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## **Non-Newtonian fluids: obtaining viscometric data for frictional pressure loss estimation for pipeflow**

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## THE PREPARATION OF THIS DATA ITEM

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## NON-NEWTONIAN FLUIDS: OBTAINING VISCOMETRIC DATA FOR FRICTIONAL PRESSURE LOSS ESTIMATION FOR PIPEFLOW

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## NON-NEWTONIAN FLUIDS: OBTAINING VISCOMETRIC DATA FOR FRICTIONAL PRESSURE LOSS ESTIMATION FOR PIPEFLOW

### 1. NOTATION AND UNITS

|            |                                                                             | Units                  |                           |
|------------|-----------------------------------------------------------------------------|------------------------|---------------------------|
|            |                                                                             | SI                     | British                   |
| $A_1, A_2$ | functions defined by Equations (6.47) and (6.48)                            | —                      | —                         |
| $B$        | function defined by Equation (6.37)                                         | —                      | —                         |
| $Br$       | Newtonian Brinkmann number as defined by Equation (6.19)                    | —                      | —                         |
| $c_v$      | total solids concentration by volume, expressed as percentage               | —                      | —                         |
| $C_e$      | correction factor for geometric effect, see Equation (6.32)                 | —                      | —                         |
| $d$        | particle diameter                                                           | m                      | ft                        |
| $D$        | pipe or tube inside diameter                                                | m                      | ft                        |
| $D_{agit}$ | agitator diameter                                                           | m                      | ft                        |
| $E$        | error, see Section 8.2                                                      | —                      | —                         |
| $f_c$      | viscosity factor, see Equation (6.33)                                       | N rad/m <sup>2</sup> φ | lbf rad/ft <sup>2</sup> φ |
| $f_e$      | edge effect factor, see Equation (6.54)                                     | —                      | —                         |
| $g$        | gravitational constant, $g = 9.81 \text{ m/s}^2, 32.174 \text{ ft/s}^2$     | m/s <sup>2</sup>       | ft/s <sup>2</sup>         |
| $g(n)$     | shear rate coefficient, see Equations (6.28) and (6.29)                     | —                      | —                         |
| $Ga$       | Galileo number, see Equation (A2.5)                                         | —                      | —                         |
| $h$        | fluid head, see Sketch 6.1                                                  | m                      | ft                        |
| $h_1$      | fluid height above bob (see Sketch 6.12a)                                   | m                      | ft                        |
| $h_2$      | height between base of bob and bottom of cup (see Sketches 6.12a and 6.12b) | m                      | ft                        |
| $H$        | parallel plate separation (gap)                                             | m                      | ft                        |

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|            |                                                                                                   |                                                       |                                     |
|------------|---------------------------------------------------------------------------------------------------|-------------------------------------------------------|-------------------------------------|
| $H_o$      | gap setting error for cone-and-plate and parallel plate viscometers                               | m                                                     | ft                                  |
| $k$        | torsion spring constant                                                                           | N m/ $\phi$                                           | ft lbf/ $\phi$                      |
| $k_s$      | proportionality constant between torque-averaged shear rate and agitator speed in Equation (B2.7) | —                                                     | —                                   |
| $k_1, k_2$ | functions defined by Equation (6.45) and (6.46)                                                   | —                                                     | —                                   |
| $K$        | consistency coefficient                                                                           | N s <sup>n</sup> /m <sup>2</sup> (Pa s <sup>n</sup> ) | lbf s <sup>n</sup> /ft <sup>2</sup> |
| $K_o$      | consistency coefficient at reference temperature, $\theta_o$                                      | N s <sup>n</sup> /m <sup>2</sup> (Pa s <sup>n</sup> ) | lbf s <sup>n</sup> /ft <sup>2</sup> |
| $K_L$      | calibration constant given by Equation (B2.3)                                                     | —                                                     | —                                   |
| $K_R$      | parameter given by $\ln(R_{cup}/R_{bob})$                                                         | —                                                     | —                                   |
| $L$        | pipe length                                                                                       | m                                                     | ft                                  |
| $L_{bob}$  | bob length                                                                                        | m                                                     | ft                                  |
| $m$        | exponent in Equation (6.6)                                                                        | —                                                     | —                                   |
| $n$        | exponent in power law or Herschel-Bulkley model                                                   | —                                                     | —                                   |
| $n_o$      | power law exponent at reference temperature, $\theta_o$                                           | —                                                     | —                                   |
| $n'$       | local slope of $\ln T$ versus $\ln \Omega$ or $\ln \tau$ versus $\ln \dot{\gamma}$                | —                                                     | —                                   |
| $n''$      | local slope of plot of $n'$ versus $\ln \Omega$ or $\ln n'$ versus $\ln \dot{\gamma}$ plot        | —                                                     | —                                   |
| $N$        | agitator rotational speed                                                                         | Hz                                                    | Hz                                  |
| $p_1$      | absolute pressure at tube inlet                                                                   | N/m <sup>2</sup> (Pa)                                 | lbf/ft <sup>2</sup>                 |
| $p_o$      | absolute applied pressure                                                                         | N/m <sup>2</sup> (Pa)                                 | lbf/ft <sup>2</sup>                 |
| $p_{atm}$  | absolute atmospheric pressure                                                                     | N/m <sup>2</sup> (Pa)                                 | lbf/ft <sup>2</sup>                 |
| $Po$       | Power number, see Equation (B2.1)                                                                 | —                                                     | —                                   |
| $Q$        | volumetric flowrate                                                                               | m <sup>3</sup> /s                                     | ft <sup>3</sup> /s                  |
| $R_{bob}$  | bob radius                                                                                        | m                                                     | ft                                  |
| $R_{cone}$ | cone radius                                                                                       | m                                                     | ft                                  |
| $R_{cup}$  | cup radius                                                                                        | m                                                     | ft                                  |

|                     |                                                                                         |               |                  |
|---------------------|-----------------------------------------------------------------------------------------|---------------|------------------|
| $R_{disc}$          | disc radius                                                                             | m             | ft               |
| $R_{plate}$         | plate radius                                                                            | m             | ft               |
| $Re$                | Reynolds number                                                                         | —             | —                |
| $Re_{agit}$         | agitator Reynolds number                                                                | —             | —                |
| $Re_c$              | critical Reynolds number, see Equation (6.30)                                           | —             | —                |
| $Re_o$              | particle Reynolds number, see Equations (A2.2) to (A2.4) and Equation (A2.6)            | —             | —                |
| $S$                 | slip factor, see Equation (6.5) or (6.6)                                                | $m^{2+m}s/kg$ | $ft^{2+m}s/slug$ |
| $T$                 | torque                                                                                  | N m           | ft lbf/rad       |
| $T_e$               | torque error due to edge/end effect                                                     | N m           | ft lbf/rad       |
| $Ta_c$              | critical Taylor number                                                                  | —             | —                |
| $T_{min}$           | minimum suspension residence time, see Equation (A2.8)                                  | s             | s                |
| $T_{max}$           | maximum suspension residence time, see Equation (A2.9)                                  | s             | s                |
| $V$                 | mean axial velocity                                                                     | m/s           | ft/s             |
| $V_o$               | particle settling velocity                                                              | m/s           | ft/s             |
| $V_{slip}$          | wall slip velocity                                                                      | m/s           | ft/s             |
| $x$                 | volume fraction (ratio of volume of component to total volume), expressed as percentage | —             | —                |
| $y$                 | factor in Equation (6.38)                                                               | —             | —                |
| $\alpha$            | cone angle                                                                              | deg           | deg              |
| $\beta$             | viscosity-temperature coefficient, see Equations (6.3) and (6.18)                       | —             | —                |
| $\dot{\gamma}$      | shear rate                                                                              | $s^{-1}$      | $s^{-1}$         |
| $\dot{\gamma}_{av}$ | average shear rate                                                                      | $s^{-1}$      | $s^{-1}$         |
| $\delta$            | $H_o/\alpha R_{cone}$ , see Section 6.4.2.7                                             | —             | —                |
| $\delta_1$          | $R_{bob}/R_{cup}$                                                                       | —             | —                |
| $\delta_2$          | $R_{cup} - R_{bob}$                                                                     | m             | m                |

