



ESDU 95010

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Computer program for estimation of spanwise loading of wings with camber and twist in subsonic attached flow (Lifting – surface theory) (With Appendix A: effect of plain flaps)

**Associated software: ESDUpacs A9510
B9510**



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COMPUTER PROGRAM FOR ESTIMATION OF SPANWISE LOADING OF WINGS WITH CAMBER AND TWIST IN SUBSONIC ATTACHED FLOW (LIFTING-SURFACE THEORY) (WITH APPENDIX A: EFFECT OF PLAIN FLAPS)

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COMPUTER PROGRAM FOR ESTIMATION OF SPANWISE LOADING OF WINGS WITH CAMBER AND TWIST IN SUBSONIC ATTACHED FLOW (Lifting-Surface Theory) (WITH APPENDIX A: EFFECT OF PLAIN FLAPS)

The program has been written in "STRICT" Microsoft FORTRAN 77 for use on machines using PC/MS DOS.

1. NOTATION AND UNITS

		SI	British
A	aspect ratio		
a_1	lift-curve slope	rad^{-1}	rad^{-1}
C_L	lift coefficient		
C_{LL}	local lift coefficient		
C_{Lspr}	lift coefficients specified in input, $1 \leq r \leq N_{sp}$		
C_m	pitching moment coefficient about wing apex, based on $\bar{c}$		
C_{mL}	local pitching moment coefficient about local quarter-chord point, based on c , positive leading edge up		
c	local chord	m	ft
$\bar{c}$	geometric mean chord	m	ft
c_0	root (centre-line) chord	m	ft
c_i	chord at crank for $1 \leq i \leq N_k$, $c_{N_k+1} \equiv c_t$, see Sketch 3.1	m	ft
c_{Ms1}	chord at first outboard collocation station	m	ft

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c_t	tip chord ($\equiv c_{N_k + 1}$)	m	ft
$\{L_m\}$	integer set indicating loading types to be calculated input $L_m = 1$ generates loading due to incidence input $L_m = 3$ generates loading due to camber at zero incidence input $L_m = 4$ generates loading due to twist at zero incidence		
M_l	free-stream Mach numbers in input, $1 \leq l \leq N_M$		
N_{cCi}	number of chordwise stations at which camber co-ordinates, ξ_{Cij} , ζ_{Cij} are defined, $1 \leq i \leq N_{sC}$		
N_k	number of cranks in half-wing planform (where there is a change in sweep in either leading edge or trailing edge or both)		
N_L	number of loading types		
N_M	number of Mach numbers		
N_{Mc}	number of Multhopp chordwise collocation stations		
N_{Ms}	number (tip to tip) of Multhopp spanwise collocation stations		
N_o	number of spanwise stations for output		
N_{sC}	number of spanwise stations at which camber co-ordinates, ξ_{Cij} , ζ_{Cij} are defined		
N_{sp}	number of specified values of incidence or overall lift coefficient		
N_{sT}	number of spanwise stations at which twist angles are defined		
n	denotes n^{th} -chord line		
P	integer selecting calculation procedure, see Section 3.1		
S	wing planform area	m ²	ft ²
s	wing semi-span ($\equiv s_{N_k + 1}$)	m	ft
s_i	spanwise position of crank for $1 \leq i \leq N_k$, $s_0 \equiv 0$ and $s_{N_k + 1} \equiv s$, see Sketch 3.1	m	ft
x_i	streamwise co-ordinate of wing leading edge at spanwise station s_i , see Sketch 3.1	m	ft
α_{spr}	angles of incidence specified in input, $1 \leq r \leq N_{sp}$	deg	deg
α_{Ti}	twist angle relative to reference plane at station η_{Ti} , $1 \leq i \leq N_{sT}$, positive leading edge up	deg	deg

ζ_{Cij}	camber ordinate, as fraction of c , at chordwise station ξ_{Cij} and spanwise station η_{Ci} , $1 \leq i \leq N_{sC}$, $1 \leq j \leq N_{cCi}$		
η	dimensionless spanwise ordinate, based on s		
η_{Ci}	dimensionless spanwise stations at which camber co-ordinates are defined $1 \leq i \leq N_{sC}$		
η_{oi}	dimensionless spanwise stations for output, $1 \leq i \leq N_o$		
η_{Ti}	dimensionless spanwise stations at which twist angles are defined, $1 \leq i \leq N_{sT}$		
Λ_n	sweep of n^{th} -chord line of straight-tapered planform	deg	deg
Λ_0	leading-edge sweep of straight-tapered planform	deg	deg
Λ_{0e}	leading-edge sweep of “equivalent” straight-tapered planform	deg	deg
λ	taper ratio (ratio of tip chord to root chord) of “equivalent” straight-tapered planform		
ξ_{Cij}	chordwise station, as fraction of c , at which camber ordinate ζ_{Cij} is defined for spanwise station η_{Ci} , $1 \leq i \leq N_{sC}$, $1 \leq j \leq N_{cCi}$		

