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One-dimensional representation of steady, spatially non-uniform flow An equivalent mean-value set for compressible flow Part 1. Implementation for an ideal, calorically-perfect gas

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ONE-DIMENSIONAL REPRESENTATION OF STEADY, SPATIALLY NON-UNIFORM FLOW. AN EQUIVALENT MEAN-VALUE SET FOR COMPRESSIBLE FLOW. PART 1. IMPLEMENTATION FOR AN IDEAL, CALORICALLY-PERFECT GAS.

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ONE-DIMENSIONAL REPRESENTATION OF STEADY, SPATIALLY NON-UNIFORM FLOW. An Equivalent Mean-value Set for Compressible Flow.

Part 1. Implementation for an ideal, calorically-perfect gas.

1. INTRODUCTION

Despite the fact that some degree of spatial non-uniformity is present in the flow in all real systems, design and performance analysis methods for ducted-flow systems often involve an explicit or implicit assumption that the flow is uniform. Consequently, it is often necessary to formulate a set of representative mean properties to describe the flow and to use those in the analysis methods.

This Data Item describes a mean-value set based on an equivalent-state, reference-mean-flow model derived from measured*, local properties at a cross section of the flow in a duct. The flow considered is a steady, spatially non-uniform (profiled), compressible flow of an ideal, calorically-perfect gas. It is assumed to be axial and free from significant swirl. Measurement uncertainties on the raw measurements are assumed to be accounted for separately so that the definition starts with a “corrected” set of measurements. The model allows for the simultaneous variation of total and static temperatures and pressures and other flow properties across the section in the flow and is applicable to both subsonic and supersonic flow.

The Item is one of a group concerned with the definition of a mean-value set for the one-dimensional representation of conditions, at a cross section of a duct, of a steady, profiled flow and their use in the analysis of system performance. Other Items in the group include Reference 2, for ideal, thermally-perfect gas flow (using a “*true-c_p*” basis), Reference 3, presenting numerical examples of the application, and Reference 4, presenting a mean-value set for incompressible flow.

The layout of the Item is as follows.

- Section 2 gives the notation used in the Item.
- Section 3 considers the equations for the entropy of the flow and introduces the logarithmic-mean values of properties.
- Section 4 defines the reference-mean flow that forms the basis of the mean-value set.
- Section 5 considers the extensive properties of the reference-mean flow and the profile factors that relate them to corresponding properties of the measured, profiled flow.
- Section 6 considers the representation of the mass flow.
- Section 7 considers the representation of the momentum and absolute stream force of the non-uniform flow.
- Section 8 summarises the derivation of the mean-value set.
- Section 9 presents selected analytical examples illustrating applications of the reference-mean-flow model to ducted flows and continuous-flow, adiabatic compression and expansion processes.
- Appendix A summarises the equations for the mean-value set.
- Appendix B provides a glossary of terms as they are used in this Data Item.

* The term “measured” may be extended to include local property values from computational methods (CFD).