|              |                                                                                 |                           |                       |
|--------------|---------------------------------------------------------------------------------|---------------------------|-----------------------|
| $\Delta H$   | increase in gap of cone-and-plate viscometer                                    | m                         | ft                    |
| $\Delta p$   | frictional pressure loss from flow through straight sections of pipework        | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\Delta p_e$ | end effect pressure loss                                                        | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\Delta p_F$ | pressure loss through pipe fittings                                             | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\Delta p_s$ | pressure change due to elevation changes (positive for increase in static head) | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\Delta p_T$ | overall pressure drop                                                           | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\phi$       | torque scale deflection reading                                                 | —                         | —                     |
| $\theta$     | temperature                                                                     | K                         | R                     |
| $\theta_o$   | reference temperature                                                           | K                         | R                     |
| $\lambda$    | thermal conductivity                                                            | W/m K                     | Btu/ft h°F            |
| $\lambda_o$  | thermal conductivity at reference temperature $\theta_o$                        | W/m K                     | Btu/ft h°F            |
| $\mu$        | dynamic viscosity                                                               | N s/m <sup>2</sup> (Pa s) | lbf s/ft <sup>2</sup> |
| $\mu_B$      | Bingham plastic viscosity                                                       | N s/m <sup>2</sup> (Pa s) | lbf s/ft <sup>2</sup> |
| $\mu_o$      | viscosity at reference temperature $\theta_o$                                   | N s/m <sup>2</sup> (Pa s) | lbf s/ft <sup>2</sup> |
| $\rho$       | fluid density                                                                   | kg/m <sup>3</sup>         | slug/ft <sup>3</sup>  |
| $\rho_m$     | suspension density                                                              | kg/m <sup>3</sup>         | slug/ft <sup>3</sup>  |
| $\rho_s$     | solids density                                                                  | kg/m <sup>3</sup>         | slug/ft <sup>3</sup>  |
| $\sigma$     | cone truncation radius                                                          | m                         | ft                    |
| $\tau$       | shear stress                                                                    | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\tau_{av}$  | average shear stress                                                            | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\tau_{yB}$  | Bingham yield stress                                                            | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\tau_{yHB}$ | Herschel-Bulkley yield stress                                                   | N/m <sup>2</sup> (Pa)     | lbf/ft <sup>2</sup>   |
| $\Omega$     | rotational speed                                                                | rad/s                     | rad/s                 |

## ***Subscripts***

|             |                                                             |
|-------------|-------------------------------------------------------------|
| <i>a, b</i> | refer to different bobs in coaxial cylinder geometries      |
| <i>A, B</i> | refer to different cylinders in coaxial cylinder geometries |
| <i>c</i>    | refers to corrected value                                   |
| <i>lim</i>  | refers to limiting value                                    |
| <i>min</i>  | refers to minimum value                                     |
| <i>max</i>  | refers to maximum value                                     |
| <i>N</i>    | refers to Newtonian fluid                                   |
| <i>nN</i>   | refers to non-Newtonian fluid                               |
| <i>nom</i>  | refers to nominal value                                     |
| <i>r</i>    | refers to rough pipe or bob                                 |
| <i>rim</i>  | refers to value at the rim of cone or plate                 |
| <i>s</i>    | refers to smooth pipe or bob                                |
| <i>tav</i>  | refers to torque-averaged value                             |
| <i>turb</i> | refers to turbulent flow                                    |
| <i>w</i>    | refers to value at pipe wall                                |
| <i>1, 2</i> | refers to values of quantities varied in two experiments    |

## 2. PURPOSE AND SCOPE

This Data Item is one of a group on the flow properties of non-Newtonian fluids. ESDU 97034<sup>31</sup> provides an introductory guide to the classification and characteristics of non-Newtonian fluids. ESDU 91025<sup>38</sup> outlines the various methods available for estimating frictional pressure losses for fully-developed isothermal flow of non-Newtonian fluids in straight pipes of uniform circular cross-section. ESDU 04005<sup>4</sup> contains a worked example that illustrates the use of a tube viscometer to measure viscometric data and develop the flow curve.

This Item is concerned with obtaining the relevant viscometric data required for pressure drop prediction using ESDU 91025. The methods are applicable to fluids that can be considered pseudohomogeneous (non-settling) under conditions of steady shear (see ESDU 97034<sup>31</sup> for clarification).

The overall pressure loss,  $\Delta p_T$ , in a length of pipework is in general given by

$$\Delta p_T = \Delta p + \Delta p_F + \Delta p_s, \quad (2.1)$$

where  $\Delta p$  is the frictional pressure loss from flow through the straight sections of the pipework,  $\Delta p_F$  is the pressure loss from flow through various types of fittings (for example, valves, bends, contractions, expansions, etc) and  $\Delta p_s$  is the pressure loss or gain resulting from elevation changes and is negative for a reduction in static head and positive for an increase in static head. The relative magnitudes of the three pressure loss terms in Equation (2.1) is dependent on the pipeline length. For short pipelines,  $\Delta p$  and  $\Delta p_F$  may be insignificant compared with  $\Delta p_s$  whereas, for very long pipelines,  $\Delta p$  is usually the main contributor.

Table 2.1 summarises various methods used for estimating  $\Delta p$ , together with the rheological data they require. This Item is relevant only to those that utilise viscometric data. These data usually comprise a flow curve for the fluid in the form of a plot of shear stress versus shear rate or alternatively a viscosity curve in the form of a plot of viscosity (the ratio of shear stress to shear rate) versus shear rate.

A primary purpose of this Data Item is to guide the user in the selection of the most appropriate viscometer for determining the flow curve for the sample under test and the pipeline application in question. The layout is as follows.

- Section 3 summarises the classification and behaviour of homogeneous and pseudohomogeneous non-Newtonian fluids.
- Section 4 describes the various flow curve models used in pipeline design.
- Section 5 states criteria used in viscometric testing to obtain rheological data for pipeline design.
- Section 6 describes five types of viscometric geometries used for flow curve determination of non-Newtonian fluids; they are tube, coaxial cylinder, rotating disc, cone-and-plate and parallel plate. With these geometries, several sources of error can arise when making flow curve measurements. To obtain the *corrected* flow curve, a step-by-step procedure is given for each geometry.
- Section 7 discusses the various material and equipment considerations which must be taken into account when carrying out viscometer selection.
- Section 8 deals with the flow curve data generation and interpretation.