Dressings

$()_C$	denotes contribution due to camber
$()_T$	denotes contribution due to twist
$()_\alpha$	denotes contribution due to incidence

2. INTRODUCTION

A computer program, ESDUpac A9510, has been prepared that uses steady lifting-surface theory based on the Multhopp-Richardson solution (Derivation 1) to provide the spanwise loading of wings with camber and twist in subsonic attached flow. The basic calculation core of the program is as reported in Derivation 2, and this has been adapted to ESDUpac format, with modified input and output routines.

The program calculates the spanwise loading distributions of local lift and pitching moment due to incidence, due to camber at zero incidence, and due to twist at zero incidence. The loadings can be obtained individually or simultaneously. The overall loading distribution can be calculated for a specified incidence or total lift coefficient.

This Item is a companion to Item No. 83040 (Reference 10) that gives a graphical means of obtaining the spanwise lift loading based on a correlation of numerous calculations by Multhopp's lifting-surface theory. In the cases of the camber and twist contributions the graphical method was based on runs of the program of Derivation 2, although the method for the loading due to incidence was based on the predictions of a different encapsulation of Multhopp's theory, see Reference 6.

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An extended program, ESDUpac B9510, that includes the effect of deflected plain leading-edge and plain trailing-edge flaps is introduced in Appendix A.

The program allows for either the input of a straight-tapered planform defined simply via aspect ratio, taper ratio and n^{th} -chord line sweep or the input of a cranked planform with straight leading-edge and trailing-edge segments. The details of how the geometry for the cranked wing is input are given subsequently.

The program outputs a wing lift-curve slope, a_1 . This is taken from the program ESDUpac A7011, described in Derivation 3, that is incorporated in ESDUpac A9510 as a subroutine. The data in ESDUpac A7011 were based on calculations reported in Reference 9 that were made using the Multhopp-Garner lifting-surface solution. This ensures consistency with other aerodynamics Items (including Item No. 83040) and provides a slightly more accurate value for a_1 than the Multhopp-Richardson solution. All the loading distributions and force and moment coefficients that are output by ESDUpac A9510 have been factored by the ratio of lift-curve slopes to incorporate this improvement in accuracy.

Because ESDUpac A7011 applies only to a straight-tapered wing, the estimation of the lift-curve slope for a cranked wing requires the program to make a prior calculation of an “equivalent” straight tapered wing by the method in Addendum A of Item No. 76003 (Derivation 4). This is the recommended procedure in Item No. 83040.

Section 3 describes the necessary input and the options that may be selected when running the program. Table 3.1 summarises all the required input. Section 4 summarises the output. The accuracy and applicability are discussed in Section 5. The Derivation and References are given in Section 6. Input and output examples are given in Section 7.

3. INPUT

3.1 Control Options and Flow Conditions

Table 3.1 lists, in order, the parameters required in the input file for the ESDUpac.

At the top of the file there is provision for the user to enter three lines of text to describe any run. In each case a blank line must be entered if there is no text.

The user may specify N_{Ms} and N_{Mc} , the number of Multhopp spanwise and chordwise collocation stations. It is recommended that N_{Ms} be taken as 33, and N_{Mc} as 3 or 5. A value of N_{Mc} less than 3 is inadequate. Only small differences in the loadings occur as N_{Mc} is increased from 3 to 5, but the latter value is more suitable for complex camber distributions or wings of low reduced aspect ratio, $\beta A \leq 2$. Increase of N_{Mc} above 5 is not recommended as there may be a loss of accuracy if the foremost chordwise collocation stations approach too close to the leading edge. (See Section 5.3 for more discussion of N_{Ms} and N_{Mc} .)

The number of loading types is specified by N_L . The type is selected by including some or all of the integers 1, 3, 4 in $\{L_m\}$ in ascending order. Any of the integers may be omitted. The presence of 3 or 4 requires the later input of a geometry data block.