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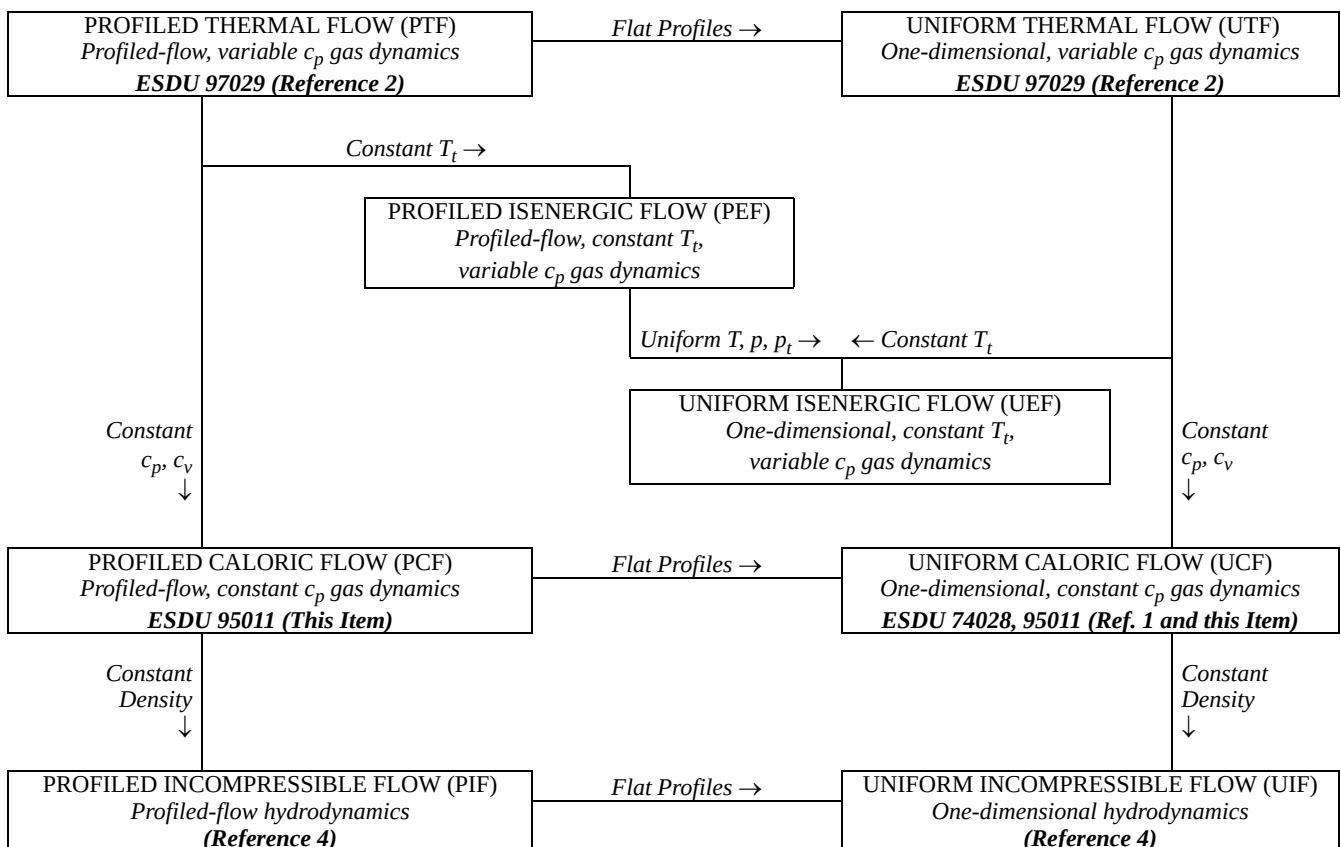
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The mean-value set provides a means of characterising a profiled flow at a particular cross section in the flow. In order to assess the performance of a system it will be necessary to make measurements at more than one cross section and derive corresponding mean-value sets at each measurement section.

The application of the reference-mean-flow model at the measurement sections in continuous flow systems leads to a correct representation of the entropy at each section and hence to a proper accounting of the reversible and actual work or heat transfer in the processes between the measurement sections. Thus a correct distribution of work and irreversibilities among the components of multi-process systems can be made, avoiding the attribution of artificial flow-mixing losses to system components upstream or downstream of a measurement section.

The reference-flow model, based on the conservation of entropy and mass flux between the profiled and reference-mean flows, provides a useful unification of the treatment of continuous, adiabatic flows as illustrated by the schema in Sketch 1.1. Calorically-perfect gas flows emerge as special cases when c_p is kept constant. Isenergetic flows emerge as special cases when the total temperature is constant everywhere. Uniform flows are special cases when all properties are uniform across the section (flat profiles). The flow model applies to incompressible flows, both profiled and uniform, when the density is made constant.



Sketch 1.1 Schema Illustrating Unification of Flow Descriptions

The present mean-value set provides a basis for the assessment and comparison of historical and alternative mean values and mean-value sets.