TABLE 2.1 Summary of Methods for Estimating  $\Delta p$

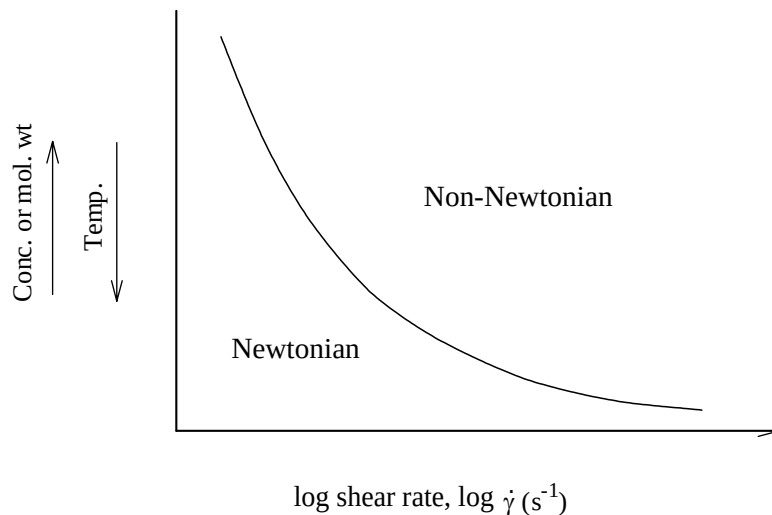
| Pipe flow regime | Pipe wall condition | Data Source                                         | Flow curve                 | Procedure in ESDU 91025 | Rheological data required    |
|------------------|---------------------|-----------------------------------------------------|----------------------------|-------------------------|------------------------------|
| Laminar          | Smooth and rough    | Viscometric data                                    | No model fit               | Appendix C2             | $\tau$ versus $\dot{\gamma}$ |
|                  |                     |                                                     | Power law model fit        | Section 5.1.1           | $K, n$                       |
|                  |                     |                                                     | Bingham plastic model fit  | Section 5.1.2           | $\tau_{yB}, \mu_B$           |
|                  |                     |                                                     | Herschel-Bulkley model fit | Section 5.1.3           | $\tau_{yHB}, K, n$           |
|                  |                     | Direct scale-up from small pipe laminar flow data   | No model fit               | Section 5.2             | $\tau_w$ versus $8V/D$       |
| Turbulent        | Smooth              | Viscometric data                                    | No model fit               | Appendix C3             | $\tau$ versus $\gamma$       |
|                  |                     |                                                     | Power law model fit        | Section 6.1.1           | $K, n$                       |
|                  |                     |                                                     | Bingham plastic model fit  | Appendix C4             | $\tau_{yB}, \mu_B$           |
|                  |                     |                                                     | Herschel-Bulkley model fit | Appendix C5             | $\tau_{yHB}, K, n$           |
|                  | Rough               | Viscometric data                                    | No model fit               | Appendix C6             | $\tau$ versus $\gamma$       |
|                  |                     |                                                     | Power law model fit        | Section 7.2             | $K, n$                       |
|                  |                     |                                                     | Bingham plastic model fit  | Section 7.2             | $\tau_{yB}, \mu_B$           |
|                  |                     |                                                     | Herschel-Bulkley model fit | Appendix C7             | $\tau_{yHB}, K, n$           |
|                  | Smooth and rough    | Direct scale-up from small pipe turbulent flow data | Bowen method               | Section 6.2             | $\tau_w, V, D$               |

## 3. NON-NEWTONIAN FLUIDS

### 3.1 Definition

Fluids, in general, may exhibit both Newtonian and non-Newtonian behaviour under different conditions. Newtonian behaviour<sup>37</sup> is observed when there is a linear relationship between the shear stress and the shear rate with a zero intercept and a slope of constant viscosity. Any deviation from this behaviour is said to be non-Newtonian.

For every fluid, there is a characteristic shear rate below which it exhibits Newtonian behaviour. In a homologous series of fluids (such as suspensions of increasing solids content, polymer solutions of increasing concentration or polymer melts of increasing molecular weight), the characteristic shear rate decreases with increasing concentration or molecular weight or with decreasing temperature. Sketch 3.1 shows that for the high concentration or high molecular weight material, the characteristic shear rate is so low that under normal conditions, only non-Newtonian behaviour would be apparent.



**Sketch 3.1 Condition for onset of non-Newtonian behaviour**

### 3.2 Examples

It is difficult to predict whether a particular fluid will exhibit non-Newtonian behaviour under particular conditions and a rheological test is the only sure method of ascertaining the type of behaviour. However, solid/liquid suspensions, high molecular weight polymer solutions and many multiphase fluids often show non-Newtonian characteristics. Table 3.1 lists some examples of fluids that may exhibit non-Newtonian behaviour.

**TABLE 3.1 Some Examples of Fluids Exhibiting Non-Newtonian Behaviour**

|                       |                   |
|-----------------------|-------------------|
| Adhesives             | Peat slurries     |
| Biological fluids     | Plastic melts     |
| Cement slurries       | Polymer solutions |
| Chalk slurries        | Printing inks     |
| Chocolate melts       | Quicksand         |
| Coal slurries         | Rock slurries     |
| Detergent slurries    | Rubber solutions  |
| Food sauces           | Sand slurries     |
| Greases               | Sewage sludges    |
| Handcreams            | Shampoo           |
| Margarine             | Soap slurries     |
| Mayonnaise            | Starch solutions  |
| Metal oxide slurries  | Tomato paste      |
| Oilwell drilling muds | Toothpaste        |
| Paints                | Wet beach sand    |
| Paper pulp            |                   |

### 3.3 Classification

In this Data Item, fluids are classified in two ways:

- by consideration of the effect of phase condition on the flow behaviour (see Section 3.3.1)
- by rheological type (see Section 3.3.2).

#### 3.3.1 Classification by phase condition

Suspensions may be either settling or non-settling. For instance, coarse-particle, granular and flocculated suspensions tend to settle whereas stabilised fine-particle suspensions have a low settling tendency. However, the distinction between non-settling and settling suspensions is not clear as a judgement has to be made about the period over which settling is deemed to be important. Guidelines on how this distinction is made with respect to pipeline design are given in Appendix A.

If the solid particles, undissolved polymer molecules or droplets of a denser, immiscible liquid in a suspending liquid medium settle sufficiently slowly under gravity or if the droplets of a lighter, immiscible liquid in a suspending liquid medium rise sufficiently slowly for them not to cause creaming, the suspension (or emulsion) can be characterised viscometrically. These 'non-settling' suspensions are considered to be pseudohomogeneous (that is, the particles, polymer or droplets are uniformly distributed throughout the suspending medium). Hence, the usual viscometric methods for measuring the flow curve of homogeneous fluids can be used for these suspensions, although it is necessary to be aware of the nature of the material when making the measurement as discussed in Section 7<sup>39</sup>.

If, however, the solid particles, undissolved polymer molecules or droplets in a suspending liquid medium undergo a rapid separation of the phases, the suspension can no longer be treated as pseudohomogeneous (that is, the suspension is heterogeneous). With these suspensions, flow curve measurements using rotational or tube viscometry are difficult and often meaningless. It may be possible instead to use the Metzner-Otto method for the determination of the flow curve. A description of this method is given in Appendix B.

This Item considers only homogeneous liquids and non-settling (pseudohomogeneous) suspensions.