For $L_m = 1$, the loading due to incidence is given in normalised form as $(C_{LL})_\alpha c/C_L \bar{c}$ together with the local centre of pressure position aft of the leading edge as a fraction of local chord. For $L_m = 3$ or 4, the loadings due to camber or twist at zero incidence are given as $(C_{LL})_C c/\bar{c}$ and $(C_{mL})_C c/\bar{c}$ or as $(C_{LL})_T c/\bar{c}$ and $(C_{mL})_T c/\bar{c}$, respectively. Integrated values of lift coefficient are provided. For each loading the centre

of pressure position aft of the wing apex is expressed as a fraction of $\bar{c}$. Also given is the spanwise centre of pressure position of the half-wing loading as a fraction of s , together with the sense of the rolling moment about the root chord.

The program calculates the desired loadings for a number of Mach numbers M_l for $1 \leq l \leq N_M$.

The number and dimensionless spanwise positions of the output may be specified through N_o and η_{oi} for $1 \leq i \leq N_o$. The user may set the integer P to perform additional calculations. For $P = 0$ there are no further calculations. For $P = 1$ the user enters the number N_{sp} of specified angles of incidence for α_{spr} and $1 \leq r \leq N_{sp}$ the program calculates the total loading distribution for each incidence

$$C_{LL} \frac{c}{\bar{c}} = \left(\frac{\alpha_{spr}}{57.3} \right) a_1 \frac{(C_{LL})_\alpha}{(C_L)_\alpha} \frac{c}{\bar{c}} + (C_{LL})_C \frac{c}{\bar{c}} + (C_{LL})_T \frac{c}{\bar{c}}, \quad (3.1)$$

$$C_{mL} \frac{c}{\bar{c}} = \left(\frac{\alpha_{spr}}{57.3} \right) a_1 \frac{(C_{mL})_\alpha}{(C_L)_\alpha} \frac{c}{\bar{c}} + (C_{mL})_C \frac{c}{\bar{c}} + (C_{mL})_T \frac{c}{\bar{c}}. \quad (3.2)$$

For $P = 2$, the number N_{sp} of specified total lift coefficients C_{Lspr} is entered and the corresponding values of α_{spr} are calculated from

$$\alpha_{spr} = 57.3(C_{Lspr} - C_{LC} - C_{LT}) / a_1. \quad (3.3)$$

The total loading distributions are then calculated by Equations (3.1) and (3.2).

3.2 Planform Geometry

For a straight-tapered planform the number of cranks N_k is entered as zero and the program requires a definition of the wing planform in terms of aspect ratio A , taper ratio λ , and the sweep of the chosen n^{th} chord line Λ_n , together with the value of n .

For a wing with N_k cranks, see Sketch 3.1, the planform is divided into $N_k + 1$ panels. Those panels are defined in dimensional terms, following the input of an integer that specifies the units system, 1 for *SI*, 2 for *British*, by