2. NOTATION

		<i>SI Unit</i>
A	cross-sectional area of flow at duct section	m^2
A_{eff}	effective cross-sectional area of flow at duct section	m^2
a	speed of sound in gas	m/s
B_t, b_t	sectional and specific availability (see Glossary, Appendix B)	J, J/kg
C_d	discharge coefficient or mass-flow profile factor	—
C_{pr}, C'_{pr}	diffuser static-pressure recovery coefficients	—
c_p	specific heat capacity at constant pressure	J/kg K
c_v	specific heat capacity at constant volume	J/kg K
$E_{\text{ex}}, e_{\text{ex}}$	sectional and specific exergy (see Glossary, Appendix B)	J, J/kg
E_i, e_i	sectional and specific internal energy	J, J/kg
E_k, e_k	sectional and specific kinetic energy	J, J/kg
E_t, e_t	sectional and specific total energy	J, J/kg
F_a	absolute stream force	N
F_M	momentum force	N
H, h	sectional and specific enthalpy	J, J/kg
I_E	kinetic-energy profile factor	—
I_M	momentum profile factor	—
I_p	static-pressure profile factor	—
$J_{Tt}, J_T,$ $J_{\rho t}, J_{\rho}, J_{pt}, J_p$	profile factors relating mass-flow-weighted-mean values to reference-mean values	—
K_{pt}, K'_{pt}, K_t	diffuser total-pressure loss coefficients	—
M	Mach number	—
m_w	molecular weight of gas	kg/kg-mol
$\dot{m}$	mass flow rate or mass flux through section	kg/s
n	exponent in polytropic compression or expansion	—
p	static pressure	$\text{Pa (N/m}^2\text{)}$
p_{res}	reservoir pressure	$\text{Pa (N/m}^2\text{)}$

p_t	total pressure (stagnation pressure)	Pa (N/m ²)
Q_1, Q_2, Q_3	mass flow functions	–
R	gas constant, $R = R_u / m_w$	J/kg K
R_u	universal gas constant, 8314 J/kg-mol K	J/kg-mol K
S, s	sectional and specific entropy	J/K, J/kg K
T	static temperature	K
T_a	reference environmental temperature	K
T_{res}	reservoir temperature	K
T_t	total temperature	K
V	axial velocity	m/s
γ	ratio of specific heat capacities, c_p/c_v	–
η	process efficiency or effectiveness	–
ρ	static density of gas	kg/m ³
ρ_{res}	reservoir density of gas	kg/m ³
ρ_t	total density of gas	kg/m ³

Subscripts

Where appropriate, subscripts are used in combination.

A	denotes integration over whole duct area at a section
a	denotes ambient or environmental conditions
aw	denotes area-weighted-mean value (see Glossary, Appendix B)
CM	denotes continuity-mean value
i	denotes value applicable to an isentropic process
lm	denotes logarithmic-mean value (see Glossary, Appendix B)
mw	denotes mass-flow-weighted-mean value (see Glossary, Appendix B)
$\dot{m}$	denotes integration over whole of mass flow at a section
res	denotes reservoir-state value
rms	denotes root-mean-square value
s	denotes static-state value
t	denotes total-state value
1 to 6	denote values at successive sections along a ducted flow

Superscripts

- ★ denotes a value at sonic conditions ($M = 1$)
- ' Section 9.1.1, denotes alternative parameter definition, Sections 9.2 to 9.4, denotes value related to temperature-mixed flow condition

Dressings

- denotes mean value of type defined by subscript
- ~ denotes reference-mean flow value

3. ENTROPY CONSIDERATIONS

In formulating mean values of flow properties at a section in a non-uniform, compressible flow, the method of accounting for the entropy of the flow is of central importance if a correct allocation is to be made of the losses associated with irreversibilities* in the flow. A particular feature of the expression for the entropy of a calorically-perfect gas is the logarithmic form of the temperature and pressure or density terms.

3.1 Streamtubes

With a proper choice of datum, the state property s_s , the specific[†] static entropy, in a streamtube of elemental area dA and mass flow $d\dot{m}$ at a section in a flow can be related to the local static temperature and static pressure or static density state properties by

$$s_s = c_p \ln T - R \ln p \text{ and } s_s = c_v \ln T - R \ln p - R \ln R. \quad (3.1)$$

Total, stagnation or reservoir conditions defined locally at a point in a flow are the conditions that would be attained if the flow were brought to rest in a hypothetical *isentropic* process. The local entropy in the reservoir, by definition the local total (or stagnation) entropy, s_t , can be expressed in terms of the local temperature and pressure or density, by definition the local total temperature, total pressure and total density, to yield

$$s_t = c_p \ln T_t - R \ln p_t \text{ and } s_t = c_v \ln T_t - R \ln \rho_t - R \ln R. \quad (3.2)$$

By definition, $s_t = s_s$, and Equations (3.1) and (3.2) can be equated to yield the isentropic relationships between local total and static pressure, density and temperature as

$$\frac{p_t}{p} = \left(\frac{T_t}{T} \right)^{\frac{\gamma}{\gamma-1}} \text{ and } \frac{\rho_t}{\rho} = \left(\frac{p_t}{p} \right)^{\frac{1}{\gamma}} = \left(\frac{T_t}{T} \right)^{\frac{1}{\gamma-1}}. \quad (3.3)$$

The above definitions for a streamtube can be extended to cover the whole flow at a section of a uniform flow since the flow properties are constant across the section.