## 3.3.2 Classification by rheological type

Homogeneous and pseudohomogeneous fluids may be further classified into various rheological types on the basis of their behaviour when subjected to shear. For these fluids, there are three main types of rheological behaviour: purely viscous, thixotropic and viscoelastic. Under steady shear, all fluids exhibit variable viscosity with shear rate which is described by the flow curve. Thixotropic behaviour is characterised, on step increase in shear rate or shear stress, by viscosity (of a well-rested material) decreasing with the time of shear. Conversely, viscoelastic behaviour is characterised by viscosity increasing with the time of shear. If this shear is held for a sufficient time, that is, minutes/hours for thixotropic materials or seconds/minutes for viscoelastic materials, the viscosity will attain a steady state value. Replotting this viscosity as a function of shear rate in the form of a shear stress versus shear rate plot will yield the equilibrium flow curve for thixotropic materials or the steady shear flow curve for viscoelastic materials.

Thixotropic behaviour is generally associated with particulate systems (that is, solid/liquid suspensions, colloids, slurries, pastes and emulsions) and viscoelastic behaviour with polymeric systems. However, both systems can, in fact, exhibit thixotropic or viscoelastic behaviour. Moreover, many fluids can show a combination of thixotropic and viscoelastic behaviour thus making time-dependent behaviour even more difficult to characterise.

Table 3.2 indicates for the three classes of fluid their rheological behaviour under the two different shear conditions relevant to pipeflow.

**TABLE 3.2 Types of Rheological Behaviour**

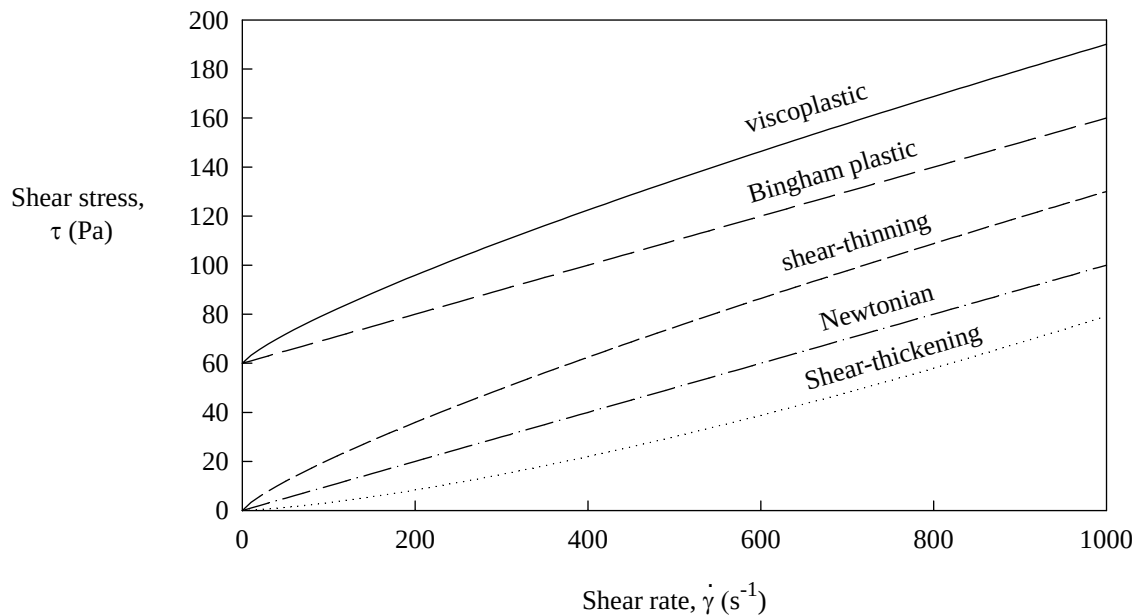
| Type of rheological behaviour | Step increase in shear<br>(Required for start up in pipeflow) | Steady-state shear<br>(Required for steady pipeflow) |
|-------------------------------|---------------------------------------------------------------|------------------------------------------------------|
| Purely viscous                | Instantaneous response.<br>Viscosity follows the flow curve   | Flow curve                                           |
| Thixotropic                   | Viscosity decreases with time                                 | Equilibrium flow curve                               |
| Viscoelastic                  | Viscosity increases with time                                 | Steady shear flow curve                              |

As this Item is concerned only with isothermal, fully-developed flow through straight pipes of circular cross-section, the relevant rheological property is the flow curve (see Section 4) obtained experimentally under steady-state shear conditions.

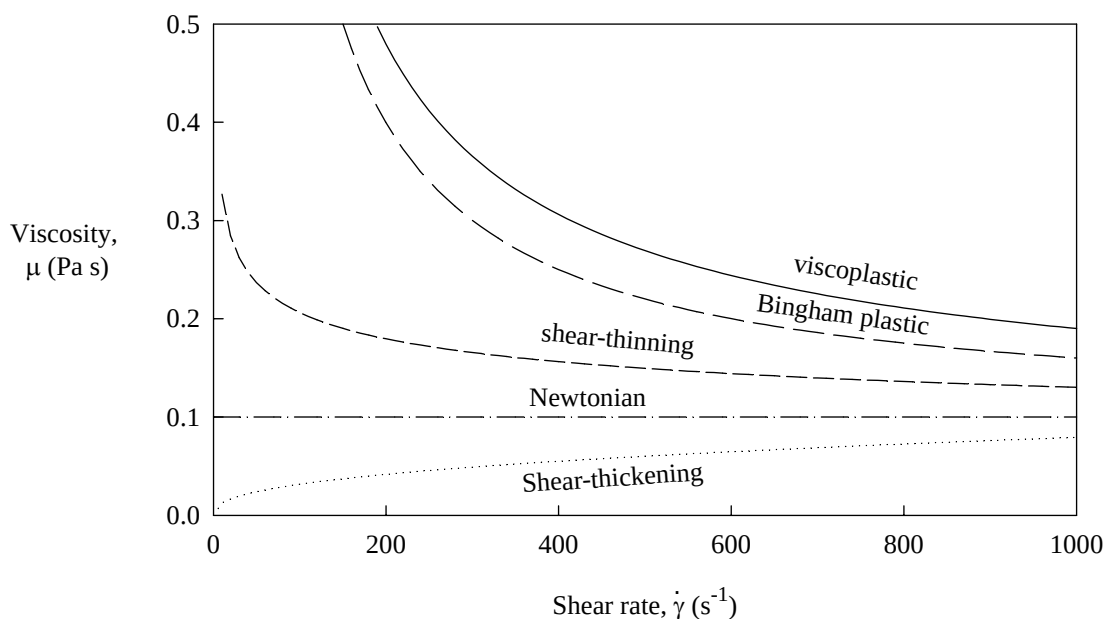
## 4. THE FLOW CURVE

Viscosity is just one of several rheological properties that can be used for material characterisation and engineering application purposes. It is defined as the ratio of shear stress to shear rate and is usually derived from the flow curve which is simply a plot of shear stress versus shear rate. Plotting this ratio as a function of shear rate will result in a viscosity curve. Instruments which measure viscosity are referred to as *viscometers* whereas those measuring other rheological properties in addition to viscosity are known as *rheometers*. These instruments can be operated either in the *controlled rate* or *controlled stress* mode to give a flow curve for the fluid under test.

The main classes of flow curve exhibited by fluids under steady-state shear are summarised in Table 4.1 and shown in Sketch 4.1, together with their corresponding viscosity curves in Sketch 4.2.