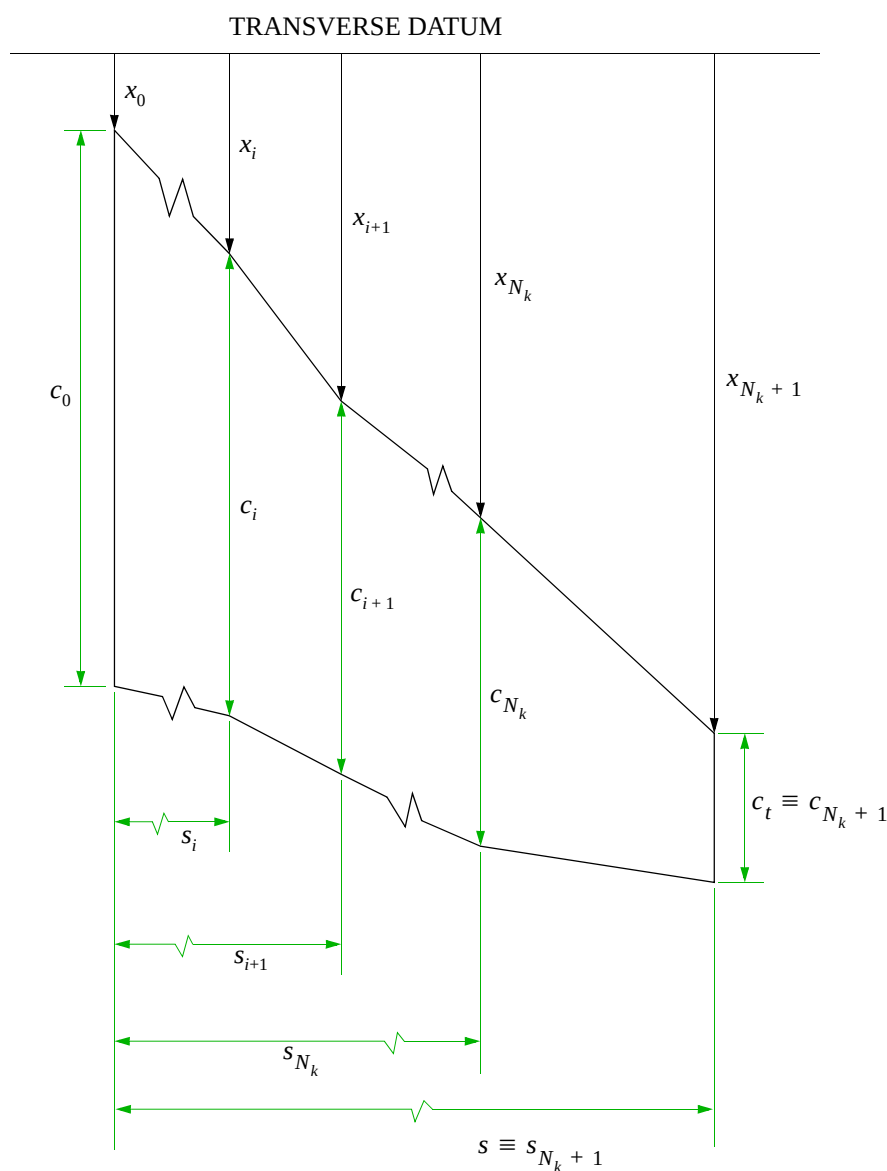
- the spanwise positions of the root chord, of each crank and of the wing tip,
 s_i for $i = 0$ to $N_k + 1$, ($s_0 \equiv 0$, $s_{N_k+1} \equiv s$)
- the streamwise co-ordinates of the wing leading edge at the spanwise stations s_i , measured positive aft from some transverse datum,
 x_i for $i = 0$ to $N_k + 1$,
- the root chord, the chord at each crank and the tip chord,
 c_i for $i = 0$ to $N_k + 1$.

3.3 Camber Geometry

The wing camber geometry, which is needed if $\{L_m\}$ contains 3, is defined at N_{sC} spanwise stations η_{Ci} , $1 \leq i \leq N_{sC}$, progressing from tip to root and including both. At each spanwise station the camber line ordinates ζ_{Cij} are given at N_{cCi} dimensionless chordwise stations ξ_{Cij} for $1 \leq j \leq N_{cCi}$, progressing from the leading edge to the trailing edge and including both.

3.4 Twist Geometry

The wing twist geometry, which is needed if $\{L_m\}$ contains 4, is defined by specifying twist angles α_{Ti} at N_{sT} spanwise stations η_{Ti} , $1 \leq i \leq N_{sT}$, progressing from tip to root and including both. The values of α_{Ti} are in degrees, positive leading edge up. They are measured with respect to the reference plane that is chosen to define the wing incidence.