* See Glossary, Appendix B.

† i.e. the value per unit mass flow.

3.2 Non-uniform Flow

In the case of non-uniform flow in a duct in which both the total temperature and total pressure vary across a section in the flow, the selection of mean values of temperature and pressure must cater for the need to account for both the sectional entropy and the mean entropy at that section. As the entropy flux is a mass-dependent extensive property, the sectional entropy can be derived by integration, with respect to mass flow, of local values across the duct. It is then appropriate, using the first mean value theorem of integrals, to define the mean entropy as the mass-flow-weighted mean value.

Thus, the sectional static entropy is given by

$$S_s = \int \dot{m} s_s d\dot{m} = \int \dot{m} (c_p \ln T - R \ln p) d\dot{m} \quad (3.4)$$

and the mass-flow-weighted, mean static entropy by

$$\bar{s}_{s, mw} = \frac{1}{\dot{m}} \int \dot{m} s_s d\dot{m} = \frac{1}{\dot{m}} \int \dot{m} (c_p \ln T - R \ln p) d\dot{m}. \quad (3.5)$$

Similarly, the sectional total entropy is given by

$$S_t = \int \dot{m} s_t d\dot{m} = \int \dot{m} (c_p \ln T_t - R \ln p_t) d\dot{m} \quad (3.6)$$

and the mass-flow-weighted, mean total entropy by

$$\bar{s}_{t, mw} = \frac{1}{\dot{m}} \int \dot{m} s_t d\dot{m} = \frac{1}{\dot{m}} \int \dot{m} (c_p \ln T_t - R \ln p_t) d\dot{m}. \quad (3.7)$$

3.2.1 Logarithmic-mean Definitions of Temperature, Pressure and Density

The logarithmic-mean definitions of static temperature and static pressure are given by

$$\ln \bar{T}_{lm} = \frac{1}{\dot{m}} \int \dot{m} \ln T d\dot{m} \text{ and } \ln \bar{p}_{tm} = \frac{1}{\dot{m}} \int \dot{m} \ln p d\dot{m} \quad (3.8)$$

and of the total temperature and total pressure by

$$\ln \bar{T}_{t, lm} = \frac{1}{\dot{m}} \int \dot{m} \ln T_t d\dot{m} \text{ and } \ln \bar{p}_{t, lm} = \frac{1}{\dot{m}} \int \dot{m} \ln p_t d\dot{m}. \quad (3.9)$$

Introducing these quantities into Equations (3.5) and (3.7) yields, for static properties,

$$\bar{s}_{s, mw} = c_p \ln \bar{T}_{lm} - R \ln \bar{p}_{lm} \quad (3.10)$$

and, for total properties,

$$\bar{s}_{t, mw} = c_p \ln \bar{T}_{t, lm} - R \ln \bar{p}_{t, lm}. \quad (3.11)$$

The above equations show that the logarithmic-mean definitions of static and total pressure emerge naturally from entropy considerations. A similar development of the entropy equations involving density leads equally to the logarithmic-mean values of static and total density, given by

$$\ln \bar{\rho}_{lm} = \frac{1}{\dot{m}} \int_m \ln \rho d\dot{m} \quad \text{and} \quad \ln \bar{\rho}_{t, lm} = \frac{1}{\dot{m}} \int_m \ln \rho_t d\dot{m}. \quad (3.12)$$

Since $\bar{s}_{s, mw} = \bar{s}_{t, mw}$, the logarithmic-mean values of total and static temperature, pressure and density can be related by isentropic relationships similar to those of Equation (3.3), so that

$$\frac{\bar{p}_{t, lm}}{\bar{p}_{lm}} = \left(\frac{\bar{T}_{t, lm}}{\bar{T}_{lm}} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad \frac{\bar{\rho}_{t, lm}}{\bar{\rho}_{lm}} = \left(\frac{\bar{p}_{t, lm}}{\bar{p}_{lm}} \right)^{\frac{1}{\gamma}} = \left(\frac{\bar{T}_{t, lm}}{\bar{T}_{lm}} \right)^{\frac{1}{\gamma-1}}. \quad (3.13)$$

4. THE EQUIVALENT-STATE, REFERENCE-MEAN-FLOW MODEL

The proposed equivalent-state, reference-mean-flow model defines uniquely a consistent thermodynamic system extending uniform-flow concepts to non-uniform flow.