**Sketch 4.1 Idealised flow curves of time-independent fluids**



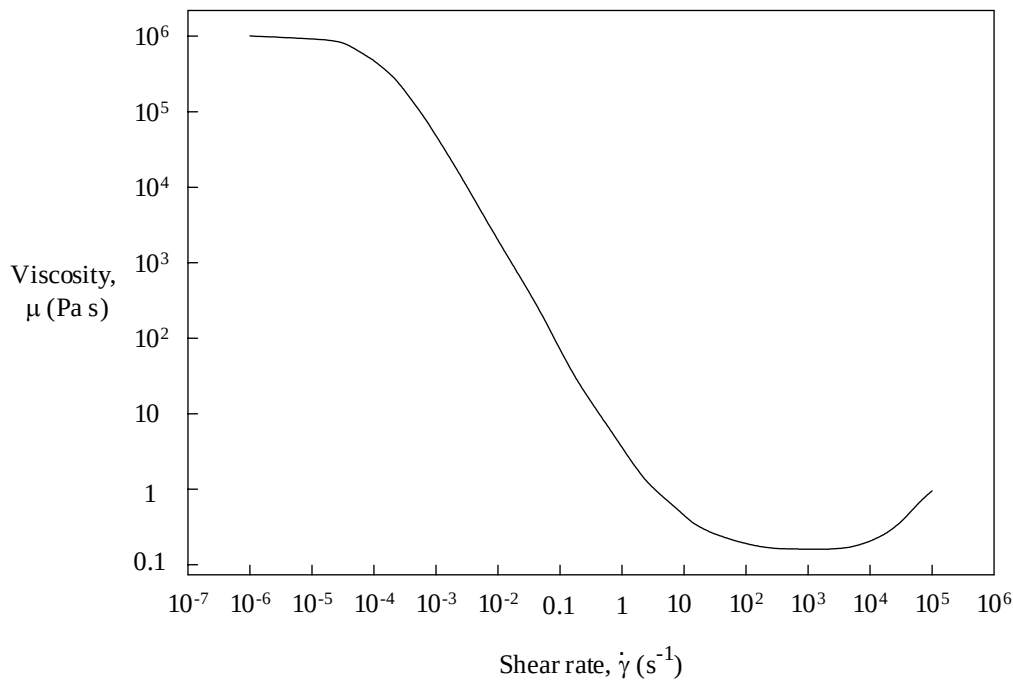
**Sketch 4.2 Idealised viscosity curves of time-independent fluids**

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**TABLE 4.1 Main Classes of Idealised Flow Curves**

| <i>Class description</i>                    | <i>Flow Curve</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Newtonian                                   | <p>The flow curve is a straight line with a zero intercept and constant slope, <math>\mu_N</math>, the viscosity. This can be written as</p> $\tau = \mu_N \dot{\gamma}. \quad (4.1)$ <p>The model is the simplest of all flow curve models.</p>                                                                                                                                                                                                                                                                                                                  |
| Shear-thinning                              | <p>The flow curve, which passes through the origin, is usually described in terms of a power law model</p> $\tau = K \dot{\gamma}^n \quad (4.2)$ <p>where <math>K</math> is the consistency coefficient (in units of <math>\text{Pa s}^n</math>) and <math>n</math>, the power law exponent, is less than 1. Here, the viscosity decreases with increasing shear rate. (This behaviour is sometimes incorrectly termed thixotropy because the equilibrium flow curve is often shear-thinning. Unlike shear-thinning behaviour, thixotropy is time-dependent.)</p> |
| Shear-thickening                            | <p>The flow curve, which passes through the origin, is usually described by the power law model given by Equation (4.2), with <math>n</math> being greater than unity. Here, the viscosity increases with increasing shear rate.</p>                                                                                                                                                                                                                                                                                                                              |
| Bingham Plastic                             | <p>These are fluids for which a finite shear stress is required to initiate flow and for which there is a linear relationship between the shear stress in excess of the Bingham yield stress, <math>\tau_{yB}</math>, and the resulting shear rate. The flow curve is described by</p> $\tau = \tau_{yB} + \mu_B \dot{\gamma} \quad (4.3)$ <p>where <math>\mu_B</math> is the Bingham plastic viscosity.</p>                                                                                                                                                      |
| Viscoplastic or Generalised Bingham Plastic | <p>These are fluids for which a finite shear stress is required to initiate flow and for which the rate of increase in shear stress, in excess of the yield stress, <math>\tau_{yHB}</math>, decreases with increasing shear rate. The flow curve may be described in terms of a three-parameter yield/power law model known as the Herschel-Bulkley model, given by</p> $\tau = \tau_{yHB} + K \dot{\gamma}^n \quad (4.4)$ <p>where <math>\tau_{yHB}</math> is the Herschel-Bulkley yield stress and <math>n &lt; 1</math>.</p>                                  |

Further discussion on these main classes of the flow curve can be found in ESDU 97034<sup>31</sup>. These are idealised representations as most fluids exhibit more than one flow curve classification over the practical shear rate range of  $10^{-6}$  to  $10^6 \text{ s}^{-1}$ . For such fluids, the general form of the flow curve is usually that shown in Sketch 4.3. This curve indicates that in the limit of very low shear rates the viscosity is constant while in the limit of high shear rates the viscosity is again constant, but at a lower level. Between these two extremes, often known as the *first* and *second Newtonian regions*, the viscosity decreases as a function of shear rate.



**Sketch 4.3 Typical viscosity curve**

It follows that a single point viscosity measurement, which is sufficient for Newtonian fluids, is inadequate to describe the flow behaviour of non-Newtonian fluids. Here, the relevant shear rate (or shear stress) range in the pipeline application must be assessed and used in determining the measurement conditions for the viscometer/rheometer (see Section 5).

## 5. FLOW CURVE MEASUREMENT USING VISCOMETRIC TESTS

### 5.1 Introduction

The following criteria are used in viscometric testing to obtain rheological data for pipeline design (see Table 2.1).

- ***Is the viscometric test suitable for the fluid under test?***

Viscometric methods for measuring the flow properties of homogeneous liquids can be used for non-settling (pseudohomogeneous) suspensions provided that the nature of the suspension is noted before a decision is made on the method to be used for measurement. Refer to Section 7 for guidance on selecting the most appropriate viscometric method(s) for the fluid under test. These methods are not suitable for settling (heterogeneous) suspensions. For these suspensions, it may be possible to use the Metzner-Otto method as described in Appendix B. Guidelines on how the distinction between non-settling and settling suspensions may be made with respect to pipeline design are given in Appendix A.

- ***Check the viscometric test is conducted at the same temperature (and possibly pressure) as that for the pipeline.***
- ***Is the viscometric test conducted at the appropriate shear rate/shear stress window over which the flow curve is required?***

The specification of the shear rate/shear stress window for the viscometric test is dependent on the type of pipeline design problem to be solved. Essentially, there are four design problems, namely:

- unknown  $\Delta p$  with  $Q$ ,  $D$ ,  $L$  and fluid properties specified,
- unknown  $Q$  with  $\Delta p$ ,  $D$ ,  $L$  and fluid properties specified,
- unknown  $D$  with  $\Delta p$ ,  $Q$ ,  $L$  and fluid properties specified,
- most economical  $D$  with  $Q$  and fluid properties specified, and  $\Delta p$  and  $L$  to be selected.

Here  $\Delta p$  is the frictional pressure loss from flow through straight sections of the pipework,  $Q$  is the fluid volumetric flowrate,  $D$  is the pipeline diameter and  $L$  is the pipeline length.