Sketch 3.1 Cranked wing geometry

TABLE 3.1 INPUT DATA

Input	Comments	
TEXT 1 TEXT 2 TEXT 3	The first three lines of the input file are read as text and are written to the output file to enable the user to describe the run. Each line may contain up to 72 characters and must end with a carriage return. In each case a blank line must be entered if there is no text.	
N_{Ms}, N_{Mc}	Integers. Number of spanwise and chordwise Multhopp collocation stations. $1 \leq N_{Ms} \leq 75$, $2 \leq N_{Mc} \leq 10$, $N_{Mc}(N_{Ms} + 1)/2 \leq 199$. (See Section 5.3 for guidance.)	
N_L	Integer. Number of loading type $1 \leq N_L \leq 3$.	
$\{L_m\}$	Integers. Defines loading type. Contains some or all of the integers 1, 3, 4 in ascending order. The presence of 3 or 4, but not 1, requires extra input blocks of geometric data.	
N_M	Integer. Number of Mach numbers $1 \leq N_M \leq 20$.	
M_l	Values of Mach numbers $0 \leq M_l \leq 1$.	
N_o	Integer. Number of spanwise stations for output $1 \leq N_o \leq 40$.	
η_{oi}	Dimensionless values of spanwise stations for output.	
P	Integer. Selects calculation mode. See footnote to Table.	
	$P = 0$	Separate spanwise loadings only.
	$P = 1$	Also calculates total loading for specified angles of incidence α_{spr} .
	$P = 2$	Also calculates total loading for specified values of overall lift coefficient C_{Lspr} .
IF $P = 1$ or 2	N_{sp}	Integer. Number of specified values of α_{spr} or C_{Lspr} , $1 \leq N_{sp} \leq 10$.
IF $P = 1$	α_{spr}	Specified incidences for loading calculation (deg), $1 \leq r \leq N_{sp}$.
IF $P = 2$	C_{Lspr}	Specified overall lift coefficients for loading calculation, $1 \leq r \leq N_{sp}$.
	N_k	Integer. Number of cranks in wing; $0 \leq N_k \leq 13$, $N_k = 0$ implies a straight-tapered planform.
IF $N_k = 0$ (planform data)	A λ Λ_n n	Aspect ratio Taper ratio Sweep of n^{th} -chord line Chord line identifier
		} input for straight-tapered wing.
IF $N_k \geq 1$ (planform data)	Units s_i x_i c_i	Integer. 1 = SI units (m), 2 = British units (ft). Spanwise stations for cranked wing Streamwise co-ordinates of wing leading edge at spanwise stations s_i Chords for cranked wing at spanwise stations s_i
		} See Sketch 3.1, $0 \leq i \leq N_k + 1$, $1 \leq N_k \leq 13$.

TABLE 3.1 INPUT DATA (continued)

	<i>Input</i>	<i>Comments</i>
IF $\{L_m\}$ contains 3 (camber data)	N_{sC}	Integer. Number of spanwise stations η_{Ci} for camber input $2 \leq N_{sC} \leq 20$.
	Repeat N_{sC} times for $i = 1, 2,$ $\dots, N_{sC}$	Dimensionless values of spanwise stations for camber input, from tip to root, as fraction of semi-span. Integer. Number of chordwise stations for camber input at η_{Ci} $2 \leq N_{cCi} \leq 20$.
	$\xi_{Ci1} \quad \zeta_{Ci1}$ $\vdots \quad \vdots$ $\xi_{Cij} \quad \zeta_{Cij}$ $\vdots \quad \vdots$ $\xi_{CiN_{cCi}} \quad \zeta_{CiN_{cCi}}$	N_{cCi} pairs dimensionless values of chordwise stations for camber input, from leading edge to trailing edge, as fraction of chord camber ordinates as fraction of chord.
IF $\{L_m\}$ contains 4 (twist data)	N_{sT}	Integer. Number of spanwise stations for twist input $2 \leq N_{sT} \leq 20$.
	$\eta_{T1} \quad \alpha_{T1}$ $\vdots \quad \vdots$ $\eta_{Ti} \quad \alpha_{Ti}$ $\vdots \quad \vdots$ $\eta_{TN_{sT}} \quad \alpha_{TN_{sT}}$	N_{sT} pairs dimensionless values of spanwise stations for twist input, from tip to root, as fraction of semi-span twist angles in degrees, positive leading edge up, measured relative to wing incidence reference plane, positive leading edge up.

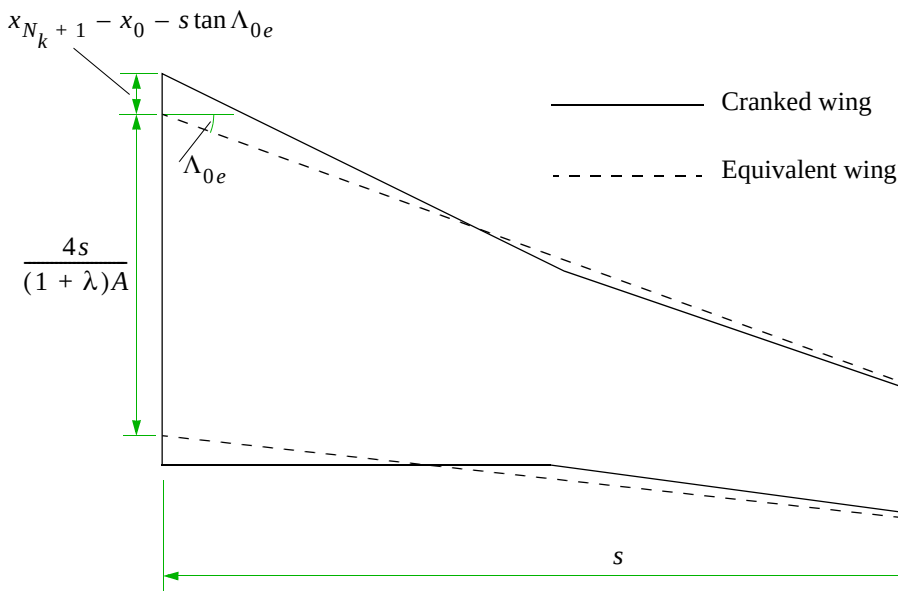
NOTES: Except for those parameters specified as integers, all data may be entered in free format, separated by one or more spaces.

