pitching moment about axis through $x = x_0$	N m	lbf ft
r	body maximum radius	m	ft
S	net wing area	m ²	ft ²
s	wing semi-span (value of $s(x)$ at station x_1), see Sketch 1.1	m	ft
$s(x)$	spanwise extent of wing at station x , see Sketch 1.1	m	ft
s'	semi-span of reference wing (value of $s'(x)$ at station x_1)	m	ft
$s'(x)$	spanwise extent of reference wing at station x , see Sketch 3.1	m	ft
t_1, t_2, t_2	transformation parameters in Appendix A		
V_∞	free-stream velocity	m/s	ft/s
x	axial distance from forebody tip	m	ft
x'_{ac}	distance of aerodynamic centre of wing-body combination aft of x_0	m	ft
$\Delta x'_{ac}$	shift of aerodynamic centre due to off-centre mounting of wing	m	ft
x_m	axial location of pitching moment axis	m	ft
α	angle of attack of body centre-line or wing alone	rad	rad
β	compressibility parameter, $ M_\infty^2 - 1 ^{1/2}$		
δ	wing-body angle, see Sketch 1.1	rad	rad
Λ_0	sweepback of wing leading edge	deg	deg

$\Lambda_{1/2}$	sweepback of wing mid-chord line	deg	deg
λ	taper ratio of net wing, tip chord/root chord		
ρ_∞	free-stream air density	kg/m ³	slug/ft ³
Φ	ϕ/π		
ϕ	angle defining wing height at station x , see Sketch 1.1	rad	rad
ϕ_1	value of ϕ at station x_1	rad	rad
ψ	$r/s(x)$		
ψ'	$r/s'(x)$		
ψ'_1	value of ψ_1 at station x_1 , r/s'		

Subscripts

B	denotes body
C	denotes wing-body combination with centrally-mounted wing
CO	denotes wing-body combination with off-centre* wing
W	denotes net wing
WB	denotes wing-body combination with centrally-mounted wing, excluding contribution from body alone
WBO	denotes wing-body combination with off-centre wing, excluding contribution from body alone
0	relates to station at most upstream part of net wing
1	relates to station at which wing attains its maximum spanwise extent
$[]_{\alpha=0}$	denotes value for $\alpha = 0$
$[]_{\delta=0}$	denotes value for $\delta = 0$

* A wing mounted above or below the body centre-line is described in this Item as "off-centre".



Sketch 1.1 Wing-Body Geometry

2. INTRODUCTION

This Item presents a theoretical method, based on slender-body theory, for estimating the effect on the lift and aerodynamic centre of mounting a wing off-centre* on a body by direct comparison with the centrally-mounted wing.

The method is derived for the case of a plane wing with a sweptback leading edge†, a local span increasing (and then possibly constant) with increase in streamwise distance, and an unswept trailing edge. The wing is mounted on a body with a circular cross-section of constant diameter over the length of the wing; the wing may be set at an angle to the body centre line, see Sketch 1.1.

It is assumed that the flow is attached throughout and that the effects of viscosity are confined to thin boundary layers on the surface and to the wake. Disturbances are assumed to be small, thus allowing the use of linearised approximations to the equations of inviscid compressible flow. The further assumption is made that the velocities change slowly in the streamwise direction relative to their rates of change across the stream; slender-body theory is then applicable.

The calculated data for use in the method are presented graphically. Equations from which they may be derived are given in Appendix A. The use of the method is illustrated by means of two worked examples in Section 6.

Methods for the prediction of the lift-curve slope and aerodynamic centre position of wing-body combinations where the wing is mounted centrally on the body are given in Item Nos 91007 and 92024 (References 6 and 7) respectively, for subsonic and supersonic speeds.

3. METHOD

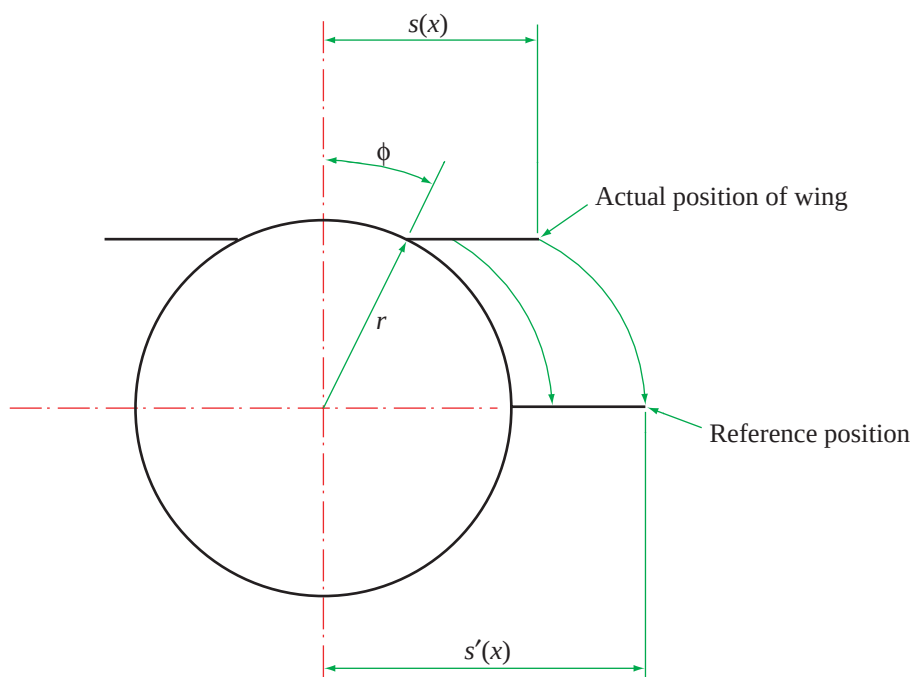
In this Item the effect on the lift of a wing-body combination due to mounting the wing off-centre is presented in the form of the ratio between the lift (excluding that on the body alone) of the actual wing body combination and one in which the net wing is removed and remounted centrally, *i.e.* $\phi = \pi/2$. Sketch 3.1 shows the relationship between the two wings, the local spanwise extents of which are connected by the equation