In a pipe, both the shear stress and shear rate values range from zero at the pipe axis to a maximum value at the pipe wall. The wall shear stress is given by<sup>39</sup>

$$\tau_w = \frac{D\Delta p}{4L}. \quad (5.1)$$

whereas the nominal wall shear rate is given by<sup>39</sup>

$$\dot{\gamma}_{nom} = \frac{8V}{D} = \frac{32Q}{\pi D^3}, \quad (5.2)$$

which is actually the wall shear rate for a Newtonian fluid in laminar flow. The true wall shear rate for a

non-Newtonian fluid is given by the Rabinowitsch-Mooney equation

$$\dot{\gamma}_w = \frac{8V}{D} \left( \frac{1 + 3n'}{4n'} \right) \quad (5.3)$$

where

$$n' = \frac{d[\ln \tau_w]}{d\left[\ln \frac{8V}{D}\right]} \quad (5.4)$$

This wall shear rate can therefore be up to about twice the nominal wall shear rate for a highly shear-thinning fluid (e.g. for  $n' \equiv 0.3$ ). The shear rate/shear stress windows described in Table 5.1 are recommended for the four design problems described above.

**TABLE 5.1**

| Design Problem | Shear rate/shear stress window                                                                                                                                                                |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (i)            | Use $V/D$ to $20V/D^*$ as the shear rate window for the viscometer.                                                                                                                           |
| (ii)           | Use $0.1\tau_w$ to $10\tau_w$ as the shear stress window for the viscometer.                                                                                                                  |
| (iii)          | Use either $V/D$ to $20V/D^\dagger$ as the shear rate window or $0.1\tau_w$ to $10\tau_w$ as the shear stress window that gives the widest range over which the flow curve is to be measured. |
| (iv)           | Use $V/D_{max}$ to $20V/D_{min}^\dagger$ as the shear rate window for the viscometer where $D_{max}$ and $D_{min}$ are the maximum and minimum pipe diameters under consideration.            |

\* The band  $V/D$  to  $20V/D$  is usually adequate although for fluids with  $n'$  values less than 0.14 it may be necessary to increase the shear rate to, say,  $50 V/D$ . This is assessed using Equation (5.3).

It should be noted that more than one viscometer may be needed to cover the required shear rate/shear stress range. Once the shear rate (or shear stress) window is estimated, a check on the capability of the viscometer to measure shear stress (or shear rate), and hence viscosity levels over the specified shear rate (or shear stress) window must be made. In some cases, the shear stress (or shear rate) and the viscosity may be too low or too high for the viscometer to measure. If this is found to be the case, a different viscometer must be selected for consideration.

- ***Is the viscometric test carried out under laminar conditions?***

Flow curve measurements are made for laminar flow conditions only. The data collected must be checked to ensure they correspond to laminar flow in the viscometer. This is achieved by calculating the laminar flow limit for the fluid under test, given in Section 6 for a range of viscometric geometries. Criteria for rejecting data subject to secondary or turbulent flow are given in Section 6. Alternatively, an experimental method using different sizes of the same viscometric geometry may be used.

- ***Is the viscometric test carried out under isothermal conditions?***

Viscometric tests should be designed to avoid significant temperature increases due to viscous heat dissipation. Viscous heating occurs when any fluid is sheared. However, serious experimental errors are generally found only with highly viscous fluids at high shear rates. The data collected must be checked to

ensure they are not affected by viscous heating. This is done by calculating the viscous heating limit following guidelines in Section 6. Alternatively, an experimental method using different sizes of viscometer of the same geometry may be used.

- ***Should the data be corrected for possible sources of experimental error?***

After ensuring the data are unaffected by secondary or turbulent flow and viscous heating, the data should be examined to see if they are affected by any of the main error sources such as end effect and wall-slip. Error correction procedures are outlined in Section 6. The likely extent of the error can be obtained by comparing the uncorrected and corrected flow curves.

- ***Check for sensitivity to changes in concentration, temperature, pressure.***

Always design for the worst case. For example, if designing a pipeline for a specific concentration, check what happens if the concentration changes.

## 5.2 Types of Viscometric Geometries

Commercial viscometers employ a wide range of geometries for flow curve measurement. The more common geometries include:

- (1) tube (see Section 6.1),
- (2) coaxial cylinder (see Section 6.2),
- (3) rotating disc (see Section 6.3),
- (4) cone-and-plate (see Section 6.4),
- (5) parallel plate (see Section 6.5).



The *type A* tube viscometer forces the fluid through a horizontal or vertical tube at a constant flow rate using a piston or ram as a pump and the resultant pressure drop is measured.

The *type B* tube viscometer often creates a near-constant pressure drop across the horizontal or vertical tube through a combination of applied compressed air pressure and fluid head, and the resultant volumetric flow rate is measured.

For vertical tubes, the pressure drop will change as the sample level in the reservoir falls. However, for wide diameter reservoirs, near-constant pressure drop can be assumed.

The following calculation procedure (in Section 6.1.1 and Section 6.1.2) illustrates the use of tube viscometers to measure viscometric data and develop the flow curve for a fluid in a specific state. This calculation procedure is illustrated by the worked example in ESDU 04005.

## 6.1.1 Calculation of shear stress and shear rate

Shear stress and shear rate are calculated at the wall of the tube. Hence, the wall shear stress is calculated from Equation (5.1) and the corresponding wall shear rate is calculated from Equation (5.3).

Prior to obtaining the flow curve the viscometer should be properly calibrated. Refer to Section 6.1.2.1 for guidance on dimensional and calibration errors. The procedure for obtaining the flow curve is given in Table 6.1.

**TABLE 6.1**

| Step  | Description                                                                                                                                                                                                                                                                                                                                                        |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (i)   | Measure $\Delta p$ as a function of $Q$ (or vice versa).                                                                                                                                                                                                                                                                                                           |
| (ii)  | Calculate $\tau_w$ values from Equation (5.1) and $8V/D$ values from Equation (5.2).                                                                                                                                                                                                                                                                               |
| (iii) | Plot $\tau_w$ versus $8V/D$ on logarithmic coordinates and determine the slope, $n'$ , at each experimental $8V/D$ value.                                                                                                                                                                                                                                          |
| (iv)  | Calculate $\dot{\gamma}_w$ values from Equation (5.3) and plot $\tau_w$ (calculated in step (ii)) against $\dot{\gamma}_w$ to give the flow curve. Note: the computer programs in ESDU 91025 require values of $8V/D$ as input data rather than the calculated values of $\dot{\gamma}_w$ . The power-law exponent $n$ is subsequently calculated by the programs. |

Note that this flow curve does not take into account any measurement errors which may arise when using tube viscometry.

The procedure for obtaining the *corrected* flow curve is given in Table 6.2.

**TABLE 6.2**

| Step | Description                                                                                                               | Section            |
|------|---------------------------------------------------------------------------------------------------------------------------|--------------------|
| (a)  | Carry out step (i) of Table 6.1.                                                                                          | —                  |
| (b)  | Reject $\Delta p$ versus $Q$ data affected by turbulent flow and/or viscous heating.                                      | 6.1.2.2<br>6.1.2.3 |
| (c)  | Correct $\Delta p$ data for end effect.                                                                                   | 6.1.2.4            |
| (d)  | Correct $Q$ data for wall slip.                                                                                           | 6.1.2.5            |
| (e)  | Using corrected $\Delta p$ versus $Q$ data, carry out steps (ii) to (iv) of Table 6.1 to obtain the corrected flow curve. | —                  |

## 6.1.2 Errors in tube viscometry

The main sources of error in tube viscometry are discussed in turn.