The calculations for the loading due to incidence are always carried out within the program but the results are output only if $L_m = 1$ is included in the input. If $L_m = 1$ is not included then setting $P = 1$ or 2 will not trigger calculations at α_{spr} or C_{Lspr} , although the inclusion of $P = 1$ or 2 , N_{sp} and α_{spr} or C_{Lspr} in the input list will not cause the program to fail.

4. OUTPUT

The output provides a standard ESDUpac header that identifies the program and its status. This is followed by a listing of the input data.

The results section begins with a re-statement of the wing planform. If the wing is cranked the planform geometry of the “equivalent” straight-tapered wing is also output. The equivalent wing has the same area, span and tip chord as the true wing. The program outputs the root chord of the equivalent wing, $4s/(1 + \lambda)A$, and the distance of the leading edge of that root chord aft of the apex of the cranked wing $x_{N_k+1} - x_0 - s \tan \Lambda_{0e}$, where Λ_{0e} is the leading-edge sweep of the equivalent wing, calculated within the program. The equivalent wing can then be constructed rapidly by joining the leading and trailing edges of the root chord to those of the tip chord. See Sketch 4.1 for example. But note that the equivalent wing is employed only in the estimation of lift-curve slope, as discussed in Section 2. All loading distributions, moments and centre of pressure positions are calculated for the true planform.



Sketch 4.1 Illustration of equivalent wing construction

The loadings due to incidence, camber and twist are output according to the values chosen for L_m as described in Section 3.1. For $P = 1$ or 2 in the input file, the total loadings are then given for specified values of incidence or total lift coefficient, provided $L_m = 1$ is included in the input list.

These calculations are repeated for the chosen number of Mach numbers.

Provision is made within the program for a number of error messages to be output when triggered. Error messages are associated with a failed or terminated run and are intended to help in pin-pointing any error or inconsistency in the input data. A number of warnings in this program may be generated during the evaluation of the lift-curve slope. They are the same as those set in ESDUpac A7011 and involve limitations on planform geometry (see Section 5.2) or cases where the value of a_1 should be treated with caution.

Table 4.1 translates the output parameters into the notation of the Item.

TABLE 4.1

Output Parameter	Notation in Item	Output Parameter	Notation in Item
a1	a_1	Nk	N_k
AlphaT	α_{Ti}	NL	N_L
Alphaspr	α_{spr}	NM	N_M
ASPECT	A	NMc	N_{Mc}
CL	C_L	NMs	N_{Ms}
CLL	C_{LL}	Nsp	N_{sp}
CLsp	C_{Lspr}	Nyo	N_o
CmL	C_{mL}	n	n
c	c	P	P
ci	$c_0, c_1, \dots, c_{N_k+1}$	S	S
cbar	$\bar{c}$	si	$s_0, s_1, \dots, s_{N_k+1}$
ELOE	Λ_{0e}	TAPER	λ
ELN	Λ_n	xi	$x_0, x_1, \dots, x_{N_k+1}$
Eta	η	x/c	ξ_{Cij}
Lm	L_m	yo/s	η_{oi}
M	M	z/c	ζ_{Cij}

5. ACCURACY AND APPLICABILITY

5.1 Central Rounding

It is fundamental to Multhopp's method that the wing loading is expressible as a double series of chordwise and spanwise functions which are appropriate to smooth planforms with smooth vertical displacements. Discontinuities in planform require the introduction of special artifices, even when the vertical displacements are smooth.