The properties of the reference-mean flow are distinguished by a tilde ($\sim$) dressing, other mean property values are distinguished by a bar ($\bar{}$) dressing and an appropriate subscript.

4.1 Definition

The basis of the reference-mean flow is as follows:-

- (i) its properties are uniform across the section,
- (ii) its mass flow is equal to the sectional mass flow of the measured, non-uniform flow,
- (iii) its total state properties are equal to those that obtain in a hypothetical reservoir that defines a quiescent end state into which the whole of the section flow is decelerated at constant total entropy and in which the equation of state applies,
- (iv) its static state properties are related isentropically to its total state properties.

Conditions (iii) and (iv) mirror those of uniform flow.

From the reservoir definition, using Section 3.2.1 and noting that the gas in the reservoir is in a uniform state, it follows that

$$S_{res} = \dot{m} s_{res} = S_t = \dot{m} \bar{s}_{t, mw}, \quad (4.1)$$

$$p_{res} = \rho_{res} R T_{res} \quad (4.2)$$

$$\text{and} \quad p_{res} = \bar{p}_{t, lm}, \quad T_{res} = \bar{T}_{t, lm} \quad \text{and} \quad \rho_{res} = \bar{\rho}_{t, lm}. \quad (4.3)$$

Using the tilde notation to denote the reference-mean flow, condition (ii) requires that

$$\tilde{m} = \dot{m}. \quad (4.4)$$

From condition (iii) and Equation (4.3), the total state intensive properties of the reference-mean flow are defined by

$$\tilde{p}_t \equiv \bar{p}_{t, lm}, \quad \tilde{T}_t \equiv \bar{T}_{t, lm} \quad \text{and} \quad \tilde{\rho}_t \equiv \bar{\rho}_{t, lm} \quad (4.5)$$

and, using Equation (4.2), the total state equation for the reference-mean flow is then

$$\tilde{p}_t = \tilde{\rho}_t R \tilde{T}_t. \quad (4.6)$$

From conditions (i) and (iii) and Equation (4.1), the sectional and specific total entropies of the reference-mean flow can be expressed as

$$\tilde{S}_t = \dot{m} [c_p \ln \tilde{T}_t - R \ln \tilde{p}_t] = S_{res} = S_t \quad (4.7)$$

and
$$\tilde{s}_t = c_p \ln \tilde{T}_t - R \ln \tilde{p}_t. \quad (4.8)$$

From condition (iv), the total and static state properties of the reference-mean flow can be related by isentropic relationships as in Equation (3.13), so that

$$\frac{\tilde{p}_t}{\tilde{p}} = \left(\frac{\tilde{T}_t}{\tilde{T}} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad \frac{\tilde{p}_t}{\tilde{p}} = \left(\frac{\tilde{p}_t}{\tilde{p}} \right)^{\frac{1}{\gamma}} = \left(\frac{\tilde{T}_t}{\tilde{T}} \right)^{\frac{1}{\gamma-1}}. \quad (4.9)$$

It follows that
$$\tilde{p} \equiv \bar{p}_{lm}, \quad \tilde{T} \equiv \bar{T}_{lm} \quad \text{and} \quad \tilde{\rho} \equiv \bar{\rho}_{lm} \quad (4.10)$$

and that the static equation of state for the reference-mean flow is

$$\tilde{p} = \tilde{\rho} R \tilde{T}. \quad (4.11)$$

The sectional and specific static entropies of the reference-mean flow can then be written as

$$\tilde{S}_s = \dot{m} [c_p \ln \tilde{T} - R \ln \tilde{p}] = S_s \quad (4.12)$$