$$s'(x) = s(x) + r(1 - \sin\phi). \quad (3.1)$$

For convenience, the local spanwise extents are expressed in non-dimensional form through the parameters $\psi (= r/s(x))$ and $\psi' (= r/s'(x))$.

* A wing mounted above or below the body centre line is described in this Item as "off-centre".

† See comments in Section 4 for application to rectangular wings.



Sketch 3.1 Relationship between actual and reference positions of wing at station x

3.1 Lift Due to Angle of Attack ($\delta = 0$)

Item No. 91007 presents the lift (L_C) on a wing-body combination with a centrally-mounted wing in the form

$$L_C = L_B + L_{W(B)} + L_{B(W)} \quad (3.2)$$

where

- (i) L_B is the lift of the body alone,
- (ii) $L_{W(B)}$ is the lift on the wing in the presence of the body

and (iii) $L_{B(W)}$ is the change of the lift on the body due to the presence of the wing.

Data for each of these components are then presented in the form of their ratios to the lift of the net wing alone (at the same angle of attack) K_B , $K_{W(B)}$ and $K_{B(W)}$.

The last two terms in Equation (3.2) may be combined in the form

$$L_{WB} = L_{W(B)} + L_{B(W)} \quad (3.3)$$

to give the lift on the wing-body combination, excluding the contribution from the body alone.

Similarly, the ratios with the lift of the net wing may be combined as

$$K_{WB} = K_{W(B)} + K_{B(W)}. \quad (3.4)$$

The value of K_{WB} resulting from slender-body theory is plotted in Figure 1 as a function of ψ'_1 . The expression from which K_{WB} was calculated is given in Equation (3.5) of Item No. 91007.

The treatment in Derivation 1, when the wing is mounted off-centre, only gives the lift in the combined form of Equation (3.3). In this Item the terms L_{CO} , L_{WBO} and K_{WBO} are used to distinguish the off-centre case from the centrally-mounted case. For the off-centre case, Equation (3.2) then becomes

$$L_{CO} = L_B + L_{WBO}. \quad (3.5)$$

Data from which to calculate $[L_{WBO}]_{\delta=0}$ are presented in this Item in the form F_{WBO} , the ratio of K_{WBO} to $K_{WB} (\equiv [L_{WBO}]_{\delta=0}/[L_{WB}]_{\delta=0})$. The ratio F_{WBO} is plotted in Figure 2 as a function of ϕ_1/π and ψ'_1 for the reference wing. Data are only presented for values of ϕ_1/π from 0 to 0.5 (the wing above the centre line) since the effect is symmetrical above and below the centre line, so the value at ϕ_1 is identical to that at $\pi - \phi_1$.

The equation from which F_{WBO} was calculated is given as Equation (A2.4) of Appendix A.

3.2 Lift Due to Wing-body Angle ($\alpha = 0$)

Item No. 91007 also presents the lift on a centrally-mounted wing due to wing-body angle, δ , in terms of the ratios

(i) $k_{W(B)}$, corresponding to the lift on the wing in the presence of the body,

and (ii) $k_{B(W)}$, corresponding to the change of the lift on the body due to the presence of the wing,

each related to the lift on the net wing alone at an angle of attack equal to the wing-body angle. These may be combined in the form

$$k_{WB} = k_{W(B)} + k_{B(W)}. \quad (3.6)$$

The value of k_{WB} resulting from slender-body theory is plotted in Figure 1 as a function of ψ'_1 . The expressions from which k_{WB} was calculated are given in Equations (A1.1) and (A1.10) of Item No. 91007.

Again, available theory for the lift of a wing-body combination with an off-centre wing (Derivation 1) gives only the combined lift. As before, the term k_{WBO} is used to distinguish the off-centre case from the centrally-mounted case. Data from which to calculate the lift are presented in this Item in the form f_{WBO} , the ratio of k_{WBO} to $k_{WB} (\equiv [L_{WBO}]_{\alpha=0}/[L_{WB}]_{\alpha=0})$. The ratio f_{WBO} is plotted in Figure 3, again as a function of ϕ_1/π and ψ'_1 for the reference wing and plotted only for values of ϕ_1/π from 0 to 0.5.

The equation from which f_{WBO} was calculated is given as Equation (A2.5) of Appendix A.