With an odd number of spanwise collocation stations, N_{Ms} , the centre-line is itself a collocation station and the central kink in the planform of a straight-edged wing is a particular problem that requires some artificial rounding of the planform at the centre-line. (The choice of an even number for N_{Ms} removes the centre-line as a collocation station and partly evades the problem, but N_{Ms} is usually set to an odd number.) In Multhopp's original work (Reference 5, Appendix vi) the central chord was equated to $(5/6)c_0 + (1/6)c_{Ms1}$, where c_{Ms1} is the chord at the first outboard collocation station, with its leading edge moved rearward by 1/6 of the streamwise distance of the leading edge of c_{Ms1} aft of the wing apex. Experience has shown that better results are obtained with weightings of 5/8 and 3/8 in place of 5/6 and 1/6. However, reliance on c_{Ms1} and its position implies that the rounding imposed depends on N_{Ms} .

5.2 Planform Restrictions

With provision made for central rounding, application of lifting-surface theory is not unduly hampered by planform geometry, provided that there are no sudden changes in chord. It is best to avoid a collocation station very close to a crank. It may be difficult to achieve convergence near a pointed tip but, since the loading there falls to zero, this is of limited importance.

The use of ESDUpac A7011 as a subroutine imposes the restrictions of that program, namely, that for a straight-tapered wing planform the following must be satisfied

$$\text{i)} \quad 0 \leq \beta A \leq 12,$$

$$\text{ii)} \quad A \tan \Lambda_{1/2} \leq 6,$$

$$\text{iii)} \quad 0 \leq \lambda \leq 1,$$

$$\text{iv)} \quad A \tan \Lambda_{1/2} \geq -2 \left(\frac{1-\lambda}{1+\lambda} \right)$$

$$\text{and v)} \quad \tan^{-1}[A \tan \Lambda_{1/2} / \beta A] \geq -20^\circ.$$

Conditions i) to iii) allow most practical geometries, but exclude reduced aspect ratios that are too high, excessive sweepback and inverse taper.

Conditions iv) and v) mean that the leading edge may not be swept forward and the transformed half-chord sweep must not be swept forward more than 20° .

5.3 Selection of Number of Collocation Stations N_{Ms} and N_{Mc}

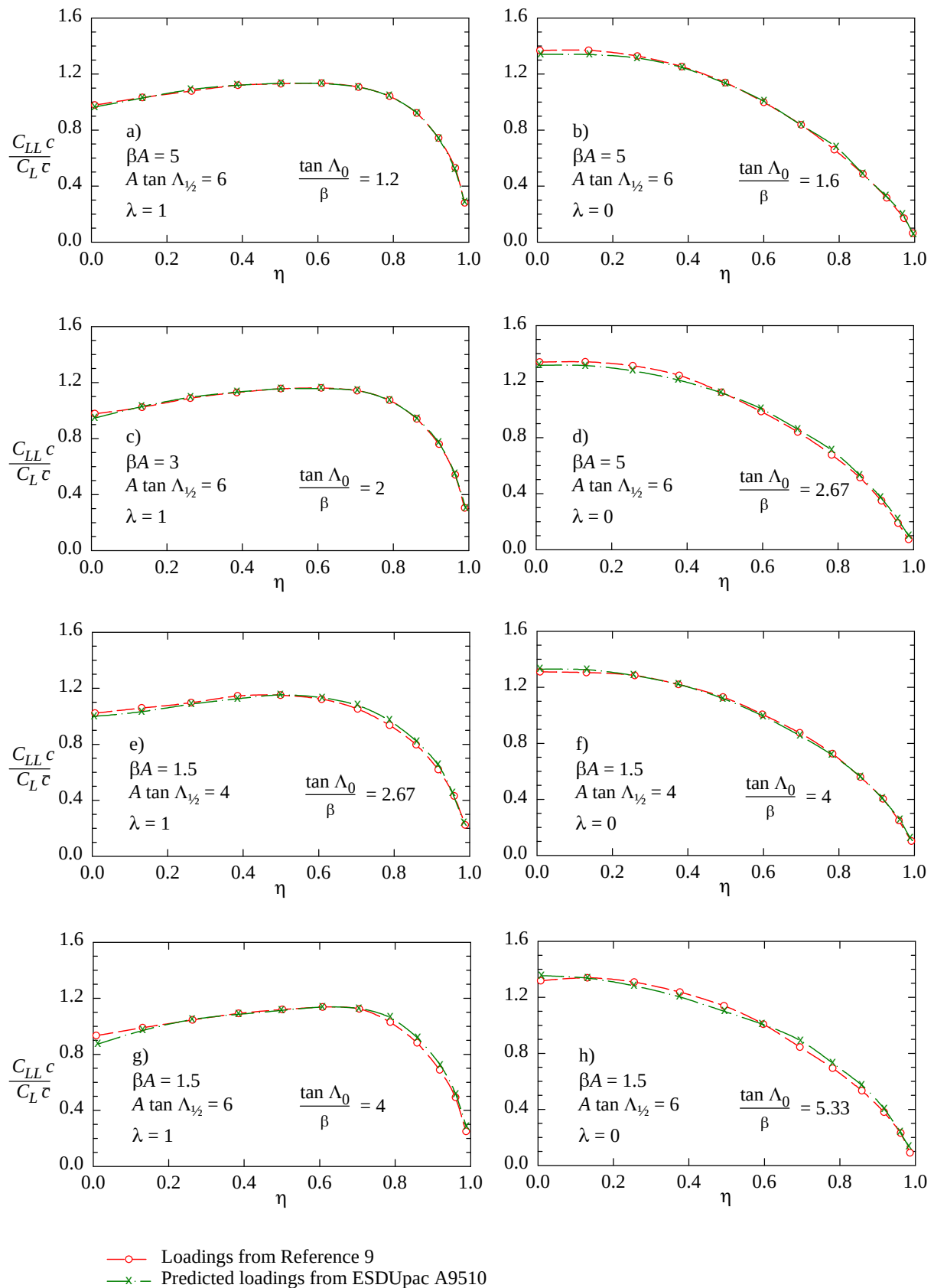
To determine a recommendation for the number of spanwise collocation stations, the predicted spanwise loadings due to incidence were compared with the tabulated values presented in Reference 9, (plus some unpublished data obtained on a similar basis at $\beta A = 12$). Numerical studies (*e.g.* References 7 and 8) and experience have shown that the latter data are reliable as a standard against which to check.