### 6.1.2.1 Dimensional and calibration errors

The tube diameter and length must be known accurately for the calculation of wall shear stress and wall shear rate. The accuracy for measuring diameter is dictated by the wall shear rate which is inversely proportional to  $D^3$ . For a 1% accuracy in shear rate,  $D$  has to be known to better than  $1/3$  %. When calculating the viscosity (that is, the ratio of wall shear stress to wall shear rate), even greater accuracy is required as shear rate is proportional to  $D^4$ : for a 1% accuracy in viscosity,  $D$  has to be known to better than  $1/4$  %.

In addition to checking the tube diameter, it is advisable to check the circularity and straightness of the tube. Use of non-circular tubes can lead to lower flowrates than obtained in circular tubes for a given pressure drop. The use of non-straight tubes can lead to either a reduction in flowrate for inelastic fluids or an increase for elastic fluids.

As the results are based on measurements of pressure and flowrate, it is advisable to calibrate the pressure and flowrate measuring devices periodically.

### 6.1.2.2 Turbulent flow effect

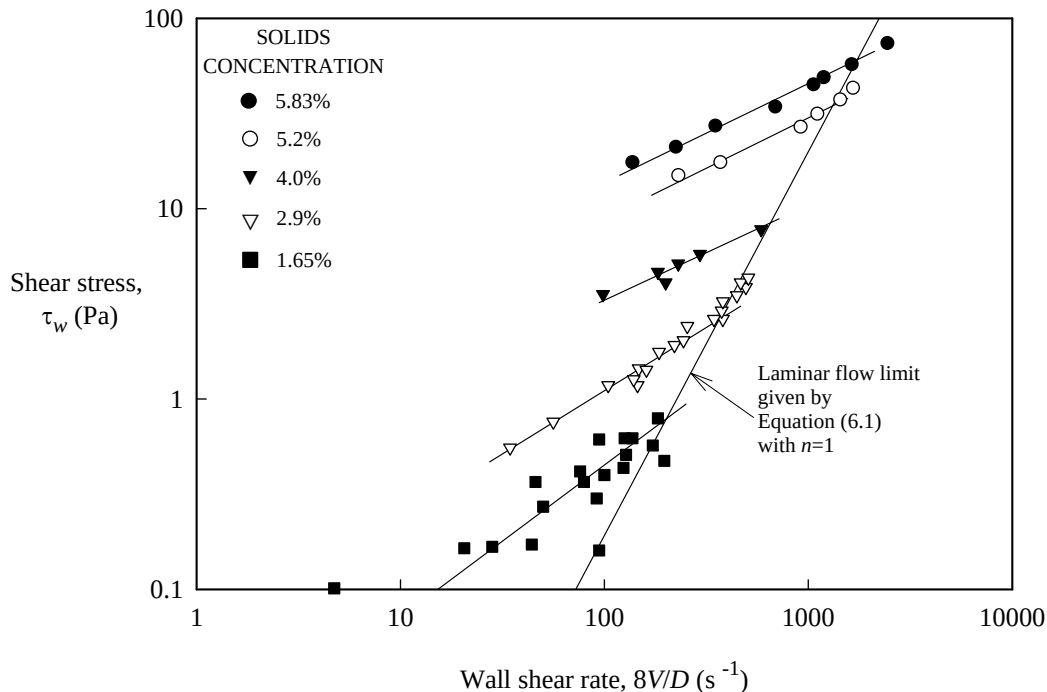
The expression for wall shear rate given by Equation (5.3) is valid only for laminar flow conditions and does not apply for turbulent flow. Hence any transitional or turbulent flow data collected during the course of an experiment must be rejected. This requires that the laminar limit for the sample under test should be calculated.

The criterion for the laminar flow limit of power law non-Newtonian fluids is given by<sup>5,35</sup>

$$\tau_w > \frac{\rho D^2}{8} \left[ \frac{n}{404} \frac{\left[ \frac{1+3n}{4n} \right]^2}{(n+2)^{\frac{n+2}{n+1}}} \right] \left[ \frac{8V}{D} \right]^2. \quad (6.1)$$

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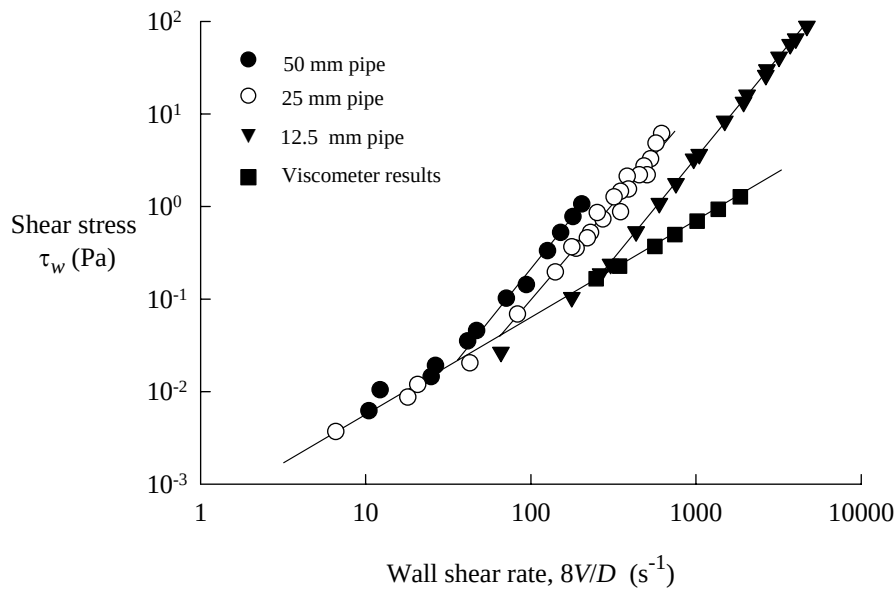
This equation can be plotted as a double logarithmic plot of  $\tau_w$  against  $8V/D$  to give the laminar limit line; data to the left of the line (which on a log-log plot has a slope of 2) correspond to laminar flow, while data lying on or to the right of the line must be rejected. Sketch 6.2 gives an example of data obtained as a function of solids concentration using a single tube size



**Sketch 6.2 Plots of  $\tau_w$  versus  $8V/D$  as a function of solids concentration with the laminar limit**

From Sketch 6.2, it can be seen that the slopes of the flow curves,  $n$ , vary with solids concentration. The laminar limit shown in Sketch 6.2 should be calculated for each flow curve using Equation (6.1) with the appropriate value of  $n$ . As Equation (6.1) is only weakly dependent on  $n$  is often convenient simply to take  $n = 1$  in Equation (6.1).

Alternatively, an experimental method can be used to assess turbulent flow effects. Here, measurements are made of pressure drop against flowrate for several different tube sizes. On a plot of  $\tau_w$  versus  $8V/D$ , assuming the end effect and wall slip effects have been corrected for, results affected by turbulent flow show up as deviations from the main curve. This occurs at lower shear rates for larger diameter tubes (see Sketch 6.3).



**Sketch 6.3 Onset of turbulent flow in pipes of different diameters**

These procedures are step (b) of Table 6.2, for the determination of the *corrected* flow curve.