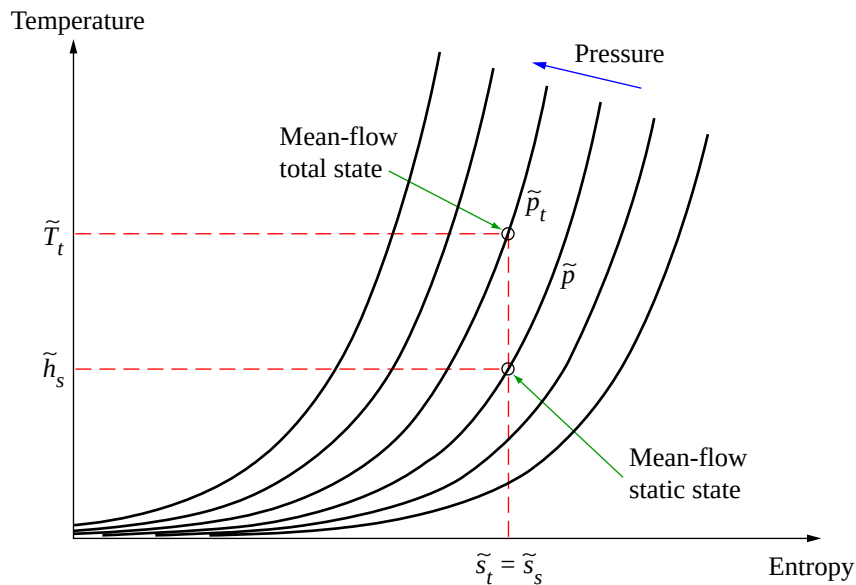
and
$$\tilde{s}_s = c_p \ln \tilde{T} - R \ln \tilde{p}. \quad (4.13)$$

4.2 The Temperature~entropy Diagram

The reference-mean-flow total and static states, defined by Equations (4.8) and (4.13), can be expressed in functional form as

$$\tilde{T}_t = T_t\{\tilde{s}_t, \tilde{p}_t\} \text{ and } \tilde{T} = T\{\tilde{s}_s, \tilde{p}\}. \quad (4.14)$$

These states can be represented on a temperature~entropy diagram for the gas, as shown in Sketch 4.1.



Sketch 4.1 Temperature~entropy diagram for a calorically-perfect gas with the reference-mean states for a profiled flow

The isentropic relationship between the reference-mean total and static pressures and the corresponding entropy functions is also shown on the sketch.

It is of fundamental importance to recognise that, unlike the reference-mean flow which can be represented by a single static thermodynamic state and a single total thermodynamic state, the actual non-uniform flow comprises an infinite number of static and total states defined by the static and total temperature and pressure profiles at the section and that these are not in equilibrium.

5. THERMODYNAMIC FLUX PROPERTIES AND RELATIONSHIPS

The thermodynamic fluxes are extensive properties and are mass flow related. These fluxes and derived properties and relationships for the reference-mean flow are defined in Section 5.1. The definition of profile factors used to relate the properties of the non-uniform flow to those of the reference-mean flow is considered in Section 5.2. The sectional thermodynamic fluxes of the non-uniform flow and their relationship to the corresponding properties of the reference-mean flow are considered in Section 5.3.

5.1 Reference-mean-flow Properties

5.1.1 Thermodynamic flux properties

Since the reference-mean flow at a section is uniform, the sectional fluxes can be expressed as the product of the corresponding specific flux and the sectional mass flow. The specific fluxes can be defined directly in terms of the intensive properties of the reference-mean flow and are summarised in Table 5.1.

TABLE 5.1

<i>Property</i>	<i>Specific flux in reference-mean flow</i>
Total entropy	$\tilde{s}_t = c_p \ln \tilde{T}_t - R \ln \tilde{p}_t$
Static entropy	$\tilde{s}_s = c_p \ln \tilde{T} - R \ln \tilde{p}$
Total enthalpy	$\tilde{h}_t = c_p \tilde{T}_t$
Static enthalpy	$\tilde{h}_s = c_p \tilde{T}$
Internal energy	$\tilde{e}_i = c_v \tilde{T}$
Total energy	$\tilde{e}_t = \tilde{e}_i + (\tilde{h}_t - \tilde{h}_s)$
Availability*	$\tilde{b}_t = \tilde{h}_t - T_a \tilde{s}_t$
Exergy*	$\tilde{e}_{ex} = \tilde{b}_t - b_{t,a}$

* See Glossary, Appendix B for definition. The terms T_a and $b_{t,a}$ apply at ambient or environmental conditions.

The forms of these equations are identical to those for uniform flow of a calorically-perfect gas, without the use of profile factors. The reference-mean intensive properties, $\tilde{p}_t$, $\tilde{T}_t$, $\tilde{p}$ and $\tilde{T}$, in these definitions are equal to the logarithmic-mean values of the profiles of properties at the section in the non-uniform flow.