A parametric series of planforms was studied with $1.5 \leq \beta A \leq 12$, $0 \leq A \tan \Lambda_{1/2} \leq 6$ and $0 \leq \lambda \leq 1$. With 3 chordwise collocation stations it was found that the best agreement throughout the range of geometries was obtained with $N_{Ms} = 33$. The agreement tended to worsen as βA decreased and $A \tan \Lambda_{1/2}$ increased or, more significantly, as the leading-edge sweep parameter $\beta^{-1} \tan \Lambda_0$ increased.

With $N_{Ms} = 33$ and $N_{Mc} = 3$, Sketches 5.1a to 5.1h illustrate the good agreement between the loadings until $\beta^{-1} \tan \Lambda_0 \geq 4$. The sketches are arranged in pairs that show the cases for $\lambda = 1$ and 0 for selected values of βA and $A \tan \Lambda_{1/2}$.

Sketches 5.2a to 5.2h repeat these comparisons with $N_{Mc} = 5$. There is a consistent small improvement when $\lambda = 1$, although comparison of Sketches 5.1h and 5.2h for $\lambda = 0$ demonstrates the uncertainty of the loading for $\beta^{-1} \tan \Lambda_0 = 5.33$. Overall the incidence loadings can be considered to be accurate with $N_{Ms} = 33$ and $N_{Mc} = 3$ for $\beta^{-1} \tan \Lambda_0 \leq 4$, with the proviso that N_{Mc} should be increased to 5 for $\beta A \leq 2$, say. Caution is required if $\beta^{-1} \tan \Lambda_0$ greatly exceeds 4, particularly for fully-tapered wings.

Within the program a Fourier-style representation is made of the camber, and this permits the substitution of effective camber slopes at the collocation stations and an efficient modelling with a small number of chordwise stations. Sketch 5.3 illustrates two cases of the loading distributions due to complex camber shapes. The increase in N_{Mc} from 3 to 5 shows only a modest effect on the magnitudes and shapes of the loadings. It is however best to take $N_{Mc} = 5$ for cambers of this degree of complexity, with $N_{Mc} = 3$ sufficing for simpler cambers, provided that $\beta A \geq 2$.



Sketch 5.1 Comparison of loadings due to incidence; $N_{Ms} = 33$, $N_{Mc} = 3$

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