### 6.1.2.3 Viscous heating effect

This is a problem found with very viscous fluids. The criterion for the viscous heating limit of power law non-Newtonian fluids is given by<sup>35</sup>

$$\tau_w < 2 \left[ \exp \left\{ \frac{0.1}{2n} \right\} - 1 \right] n \lambda \frac{\theta_o}{\beta} \left( \frac{1 + 3n}{4n} \right)^n \left[ \frac{4^{n+2}}{D^2} \right] \frac{1}{8V/D} \quad (6.2)$$

where  $\lambda$  is the thermal conductivity and  $\beta$  is the viscosity-temperature coefficient obtained from

$$\beta = \frac{\ln(K/K_o)}{1 - \theta/\theta_o} \quad (6.3)$$

in which  $K_o$  is the consistency coefficient at some reference temperature,  $\theta_o$ . This criterion is based on the maximum value of temperature not exceeding the reference temperature by more than 10% (that is,  $\theta/\theta_o \leq 1.1$ ). Equation (6.2) can be expressed as a plot of  $\ln \tau_w$  against  $\ln 8V/D$  to give the viscous heating limit line; data below the line, which has a slope of  $-1$  on a log-log plot, are free from the viscous heating effect.

Alternatively, an experimental method can be used to assess the viscous heating effect. Here, measurements are made of pressure drop against flowrate for several different tube sizes. On a plot of  $\tau_w$  versus  $8V/D$  (assuming the end effect and wall slip effect have been corrected for), the results affected by viscous heating will show up as deviations from the main curve.

These procedures are also step (b) of Table 6.2 for the determination of the *corrected* flow curve.

## 6.1.2.4 End effect

This error can be minimised by using large tube length-to-diameter ratios<sup>40</sup>. It arises in the form of additional pressure losses at the entrance to and exit from the tube. These losses result from

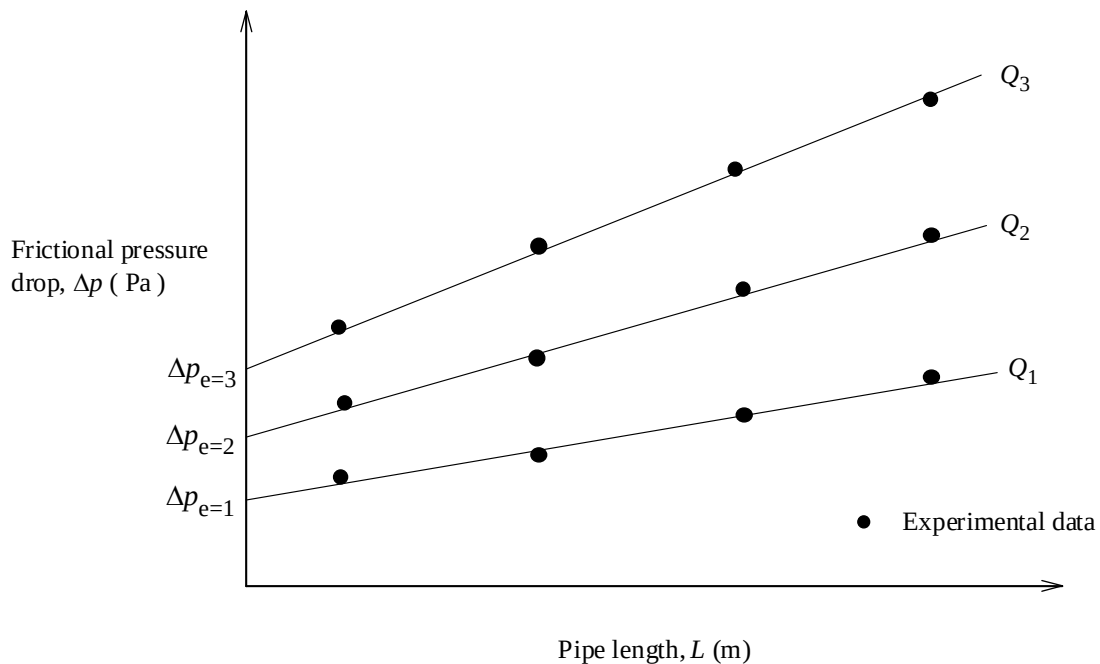
- (a) energy losses arising from viscous or elastic behaviour as the fluid converges at the entrance to the tube or diverges at the end of the tube,
- (b) losses in the kinetic energy of the fluid brought about by rearrangement of the fluid streamlines on entry and exit,
- (c) time-dependent properties of the fluid which result in an additional variation of wall shear stress along the tube entrance or exit length for steady flow, and
- (d) energy losses, by elastic deformation of viscoelastic fluids or by structural change of thixotropic fluids, not recovered during flow in the tube.

It is usual to treat the losses at the exit as being negligible compared to those at the entrance. Equations are available for estimating the entrance length required to obtain 98% of fully-developed flow of viscoplastic and Bingham plastic fluids<sup>40</sup>. However, it is preferable to correct for end effect using the procedure of Table 6.3

**TABLE 6.3**

| Step  | Description                                                                                                                                                                                        |
|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (i)   | Obtain $\Delta p$ versus $Q$ data for a number of tubes of the same diameter but of different lengths.                                                                                             |
| (ii)  | Plot $\Delta p$ against $L$ at constant $Q$ (see Sketch 6.4).                                                                                                                                      |
| (iii) | Draw a straight line through each set of constant $Q$ data. Do not include data at low values of $L$ that show curvature caused by insufficient entrance length required for fully-developed flow. |
| (iv)  | Extrapolate the straight line for each $Q$ to zero $L$ .                                                                                                                                           |
| (v)   | Obtain the end effect pressure loss, $\Delta p_e$ , from the intercept for each $Q$ .                                                                                                              |
| (vi)  | Calculate the corrected pressure loss using, for each value of $Q$ ,<br>$\Delta p_c = \Delta p - \Delta p_e \quad (6.4)$                                                                           |

This procedure is step (c) of Table 6.2, for the determination of the *corrected* flow curve.



**Sketch 6.4 Determination of end effect in tube viscometry**

## 6.1.2.5 Wall slip effect

Wall slip may occur when a multiphase material is sheared. This effect gives a resultant shear stress at a given shear rate lower than expected due to the formation of a thin layer of fluid (caused by the depletion of the dispersed phase at or near the shearing surface) having a viscosity lower than the bulk of the fluid. In tube viscometry, wall slip is present when curves of wall shear stress versus nominal wall shear rate obtained with different tube diameters do not superimpose after all other corrections to the data have been made. This effect can be quantified through a wall slip velocity,  $V_{slip}$ .

In the first instance,  $V_{slip}$  is assumed to be given by

$$V_{slip} = \frac{S\tau_w}{D} \quad (6.5)$$

and this assumption is validated when the plots of  $8V/D$  versus  $1/D$  for various constant values of  $\tau_w$  are straight lines. Otherwise,  $V_{slip}$  is given by<sup>12,32</sup>

$$V_{slip} = \frac{S\tau_w}{D^m} \quad (6.6)$$

where  $m$  takes a value such that plots of  $8V/D$  versus  $1/D^{m+1}$  for various constant values of  $\tau_w$  give straight lines; this condition is usually found by trial-and-error, assigning different values of  $m$ . It should be noted that the slip factor,  $S$ , may be a function of  $\tau_w$ .

To check whether wall slip is present, the procedure of Table 6.4 should be followed.